156. If you are given a quadratic equation, how do you determine which method to use to solve it?

157. In solving $\sqrt{2x-1}+2=x$, why is it a good idea to isolate the radical term? What if we don't do this and simply square each side? Describe what happens.

158. What is an extraneous solution to a radical equation?

Critical Thinking Exercises

Make Sense? *In Exercises 159–162, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

159. The model $P=-0.18n+2.1$ describes the number of pay phones, P, in millions, n years after 2000, so I have to solve a linear equation to determine the number of pay phones in 2006.

160. Although I can solve $3x+\frac{1}{5}=\frac{1}{4}$ by first subtracting $\frac{1}{5}$ from both sides, I find it easier to begin by multiplying both sides by 20, the least common denominator.

161. Because I want to solve $25x^2-169=0$ fairly quickly, I'll use the quadratic formula.

162. When checking a radical equation's proposed solution, I can substitute into the original equation or any equation that is part of the solution process.

In Exercises 163–166, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

163. The equation $(2x-3)^2=25$ is equivalent to $2x-3=5$.

164. Every quadratic equation has two distinct numbers in its solution set.

165. The equations $3y-1=11$ and $3y-7=5$ are equivalent.

166. The equation $ax^2+c=0$, $a\neq 0$, cannot be solved by the quadratic formula.

167. Find b such that $\frac{7x+4}{b}+13=x$ will have a solution set given by $\{-6\}$.

168. Write a quadratic equation in general form whose solution set is $\{-3, 5\}$.

169. Solve for C: $V=C-\frac{C-S}{L}N$.

170. Solve for t: $s=-16t^2+v_0t$.

Preview Exercises

Exercises 171–173 will help you prepare for the material covered in the next section.

171. Jane's salary exceeds Jim's by \$150 per week. If x represents Jim's weekly salary, write an algebraic expression that models Jane's weekly salary.

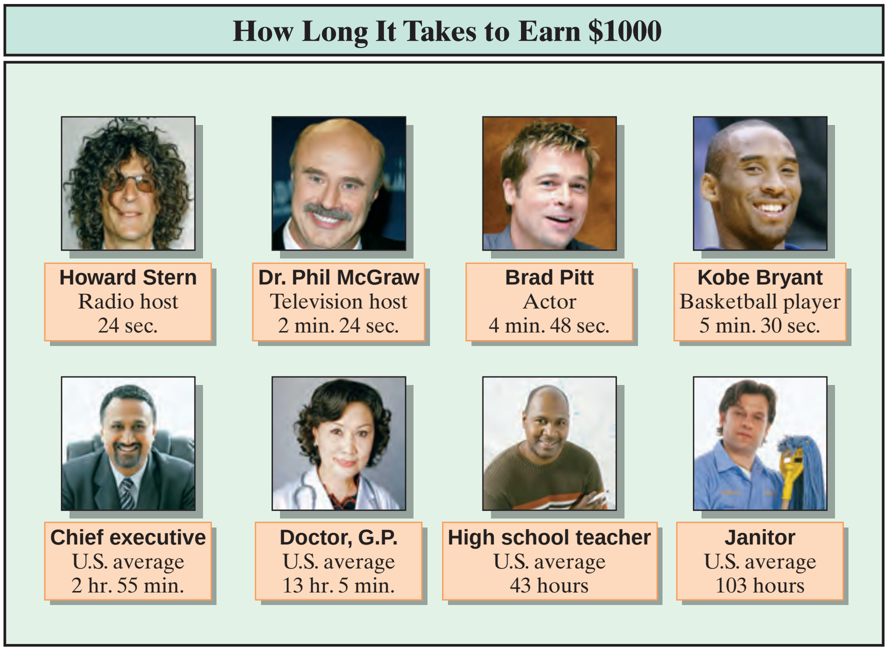
172. A long-distance telephone plan has a monthly fee of \$20 with a charge of \$0.05 per minute for all long-distance calls. Write an algebraic expression that models the plan's monthly cost for x minutes of long-distance calls.

173. If the width of a rectangle is represented by x and the length is represented by $x+200$, write a simplified algebraic expression that models the rectangle's perimeter.

Section P.8 Modeling with Equations

Objective

1. Use equations to solve problems.



Source: Time

In this section, you'll see examples and exercises focused on how much money Americans earn. These situations illustrate a step-by-step strategy for solving problems. As you become familiar with this strategy, you will learn to solve a wide variety of problems.

1 Use equations to solve problems.

Problem Solving with Equations

We have seen that a model is a mathematical representation of a real-world situation. In this section, we will be solving problems that are presented in English. This means that we must obtain models by translating from the ordinary language of English into the language of algebraic equations. To translate, however, we must understand the English prose and be familiar with the forms of algebraic language. Here are some general steps we will follow in solving word problems:

Study Tip

When solving word problems, particularly problems involving geometric figures, drawing a picture of the situation is often helpful. Label x on your drawing and, where appropriate, label other parts of the drawing in terms of x.

Strategy for Solving Word Problems

Step 1 Read the problem carefully. Attempt to state the problem in your own words and state what the problem is looking for. Let x (or any variable) represent one of the unknown quantities in the problem.

Step 2 If necessary, write expressions for any other unknown quantities in the problem in terms of x.

Step 3 Write an equation in x that models the verbal conditions of the problem.

Step 4 Solve the equation and answer the problem's question.

Step 5 Check the solution *in the original wording* of the problem, not in the equation obtained from the words.

EXAMPLE 1 Celebrity Earnings

Forbes magazine published a list of the highest paid TV celebrities between June 2006 and June 2007. The results are shown in **Figure P.14**.

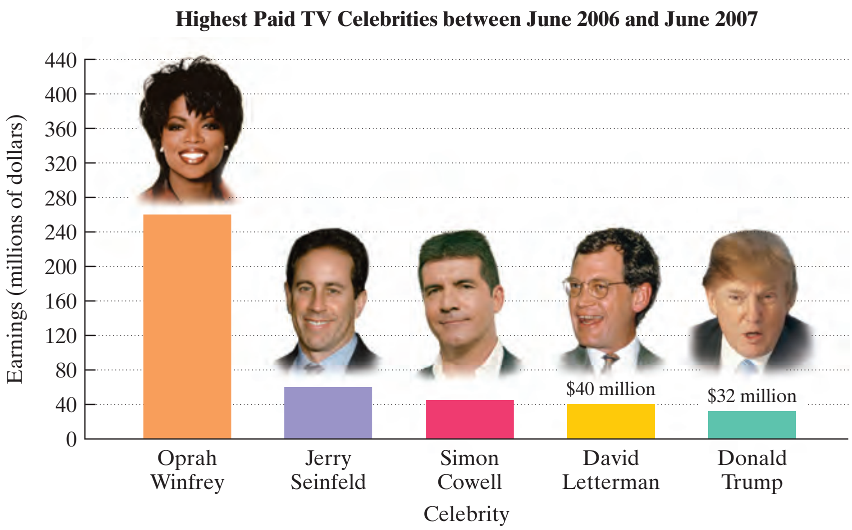


Figure P.14
Source: Forbes

The bar heights indicate that nobody came close to Oprah, who earned over four times more than any of the other TV stars. Although Seinfeld earned $15 million more than Cowell, Oprah's earnings exceeded Cowell's by $215 million. Combined, these three celebrities earned $365 million. How much did each of them earn?

Solution

Step 1 Let x represent one of the unknown quantities. We know something about Seinfeld's earnings and Oprah's earnings: Seinfeld earned $15 million more than Cowell, and Oprah's earnings exceeded Cowell's by $215 million. We will let

$$x = \text{Cowell's earnings (in millions of dollars).}$$

Step 2 Represent other unknown quantities in terms of *x*. Because Seinfeld earned \$15 million more than Cowell, let

$$x + 15 = \text{Seinfeld's earnings.}$$

Because Oprah's earnings exceeded Cowell's by \$215 million, let

$$x + 215 = \text{Oprah's earnings.}$$

Step 3 Write an equation in *x* that models the conditions. Combined, the three celebrities earned \$365 million.

Oprah's earnings	plus	Seinfeld's earnings	plus	Cowell's earnings	equal	\$365 million.
$(x + 215)$	$+$	$(x + 15)$	$+$	x	$=$	365

Step 4 Solve the equation and answer the question.

$$(x + 215) + (x + 15) + x = 365$$ This is the equation that models the problem's conditions.

$$3x + 230 = 365$$ Remove parentheses, regroup, and combine like terms.

$$3x = 135$$ Subtract 230 from both sides.

$$x = 45$$ Divide both sides by 3.

Thus,

$$\text{Cowell's earnings} = x = 45$$
$$\text{Seinfeld's earnings} = x + 15 = 45 + 15 = 60$$
$$\text{Oprah's earnings} = x + 215 = 45 + 215 = 260.$$

Between June 2006 and June 2007, Oprah earned \$260 million, Seinfeld earned \$60 million, and Cowell earned \$45 million.

Step 5 Check the proposed solution in the original wording of the problem. The problem states that combined, the three celebrities earned \$365 million. Using the earnings we determined in step 4, the sum is

$$\$45 \text{ million} + \$60 \text{ million} + \$260 \text{ million},$$

or \$365 million, which verifies the problem's conditions.

Study Tip

Modeling with the word *exceeds* can be a bit tricky. It's helpful to identify the smaller quantity. Then add to this quantity to represent the larger quantity. For example, suppose that Tim's height exceeds Tom's height by a inches. Tom is the shorter person. If Tom's height is represented by x, then Tim's height is represented by $x + a$.

Check Point 1 According to the U.S. Department of Education (2007 data), there is a gap between teaching salaries for men and women at private colleges and universities. The average salary for men exceeds the average salary for women by \$14,037. Combined, their average salaries are \$130,015. Determine the average teaching salaries at private colleges for women and for men.

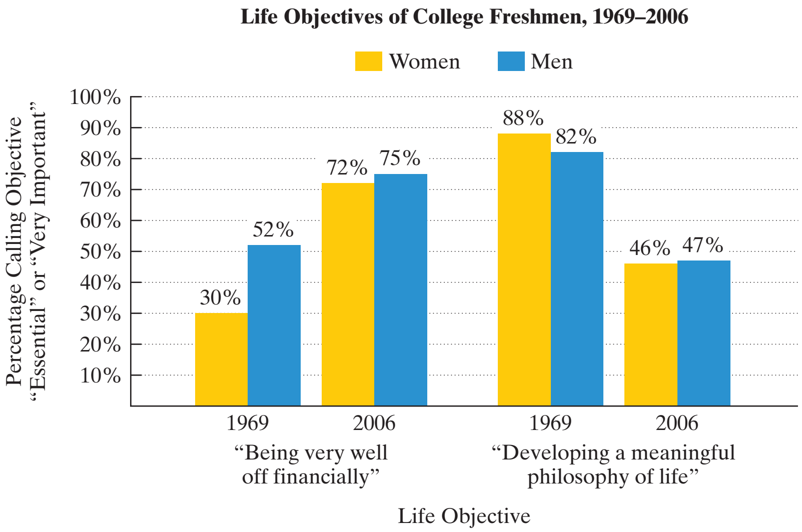


Figure P.15

Source: John Macionis, *Sociology*, Twelfth Edition, Prentice Hall, 2008

EXAMPLE 2 Modeling Attitudes of College Freshmen

Researchers have surveyed college freshmen every year since 1969. **Figure P.15** shows that attitudes about some life goals have changed dramatically. **Figure P.15** shows that the freshmen class of 2006 was more interested in making money than the freshmen of 1969 had been. In 1969, 52% of first-year college men considered "being very well off financially" essential or very important. For the period from 1969 through 2006, this percentage increased by approximately 0.6 each year. If this trend continues, by which year will all male freshmen consider "being very well off financially" essential or very important?

Solution

Step 1 Let x represent one of the unknown quantities. We are interested in the year when all male freshmen, or 100% of the men, will consider this life objective essential or very important. Let

x = the number of years after 1969 when all male freshmen will consider "being very well off financially" essential or very important.

Step 2 Represent other unknown quantities in terms of x. There are no other unknown quantities to find, so we can skip this step.

Step 3 Write an equation in x that models the conditions.

The 1969 percentage	increased by	0.6 each year for x years	equals	100% of the male freshmen.
52	+	$0.6x$	=	100

Step 4 Solve the equation and answer the question.

$$52 + 0.6x = 100$$ This is the equation that models the problem's conditions.

$$52 - 52 + 0.6x = 100 - 52$$ Subtract 52 from both sides.

$$0.6x = 48$$ Simplify.

$$\frac{0.6x}{0.6} = \frac{48}{0.6}$$ Divide both sides by 0.6.

$$x = 80$$ Simplify.

Using current trends, by 80 years after 1969, or in 2049, all male freshmen will consider "being very well off financially" essential or very important. (Do you agree with this projection that extends so far into the future? Are there unexpected events that might cause model breakdown to occur?)

Step 5 Check the proposed solution in the original wording of the problem. The problem states that all men (100%, represented by 100 using the model) will consider the objective essential or very important. Does this occur if we increase the 1969 percentage, 52%, by 0.6 each year for 80 years, our proposed solution?

$$52 + 0.6(80) = 52 + 48 = 100$$

This verifies that using trends shown in **Figure P.15**, all first-year college men will consider the objective essential or very important 80 years after 1969.

Check Point 2 **Figure P.15** shows that the freshmen class of 2006 was less interested in developing a philosophy of life than the freshmen of 1969 had been. In 1969, 88% of the women considered this objective essential or very important. Since then, this percentage has decreased by approximately 1.1 each year. If this trend continues, by which year will only 33% of female freshmen consider "developing a meaningful philosophy of life" essential or very important?

EXAMPLE 3 A Price Reduction on a Digital Camera

Your local computer store is having a terrific sale on digital cameras. After a 40% price reduction, you purchase a digital camera for \$276. What was the camera's price before the reduction?

Solution

Step 1 Let x represent one of the unknown quantities. We will let

x = the original price of the digital camera prior to the reduction.

Step 2 Represent other unknown quantities in terms of x. There are no other unknown quantities to find, so we can skip this step.

Step 3 Write an equation in x that models the conditions. The camera's original price minus the 40% reduction is the reduced price, \$276.

Study Tip

Observe that the original price, x, reduced by 40% is $x - 0.4x$ and *not* $x - 0.4$.

Original price | minus | the reduction (40% of the original price) | is | the reduced price, \$276.

$$x \quad - \quad 0.4x \quad = \quad 276$$

Step 4 Solve the equation and answer the question.

$$x - 0.4x = 276$$ This is the equation that models the problem's conditions.

$$0.6x = 276$$ Combine like terms: $x - 0.4x = 1x - 0.4x = 0.6x$.

$$\frac{0.6x}{0.6} = \frac{276}{0.6}$$ Divide both sides by 0.6.

$$x = 460$$ Simplify: $0.6\overline{)276.0}$ = 460.

The digital camera's price before the reduction was \$460.

Step 5 Check the proposed solution in the original wording of the problem. The price before the reduction, \$460, minus the 40% reduction should equal the reduced price given in the original wording, \$276:

$$460 - 40\% \text{ of } 460 = 460 - 0.4(460) = 460 - 184 = 276.$$

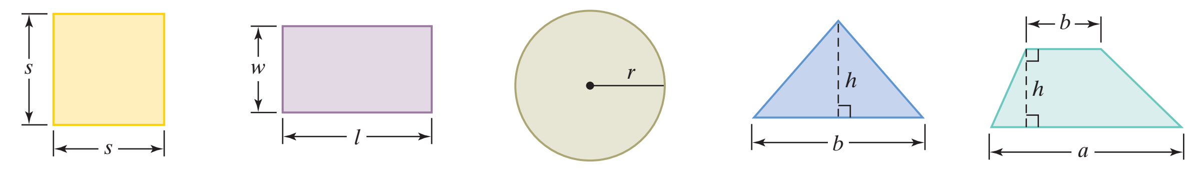
This verifies that the digital camera's price before the reduction was \$460.

Check Point 3 After a 30% price reduction, you purchase a new computer for \$840. What was the computer's price before the reduction?

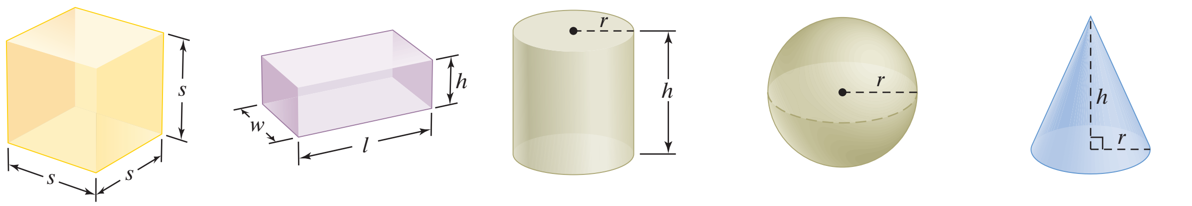
Solving geometry problems usually requires a knowledge of basic geometric ideas and formulas. Formulas for area, perimeter, and volume are given in **Table P.6**.

Table P.6 Common Formulas for Area, Perimeter, and Volume

Square	Rectangle	Circle	Triangle	Trapezoid
$A = s^2$ $P = 4s$	$A = lw$ $P = 2l + 2w$	$A = \pi r^2$ $C = 2\pi r$	$A = \frac{1}{2}bh$	$A = \frac{1}{2}h(a + b)$



Cube	Rectangular Solid	Circular Cylinder	Sphere	Cone
$V = s^3$	$V = lwh$	$V = \pi r^2 h$	$V = \frac{4}{3}\pi r^3$	$V = \frac{1}{3}\pi r^2 h$



We will be using the formula for the perimeter of a rectangle, $P = 2l + 2w$, in our next example. The formula states that a rectangle's perimeter is the sum of twice its length and twice its width.

EXAMPLE 4 Finding the Dimensions of an American Football Field

The length of an American football field is 200 feet more than the width. If the perimeter of the field is 1040 feet, what are its dimensions?

Solution

Step 1 Let x represent one of the unknown quantities. We know something about the length; the length is 200 feet more than the width. We will let

$$x = \text{the width.}$$

Step 2 Represent other unknown quantities in terms of x. Because the length is 200 feet more than the width, we add 200 to the width to represent the length. Thus,

$$x + 200 = \text{the length.}$$

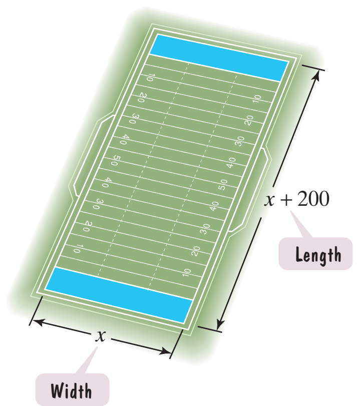


Figure P.16 An American football field

Figure P.16 illustrates an American football field and its dimensions.

Step 3 Write an equation in x that models the conditions. Because the perimeter of the field is 1040 feet,

Twice the length	plus	twice the width	is	the perimeter.
$2(x + 200)$	$+$	$2x$	$=$	$1040.$

Step 4 Solve the equation and answer the question.

$$2(x + 200) + 2x = 1040$$ This is the equation that models the problem's conditions.

$$2x + 400 + 2x = 1040$$ Apply the distributive property.

$$4x + 400 = 1040$$ Combine like terms: $2x + 2x = 4x$.

$$4x = 640$$ Subtract 400 from both sides.

$$x = 160$$ Divide both sides by 4.

Thus,

$$\text{width} = x = 160.$$

$$\text{length} = x + 200 = 160 + 200 = 360.$$

The dimensions of an American football field are 160 feet by 360 feet. (The 360-foot length is usually described as 120 yards.)

Step 5 Check the proposed solution in the original wording of the problem. The perimeter of the football field using the dimensions that we found is

$$2(360 \text{ feet}) + 2(160 \text{ feet}) = 720 \text{ feet} + 320 \text{ feet} = 1040 \text{ feet}.$$

Because the problem's wording tells us that the perimeter is 1040 feet, our dimensions are correct.

Check Point 4 The length of a rectangular basketball court is 44 feet more than the width. If the perimeter of the basketball court is 288 feet, what are its dimensions?

We will use the formula for the area of a rectangle, $A = lw$, in our next example. The formula states that a rectangle's area is the product of its length and its width.

EXAMPLE 5 Solving a Problem Involving Landscape Design

A rectangular garden measures 80 feet by 60 feet. A large path of uniform width is to be added along both shorter sides and one longer side of the garden. The landscape designer doing the work wants to double the garden's area with the addition of this path. How wide should the path be?

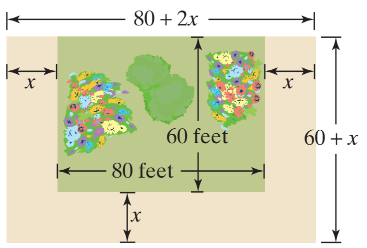


Figure P.17 The garden's area is to be doubled by adding the path.

Solution

Step 1 Let x represent one of the unknown quantities. We will let

$$x = \text{the width of the path.}$$

The situation is illustrated in **Figure P.17**. The figure shows the original 80-by-60 foot rectangular garden and the path of width x added along both shorter sides and one longer side.

Step 2 Represent other unknown quantities in terms of x. Because the path is added along both shorter sides and one longer side, **Figure P.17** shows that

$$80 + 2x = \text{the length of the new, expanded rectangle}$$

$$60 + x = \text{the width of the new, expanded rectangle.}$$

Step 3 Write an equation in x that models the conditions. The area of the rectangle must be doubled by the addition of the path.

The area, or length times width, of the new, expanded rectangle | must be | twice that of | the area of the garden.

$$(80 + 2x)(60 + x) = 2 \cdot 80 \cdot 60$$

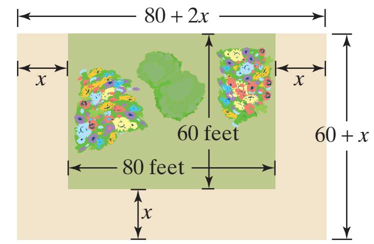


Figure P.17 (repeated)

Step 4 Solve the equation and answer the question.

$$(80 + 2x)(60 + x) = 2 \cdot 80 \cdot 60$$ This is the equation that models the problem's conditions.

$$4800 + 200x + 2x^2 = 9600$$ Multiply. Use FOIL on the left side.

$$2x^2 + 200x - 4800 = 0$$ Subtract 9600 from both sides and write the quadratic equation in general form.

$$2(x^2 + 100x - 2400) = 0$$ Factor out 2, the GCF.

$$2(x - 20)(x + 120) = 0$$ Factor the trinomial.

$$x - 20 = 0 \quad \text{or} \quad x + 120 = 0$$ Set each variable factor equal to 0.

$$x = 20 \qquad\qquad x = -120$$ Solve for x.

The path cannot have a negative width. Because -120 is geometrically impossible, we use $x = 20$. The width of the path should be 20 feet.

Step 5 Check the proposed solution in the original wording of the problem. Has the landscape architect doubled the garden's area with the 20-foot-wide path? The area of the garden is 80 feet times 60 feet, or 4800 square feet. Because $80 + 2x$ and $60 + x$ represent the length and width of the expanded rectangle,

$$80 + 2x = 80 + 2 \cdot 20 = 120 \text{ feet is the expanded rectangle's length.}$$

$$60 + x = 60 + 20 = 80 \text{ feet is the expanded rectangle's width.}$$

The area of the expanded rectangle is 120 feet times 80 feet, or 9600 square feet. This is double the area of the garden, 4800 square feet, as specified by the problem's conditions.

Check Point 5 A rectangular garden measures 16 feet by 12 feet. A path of uniform width is to be added so as to surround the entire garden. The landscape artist doing the work wants the garden and path to cover an area of 320 square feet. How wide should the path be?

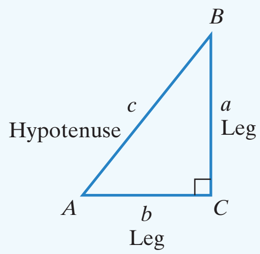
The solution to our next problem relies on knowing the **Pythagorean Theorem**. The theorem relates the lengths of the three sides of a **right triangle**, a triangle with one angle measuring 90°. The side opposite the 90° angle is called the **hypotenuse**. The other sides are called **legs**. The legs form the two sides of the right angle.

The Pythagorean Theorem

The sum of the squares of the lengths of the legs of a right triangle equals the square of the length of the hypotenuse.

If the legs have lengths a and b, and the hypotenuse has length c, then

$$a^2 + b^2 = c^2.$$



EXAMPLE 6 Using the Pythagorean Theorem

a. A wheelchair ramp with a length of 122 inches has a horizontal distance of 120 inches. What is the ramp's vertical distance?

b. Construction laws are very specific when it comes to access ramps for the disabled. Every vertical rise of 1 inch requires a horizontal run of 12 inches. Does this ramp satisfy the requirement?

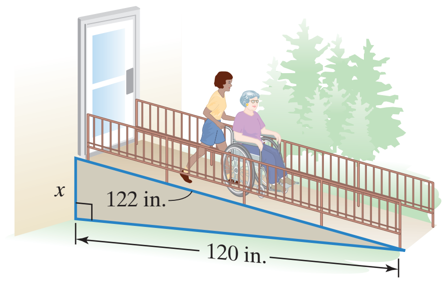


Figure P.18

Solution

a. Step 1 Let x represent one of the unknown quantities. We will let

$$x = \text{the ramp's vertical distance.}$$

The situation is illustrated in **Figure P.18**.

Step 2 Represent other unknown quantities in terms of x. There are no other unknown quantities, so we can skip this step.

Step 3 Write an equation in x that models the conditions. **Figure P.18** shows the right triangle that is formed by the ramp, the wall, and the ground. We can find x, the ramp's vertical distance, using the Pythagorean Theorem.

(leg)²	plus	(leg)²	equals	(hypotenuse)²
x^2	$+$	120^2	$=$	122^2

Step 4 Solve the equation and answer the question. The quadratic equation $x^2 + 120^2 = 122^2$ can be solved most efficiently by the square root property.

$$x^2 + 120^2 = 122^2$$ This is the equation resulting from the Pythagorean Theorem.

$$x^2 + 14{,}400 = 14{,}884$$ Square 120 and 122.

$$x^2 = 484$$ Isolate x^2 by subtracting 14,400 from both sides.

$$x = \sqrt{484} \quad \text{or} \quad x = -\sqrt{484}$$ Apply the square root property.

$$x = 22 \qquad\qquad x = -22$$ Simplify.

Because x represents the ramp's vertical distance, this measurement must be positive. We reject -22. Thus, the ramp's vertical distance is 22 inches.

Step 5 Check the proposed solution in the original wording of the problem. The problem's wording implies that the ramp, the wall, and the ground form a right triangle. This can be checked using the **converse of the Pythagorean Theorem**: If a triangle has sides of lengths a, b, and c, where c is the length of the longest side, and if $a^2 + b^2 = c^2$, then the triangle is a right triangle. Let's check that a vertical distance of 22 inches forms a right triangle with the ramp's length of 122 inches and its horizontal distance of 120 inches. Is $22^2 + 120^2 = 122^2$? Simplifying the arithmetic, we obtain the true statement $14{,}884 = 14{,}884$. Thus, a vertical distance of 22 inches forms a right triangle.

b. Every vertical rise of 1 inch requires a horizontal run of 12 inches. Because the ramp has a vertical distance of 22 inches, it requires a horizontal distance of 22(12) inches, or 264 inches. The horizontal distance is only 120 inches, so this ramp does not satisfy construction laws for access ramps for the disabled.

Study Tip

The Pythagorean Theorem is an *if . . . then* statement: If a triangle is a right triangle, then $a^2 + b^2 = c^2$. The **converse** of *if p then q* is *if q then p*. Although the converse of a true statement may not be true, the converse of the Pythagorean Theorem is also a true statement: If $a^2 + b^2 = c^2$, then a triangle is a right triangle.

Check Point 6 A radio tower is supported by two wires that are each 130 yards long and attached to the ground 50 yards from the base of the tower. How tall is the tower?

Study Tip

There is great value in reasoning through a word problem. This value comes from the problem-solving skills that are attained and is often more important than the specific problem or its solution.

In our final example, the conditions are modeled by a rational equation.

EXAMPLE 7 Dividing the Cost of a Yacht

A group of friends agrees to share the cost of a \$50,000 yacht equally. Before the purchase is made, one more person joins the group and enters the agreement. As a result, each person's share is reduced by \$2500. How many people were in the original group?

Solution

Step 1 Let x represent one of the unknown quantities. We will let

$$x = \text{the number of people in the original group.}$$

Step 2 Represent other unknown quantities in terms of *x*. Because one more person joined the original group, let

$x + 1 =$ the number of people in the final group.

Step 3 Write an equation in *x* that models the conditions. As a result of one more person joining the original group, each person's share is reduced by \$2500.

Original cost per person | minus | \$2500 | equals | the final cost per person.

$$\frac{50{,}000}{x} - 2500 = \frac{50{,}000}{x+1}$$

This is the yacht's cost, \$50,000, divided by the number of people, x.

This is the yacht's cost, \$50,000, divided by the number of people, $x + 1$.

Step 4 Solve the equation and answer the question.

$$\frac{50{,}000}{x} - 2500 = \frac{50{,}000}{x+1}$$

This is the equation that models the problem's conditions.

$$x(x+1)\left(\frac{50{,}000}{x} - 2500\right) = x(x+1)\cdot\frac{50{,}000}{x+1}$$

Multiply both sides by $x(x + 1)$, the LCD.

$$\cancel{x}(x+1)\cdot\frac{50{,}000}{\cancel{x}} - x(x+1)2500 = x\cancel{(x+1)}\cdot\frac{50{,}000}{\cancel{(x+1)}}$$

Use the distributive property and divide out common factors.

$$50{,}000(x+1) - 2500x(x+1) = 50{,}000x$$

Simplify.

$$50{,}000x + 50{,}000 - 2500x^2 - 2500x = 50{,}000x$$

Use the distributive property.

$$-2500x^2 + 47{,}500x + 50{,}000 = 50{,}000x$$

Combine like terms: $50{,}000x - 2500x = 47{,}500x$.

$$-2500x^2 - 2500x + 50{,}000 = 0$$

Write the quadratic equation in general form, subtracting $50{,}000x$ from both sides.

$$-2500(x^2 + x - 20) = 0$$

Factor out -2500.

$$-2500(x+5)(x-4) = 0$$

Factor completely.

$$x + 5 = 0 \quad \text{or} \quad x - 4 = 0$$

Set each variable factor equal to zero.

$$x = -5 \qquad\qquad x = 4$$

Solve the resulting equations.

Because x represents the number of people in the original group, x cannot be negative. Thus, there were four people in the original group.

Step 5 Check the proposed solution in the original wording of the problem.

$$\text{original cost per person} = \frac{\$50{,}000}{4} = \$12{,}500$$

$$\text{final cost per person} = \frac{\$50{,}000}{5} = \$10{,}000$$

We see that each person's share is reduced by \$12,500 − \$10,000, or \$2500, as specified by the problem's conditions.

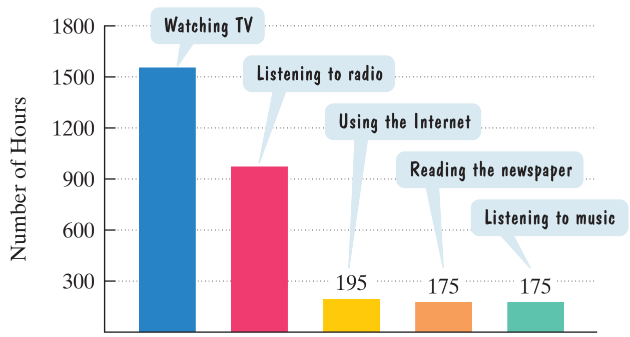
Check Point 7 A group of people share equally in a $5,000,000 lottery. Before the money is divided, three more winning ticket holders are declared. As a result, each person's share is reduced by $375,000. How many people were in the original group of winners?

Exercise Set P.8

Practice and Application Exercises

1. The bar graph shows the time Americans spent using various media in 2007.

Average Number of Hours Americans Used Various Media in 2007

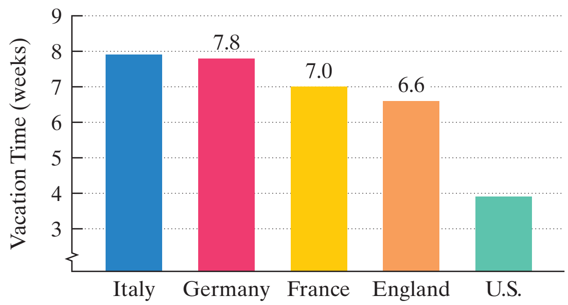


Source: Communication Industry Forecast and Report

Time spent watching TV exceeded time spent listening to the radio by 581 hours. The combined time devoted to these two media was 2529 hours. In 2007, how many hours did Americans spend listening to the radio and how many hours were spent watching TV?

2. Compared with Europeans, American employees use less vacation time.

Average Vacation Time for Europeans and Americans

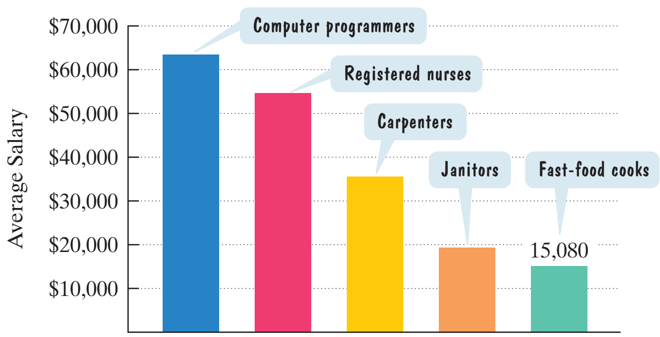


Source: The State of Working America 2006/2007

The average time Italians spend on vacation exceeds the average American vacation time by 4 weeks. The combined average vacation time for Americans and Italians is 11.8 weeks. On average, how many weeks do Americans spend on vacation and how many weeks do Italians spend on vacation?

Exercises 3–4 involve the average salaries represented by the bar graph.

Average Salaries for Various Jobs



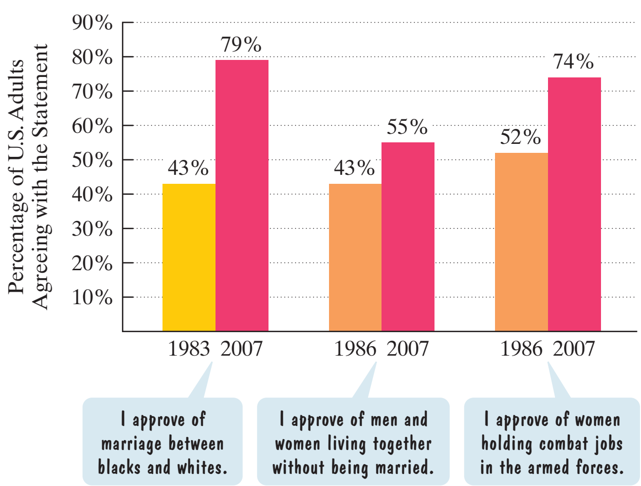
Source: 2007 data from salary.com

3. The average salary for computer programmers is $7740 less than twice the average salary for carpenters. Combined, their average salaries are $99,000. Determine the average salary for each of these jobs.

4. The average salary for registered nurses is $3500 less than three times the average salary for janitors. Combined, their average salaries are $74,060. Determine the average salary for each of these jobs.

The bar graph indicates that American attitudes have become more tolerant over two decades on a variety of issues. Exercises 5–6 are based on the data displayed by the graph.

Changing Attitudes in the United States



Source: USA Today

(Exercises 5–6 are illustrated by the bar graph at the bottom of the previous page.)

5. In 1983, 43% of U.S. adults approved of marriage between blacks and whites. For the period from 1983 through 2007, the percentage approving of interracial marriage increased on average by 1.5 each year. If this trend continues, by which year will all American adults approve of interracial marriage?

6. In 1986, 43% of U.S. adults approved of men and women living together without being married. For the period from 1986 through 2007, the percentage approving of cohabitation increased on average by approximately 0.6 each year. If this trend continues, by which year will 61% of all American adults approve of cohabitation?

7. A new car worth $24,000 is depreciating in value by $3000 per year.

a. Write a formula that models the car's value, y, in dollars, after x years.

b. Use the formula from part (a) to determine after how many years the car's value will be $9000.

8. A new car worth $45,000 is depreciating in value by $5000 per year.

a. Write a formula that models the car's value, y, in dollars, after x years.

b. Use the formula from part (a) to determine after how many years the car's value will be $10,000.

9. In 2005, there were 13,300 students at college A, with a projected enrollment increase of 1000 students per year. In the same year, there were 26,800 students at college B, with a projected enrollment decline of 500 students per year. According to these projections, when will the colleges have the same enrollment? What will be the enrollment in each college at that time?

10. In 2000, the population of Greece was 10,600,000, with projections of a population decrease of 28,000 people per year. In the same year, the population of Belgium was 10,200,000, with projections of a population decrease of 12,000 people per year. (*Source*: United Nations) According to these projections, when will the two countries have the same population? What will be the population at that time?

11. After a 20% reduction, you purchase a television for $336. What was the television's price before the reduction?

12. After a 30% reduction, you purchase a dictionary for $30.80. What was the dictionary's price before the reduction?

13. Including 8% sales tax, an inn charges $162 per night. Find the inn's nightly cost before the tax is added.

14. Including 5% sales tax, an inn charges $252 per night. Find the inn's nightly cost before the tax is added.

Exercises 15–16 involve markup, the amount added to the dealer's cost of an item to arrive at the selling price of that item.

15. The selling price of a refrigerator is $584. If the markup is 25% of the dealer's cost, what is the dealer's cost of the refrigerator?

16. The selling price of a scientific calculator is $15. If the markup is 25% of the dealer's cost, what is the dealer's cost of the calculator?

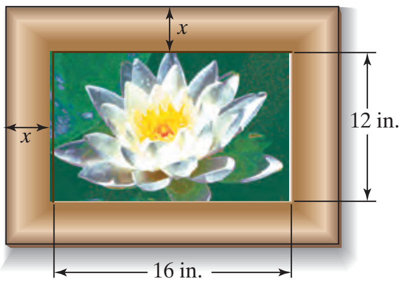
17. A rectangular soccer field is twice as long as it is wide. If the perimeter of the soccer field is 300 yards, what are its dimensions?

18. A rectangular swimming pool is three times as long as it is wide. If the perimeter of the pool is 320 feet, what are its dimensions?

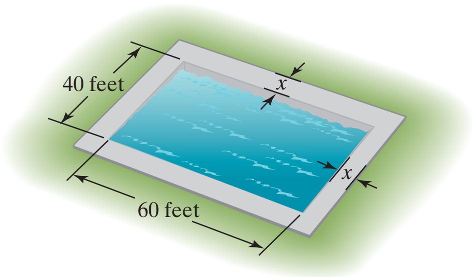
19. The length of the rectangular tennis court at Wimbledon is 6 feet longer than twice the width. If the court's perimeter is 228 feet, what are the court's dimensions?

20. The length of a rectangular pool is 6 meters less than twice the width. If the pool's perimeter is 126 meters, what are its dimensions?

21. The rectangular painting in the figure shown measures 12 inches by 16 inches and includes a frame of uniform width around the four edges. The perimeter of the rectangle formed by the painting and its frame is 72 inches. Determine the width of the frame.



22. The rectangular swimming pool in the figure shown measures 40 feet by 60 feet and includes a path of uniform width around the four edges. The perimeter of the rectangle formed by the pool and the surrounding path is 248 feet. Determine the width of the path.



23. The length of a rectangular sign is 3 feet longer than the width. If the sign's area is 54 square feet, find its length and width.

24. A rectangular parking lot has a length that is 3 yards greater than the width. The area of the parking lot is 180 square yards. Find the length and the width.

25. Each side of a square is lengthened by 3 inches. The area of this new, larger square is 64 square inches. Find the length of a side of the original square.

26. Each side of a square is lengthened by 2 inches. The area of this new, larger square is 36 square inches. Find the length of a side of the original square.

27. A pool measuring 10 meters by 20 meters is surrounded by a path of uniform width. If the area of the pool and the path combined is 600 square meters, what is the width of the path?

28. A vacant rectangular lot is being turned into a community vegetable garden measuring 15 meters by 12 meters. A path of uniform width is to surround the garden. If the area of the lot is 378 square meters, find the width of the path surrounding the garden.

29. As part of a landscaping project, you put in a flower bed measuring 20 feet by 30 feet. To finish off the project, you are putting in a uniform border of pine bark around the outside of the rectangular garden. You have enough pine bark to cover 336 square feet. How wide should the border be?

30. As part of a landscaping project, you put in a flower bed measuring 10 feet by 12 feet. You plan to surround the bed with a uniform border of low-growing plants that require 1 square foot each when mature. If you have 168 of these plants, how wide a strip around the flower bed should you prepare for the border?

31. A 20-foot ladder is 15 feet from a house. How far up the house, to the nearest tenth of a foot, does the ladder reach?

32. The base of a 30-foot ladder is 10 feet from a building. If the ladder reaches the flat roof, how tall, to the nearest tenth of a foot, is the building?

33. A tree is supported by a wire anchored in the ground 5 feet from its base. The wire is 1 foot longer than the height that it reaches on the tree. Find the length of the wire.

34. A tree is supported by a wire anchored in the ground 15 feet from its base. The wire is 4 feet longer than the height that it reaches on the tree. Find the length of the wire.

35. A rectangular piece of land whose length its twice its width has a diagonal distance of 64 yards. How many yards, to the nearest tenth of a yard, does a person save by walking diagonally across the land instead of walking its length and its width?

36. A rectangular piece of land whose length is three times its width has a diagonal distance of 92 yards. How many yards, to the nearest tenth of a yard, does a person save by walking diagonally across the land instead of walking its length and its width?

37. A group of people share equally in a \$20,000,000 lottery. Before the money is divided, two more winning ticket holders are declared. As a result, each person's share is reduced by \$500,000. How many people were in the original group of winners?

38. A group of friends agrees to share the cost of a \$480,000 vacation condominium equally. Before the purchase is made, four more people join the group and enter the agreement. As a result, each person's share is reduced by \$32,000. How many people were in the original group?

In Exercises 39–42, use the formula

$$\text{Time traveled} = \frac{\text{Distance traveled}}{\text{Average velocity}}.$$

39. A car can travel 300 miles in the same amount of time it takes a bus to travel 180 miles. If the average velocity of the bus is 20 miles per hour slower than the average velocity of the car, find the average velocity for each.

40. A passenger train can travel 240 miles in the same amount of time it takes a freight train to travel 160 miles. If the average velocity of the freight train is 20 miles per hour slower than the average velocity of the passenger train, find the average velocity of each.

41. You ride your bike to campus a distance of 5 miles and return home on the same route. Going to campus, you ride mostly downhill and average 9 miles per hour faster than on your return trip home. If the round trip takes one hour and ten minutes—that is $\frac{7}{6}$ hours—what is your average velocity on the return trip?

42. An engine pulls a train 140 miles. Then a second engine, whose average velocity is 5 miles per hour faster than the first engine, takes over and pulls the train 200 miles. The total time required for both engines is 9 hours. Find the average velocity of each engine.

43. An automobile repair shop charged a customer \$448, listing \$63 for parts and the remainder for labor. If the cost of labor is \$35 per hour, how many hours of labor did it take to repair the car?

44. A repair bill on a sailboat came to \$1603, including \$532 for parts and the remainder for labor. If the cost of labor is \$63 per hour, how many hours of labor did it take to repair the sailboat?

45. An HMO pamphlet contains the following recommended weight for women: "Give yourself 100 pounds for the first 5 feet plus 5 pounds for every inch over 5 feet tall." Using this description, what height corresponds to a recommended weight of 135 pounds?

46. A job pays an annual salary of \$33,150, which includes a holiday bonus of \$750. If paychecks are issued twice a month, what is the gross amount for each paycheck?

47. You have 35 hits in 140 times at bat. Your batting average is $\frac{35}{140}$, or 0.25. How many consecutive hits must you get to increase your batting average to 0.30?

48. You have 30 hits in 120 times at bat. Your batting average is $\frac{30}{120}$, or 0.25. How many consecutive hits must you get to increase your batting average to 0.28?

Writing in Mathematics

49. In your own words, describe a step-by-step approach for solving algebraic word problems.

50. Write an original word problem that can be solved using an equation. Then solve the problem.

51. In your own words, state the Pythagorean Theorem.

52. In the 1939 movie *The Wizard of Oz*, upon being presented with a Th.D. (Doctor of Thinkology), the Scarecrow proudly exclaims, "The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side." Did the Scarecrow get the Pythagorean Theorem right? In particular, describe four errors in the Scarecrow's statement.

Critical Thinking Exercises

Make Sense? *In Exercises 53–56, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

53. By modeling attitudes of college freshmen from 1969 through 2006, I can make precise predictions about the attitudes of the freshman class of 2020.

54. I find the hardest part in solving a word problem is writing the equation that models the verbal conditions.

55. After a 35% reduction, a computer's price is \$780, so I determined the original price, x, by solving $x - 0.35 = 780$.

56. When I use the square root property to determine the length of a right triangle's side, I don't even bother to list the negative square root.

57. The perimeter of a plot of land in the shape of a right triangle is 12 miles. If one leg of the triangle exceeds the other leg by 1 mile, find the length of each boundary of the land.

58. The price of a dress is reduced by 40%. When the dress still does not sell, it is reduced by 40% of the reduced price. If the price of the dress after both reductions is \$72, what was the original price?

59. In a film, the actor Charles Coburn plays an elderly "uncle" character criticized for marrying a woman when he is 3 times her age. He wittily replies, "Ah, but in 20 years time I shall only be twice her age." How old is the "uncle" and the woman?

60. Suppose that we agree to pay you 8¢ for every problem in this chapter that you solve correctly and fine you 5¢ for every problem done incorrectly. If at the end of 26 problems we do not owe each other any money, how many problems did you solve correctly?

61. It was wartime when the Ricardos found out Mrs. Ricardo was pregnant. Ricky Ricardo was drafted and made out a will, deciding that \$14,000 in a savings account was to be divided between his wife and his child-to-be. Rather strangely, and certainly with gender bias, Ricky stipulated that if the child were a boy, he would get twice the amount of the mother's portion. If it were a girl, the mother would get twice the amount the girl was to receive. We'll never know what Ricky was thinking of, for (as fate would have it) he did not return from war. Mrs. Ricardo gave birth to twins—a boy and a girl. How was the money divided?

62. A thief steals a number of rare plants from a nursery. On the way out, the thief meets three security guards, one after another. To each security guard, the thief is forced to give one-half the plants that he still has, plus 2 more. Finally, the thief leaves the nursery with 1 lone palm. How many plants were originally stolen?

Group Exercise

63. One of the best ways to learn how to *solve* a word problem in algebra is to *design* word problems of your own. Creating a word problem makes you very aware of precisely how much information is needed to solve the problem. You must also focus on the best way to present information to a reader and on how much information to give. As you write your problem, you gain skills that will help you solve problems created by others.

The group should design five different word problems that can be solved using equations. All of the problems should be on different topics. For example, the group should not have more than one problem on the perimeter of a rectangle. The group should turn in both the problems and their algebraic solutions.

(If you're not sure where to begin, consider the graph for Exercises 5–6 and the data that we did not use regarding attitudes about women in combat.)

Preview Exercises

Exercises 64–66 will help you prepare for the material covered in the next section.

64. Is -1 a solution of $3 - 2x \leq 11$?

65. Solve: $-2x - 4 = x + 5$.

66. Solve: $\frac{x+3}{4} = \frac{x-2}{3} + \frac{1}{4}$.

Section P.9 Linear Inequalities and Absolute Value Inequalities

Objectives

1. Use interval notation.
2. Find intersections and unions of intervals.
3. Solve linear inequalities.
4. Solve compound inequalities.
5. Solve absolute value inequalities.

Rent-a-Heap, a car rental company, charges \$125 per week plus \$0.20 per mile to rent one of their cars. Suppose you are limited by how much money you can spend for the week: You can spend at most \$335. If we let x represent the number of miles you drive the heap in a week, we can write an inequality that models the given conditions:

The weekly charge of \$125 | plus | the charge of \$0.20 per mile for x miles | must be less than or equal to | \$335.

$$125 + 0.20x \leq 335.$$