Critical Thinking Exercises

Make Sense? *In Exercises 130–133, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

130. Although $20x^3$ appears in both $20x^3 + 8x^2$ and $20x^3 + 10x$, I'll need to factor $20x^3$ in different ways to obtain each polynomial's factorization.

131. You grouped the polynomial's terms using different groupings than I did, yet we both obtained the same factorization.

132. I factored $4x^2 - 100$ completely and obtained $(2x + 10)(2x - 10)$.

133. First factoring out the greatest common factor makes it easier for me to determine how to factor the remaining factor, assuming that it is not prime.

In Exercises 134–137, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

134. $x^4 - 16$ is factored completely as $(x^2 + 4)(x^2 - 4)$.

135. The trinomial $x^2 - 4x - 4$ is a prime polynomial.

136. $x^2 + 36 = (x + 6)^2$

137. $x^3 - 64 = (x + 4)(x^2 + 4x - 16)$

In Exercises 138–141, factor completely.

138. $x^{2n} + 6x^n + 8$

139. $-x^2 - 4x + 5$

140. $x^4 - y^4 - 2x^3y + 2xy^3$

141. $(x - 5)^{-\frac{1}{2}}(x + 5)^{-\frac{1}{2}} - (x + 5)^{\frac{1}{2}}(x - 5)^{-\frac{3}{2}}$

In Exercises 142–143, find all integers b so that the trinomial can be factored.

142. $x^2 + bx + 15$

143. $x^2 + 4x + b$

Preview Exercises

Exercises 144–146 will help you prepare for the material covered in the next section.

144. Factor the numerator and the denominator. Then simplify by dividing out the common factor in the numerator and the denominator.

$$\frac{x^2 + 6x + 5}{x^2 - 25}$$

In Exercises 145–146, perform the indicated operation. Where possible, reduce the answer to its lowest terms.

145. $\frac{5}{4} \cdot \frac{8}{15}$

146. $\frac{1}{2} + \frac{2}{3}$

Chapter P Mid-Chapter Check Point

What You Know: We defined the real numbers $[\{x \mid x \text{ is rational}\} \cup \{x \mid x \text{ is irrational}\}]$ and graphed them as points on a number line. We reviewed the basic rules of algebra, using these properties to simplify algebraic expressions. We expanded our knowledge of exponents to include exponents other than natural numbers:

$$b^0 = 1; \quad b^{-n} = \frac{1}{b^n}; \quad \frac{1}{b^{-n}} = b^n; \quad b^{\frac{1}{n}} = \sqrt[n]{b};$$

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}; \quad b^{-\frac{m}{n}} = \frac{1}{b^{\frac{m}{n}}}.$$

We used properties of exponents to simplify exponential expressions and properties of radicals to simplify radical expressions. Finally, we performed operations with polynomials. We used a number of fast methods for finding products of polynomials, including the FOIL method for multiplying binomials, a special-product formula for the product of the sum and difference of two terms $[(A + B)(A - B) = A^2 - B^2]$, and special-product formulas for squaring binomials $[(A + B)^2 = A^2 + 2AB + B^2;$ $(A - B)^2 = A^2 - 2AB + B^2]$. We reversed the direction of these formulas and reviewed how to factor polynomials. We used a general strategy, summarized in the box on page 63, for factoring a wide variety of polynomials.

In Exercises 1–27, simplify the given expression or perform the indicated operation (and simplify, if possible), whichever is appropriate.

1. $(3x + 5)(4x - 7)$

2. $(3x + 5) - (4x - 7)$

3. $\sqrt{6} + 9\sqrt{6}$

4. $3\sqrt{12} - \sqrt{27}$

5. $7x + 3[9 - (2x - 6)]$

6. $(8x - 3)^2$

7. $\left(x^{\frac{1}{3}}y^{-\frac{1}{2}}\right)^6$

8. $\left(\frac{2}{7}\right)^0 - 32^{-\frac{2}{5}}$

9. $(2x - 5) - (x^2 - 3x + 1)$

10. $(2x - 5)(x^2 - 3x + 1)$

11. $x^3 + x^3 - x^3 \cdot x^3$

12. $(9a - 10b)(2a + b)$

13. $\{a, c, d, e\} \cup \{c, d, f, h\}$

14. $\{a, c, d, e\} \cap \{c, d, f, h\}$

15. $(3x^2y^3 - xy + 4y^2) - (-2x^2y^3 - 3xy + 5y^2)$

16. $\frac{24x^2y^{13}}{-2x^5y^{-2}}$

17. $\left(\frac{1}{3}x^{-5}y^4\right)(18x^{-2}y^{-1})$

18. $\sqrt[12]{x^4}$

19. $[4y - (3x + 2)][4y + (3x + 2)]$

20. $(x - 2y - 1)^2$

21. $\frac{24 \times 10^3}{2 \times 10^6}$ (Express the answer in scientific notation.)

22. $\frac{\sqrt[3]{32}}{\sqrt[3]{2}}$

23. $(x^3 + 2)(x^3 - 2)$

24. $(x^2 + 2)^2$

25. $\sqrt{50} \cdot \sqrt{6}$

26. $\dfrac{11}{7 - \sqrt{3}}$

27. $\dfrac{11}{\sqrt{3}}$

In Exercises 28–34, factor completely, or state that the polynomial is prime.

28. $7x^2 - 22x + 3$

29. $x^2 - 2x + 4$

30. $x^3 + 5x^2 + 3x + 15$

31. $3x^2 - 4xy - 7y^2$

32. $64y - y^4$

33. $50x^3 + 20x^2 + 2x$

34. $x^2 - 6x + 9 - 49y^2$

In Exercises 35–36, factor and simplify each algebraic expression.

35. $x^{-\frac{3}{2}} - 2x^{-\frac{1}{2}} + x^{\frac{1}{2}}$

36. $(x^2 + 1)^{\frac{1}{2}} - 10(x^2 + 1)^{-\frac{1}{2}}$

37. List all the rational numbers in this set:

$$\left\{-11, -\frac{3}{7}, 0, 0.45, \sqrt{23}, \sqrt{25}\right\}.$$

In Exercises 38–39, rewrite each expression without absolute value bars.

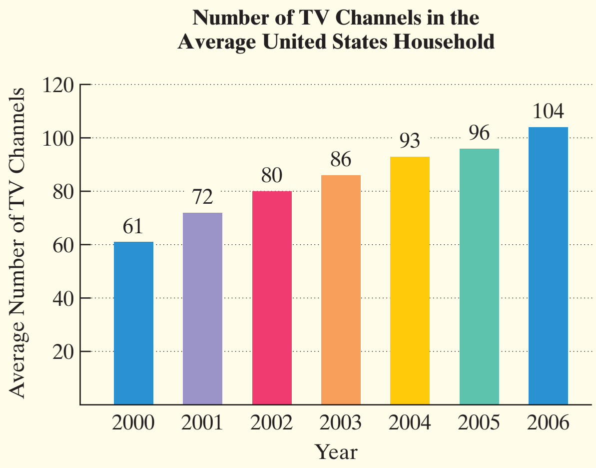
38. $|2 - \sqrt{13}|$

39. $x^2|x|$ if $x < 0$

40. If the population of the United States is approximately 3.0×10^8 and each person spends about \$140 per year on ice cream, express the total annual spending on ice cream in scientific notation.

41. A human brain contains 3×10^{10} neurons and a gorilla brain contains 7.5×10^9 neurons. How many times as many neurons are in the brain of a human as in the brain of a gorilla?

42. The number of TV channels is increasing. The bar graph shows the total channels available in the average U.S. household from 2000 through 2006.



Source: Nielsen Media Research

Here are two mathematical models for the data shown by the graph. In each formula, N represents the number of TV channels in the average U.S. household x years after 2000.

Model 1 $N = 6.8x + 64$

Model 2 $N = -0.5x^2 + 9.5x + 62$

a. Which model best describes the data for 2000?

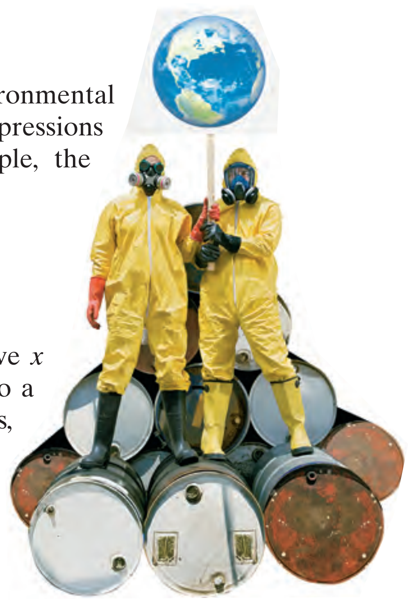
b. Does the polynomial model of degree 2 underestimate or overestimate the number of channels for 2006? By how many channels?

c. According to the polynomial model of degree 1, how many channels will the average household have in 2010?

Section P.6 Rational Expressions

Objectives

1. Specify numbers that must be excluded from the domain of a rational expression.
2. Simplify rational expressions.
3. Multiply rational expressions.
4. Divide rational expressions.
5. Add and subtract rational expressions.
6. Simplify complex rational expressions.
7. Simplify fractional expressions that occur in calculus.
8. Rationalize numerators.



How do we describe the costs of reducing environmental pollution? We often use algebraic expressions involving quotients of polynomials. For example, the algebraic expression

$$\frac{250x}{100 - x}$$

describes the cost, in millions of dollars, to remove x percent of the pollutants that are discharged into a river. Removing a modest percentage of pollutants, say 40%, is far less costly than removing a substantially greater percentage, such as 95%. We see this by evaluating the algebraic expression for $x = 40$ and $x = 95$.