107. I multiplied two complex numbers in polar form by first multiplying the moduli and then multiplying the arguments.

108. The proof of the formula for the product of two complex numbers in polar form uses the sum formulas for cosines and sines that I studied in the previous chapter.

109. My work with complex numbers verified that the only possible cube root of 8 is 2.

110. Prove the rule for finding the quotient of two complex numbers in polar form. Begin the proof as follows, using the conjugate of the denominator's second factor:

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} \cdot \frac{(\cos\theta_2 - i\sin\theta_2)}{(\cos\theta_2 - i\sin\theta_2)}.$$

Perform the indicated multiplications. Then use the difference formulas for sine and cosine.

111. Plot each of the complex fourth roots of 1.

Group Exercise

112. Group members should prepare and present a seminar on mathematical chaos. Include one or more of the following topics in your presentation: fractal images, the role of complex numbers in generating fractal images, algorithms, iterations, iteration number, and fractals in nature. Be sure to include visual images that will intrigue your audience.

Preview Exercises

Exercises 113–115 will help you prepare for the material covered in the next section.

113. Use the distance formula to determine if the line segment with endpoints $(-3, -3)$ and $(0, 3)$ has the same length as the line segment with endpoints $(0, 0)$ and $(3, 6)$.

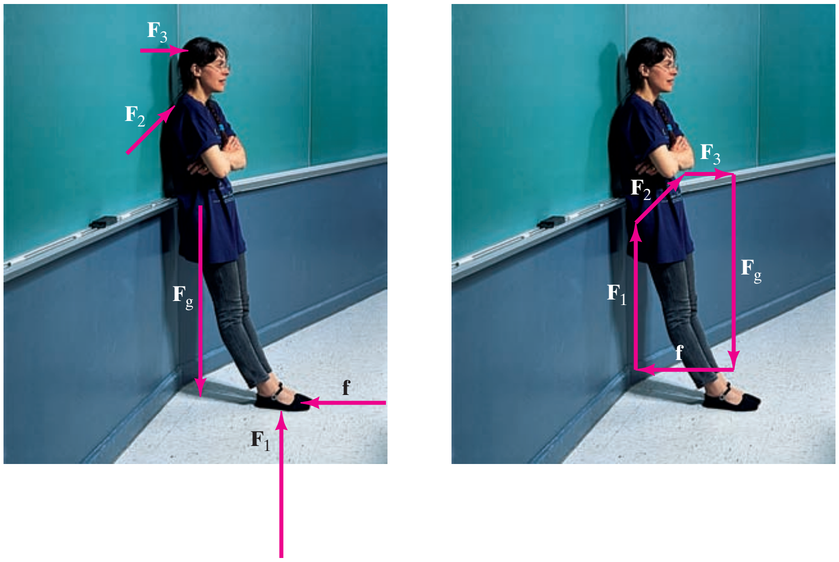
114. Use slope to determine if the line through $(-3, -3)$ and $(0, 3)$ is parallel to the line through $(0, 0)$ and $(3, 6)$.

115. Simplify: $4(5x + 4y) - 2(6x - 9y)$.

Section 6.6 Vectors

Objectives

1. Use magnitude and direction to show vectors are equal.
2. Visualize scalar multiplication, vector addition, and vector subtraction as geometric vectors.
3. Represent vectors in the rectangular coordinate system.
4. Perform operations with vectors in terms of **i** and **j**.
5. Find the unit vector in the direction of **v**.
6. Write a vector in terms of its magnitude and direction.
7. Solve applied problems involving vectors.



It's been a dynamic lecture, but now that it's over it's obvious that my professor is exhausted. She's slouching motionless against the board and—what's that? The forces acting against her body, including the pull of gravity, are appearing as arrows. I know that mathematics reveals the hidden patterns of the universe, but this is ridiculous. Does the arrangement of the arrows on the right have anything to do with the fact that my wiped-out professor is not sliding down the wall?

Ours is a world of pushes and pulls. For example, suppose you are pulling a cart up a 30° incline, requiring an effort of 100 pounds. This quantity is described by giving its magnitude (a number indicating size, including a unit of measure) and also its direction. The magnitude is 100 pounds and the direction is 30° from the horizontal. Quantities that involve both a magnitude and a direction are called **vector quantities**, or **vectors** for short. Here is another example of a vector:

> You are driving due north at 50 miles per hour. The magnitude is the speed, 50 miles per hour. The direction of motion is due north.

This sign shows a distance and direction for each city. Thus, the sign defines a vector for each destination.

Some quantities can be completely described by giving only their magnitudes. For example, the temperature of the lecture room that you just left is 75°. This temperature has magnitude, 75°, but no direction. Quantities that involve magnitude, but no direction, are called **scalar quantities**, or **scalars** for short. Thus, a scalar has only a numerical value. Another example of a scalar is your professor's height, which you estimate to be 5.5 feet.

In this section and the next, we introduce the world of vectors, which literally surround your every move. Because vectors have nonnegative magnitude as well as direction, we begin our discussion with directed line segments.

Directed Line Segments and Geometric Vectors

A line segment to which a direction has been assigned is called a **directed line segment**. **Figure 6.48** shows a directed line segment from P to Q. We call P the **initial point** and Q the **terminal point**. We denote this directed line segment by

$$\overrightarrow{PQ}.$$

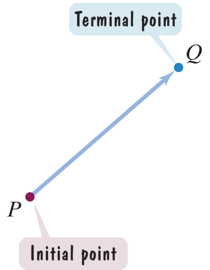


Figure 6.48 A directed line segment from P to Q

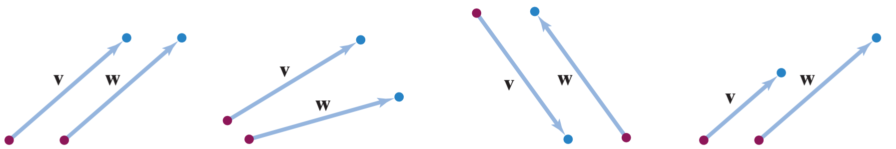
The **magnitude** of the directed line segment $\overrightarrow{PQ}$ is its length. We denote this by $\|\overrightarrow{PQ}\|$. Thus, $\|\overrightarrow{PQ}\|$ is the distance from point P to point Q. Because distance is nonnegative, vectors do not have negative magnitudes.

Geometrically, a **vector** is a directed line segment. Vectors are often denoted by boldface letters, such as **v**. If a vector **v** has the same magnitude and the same direction as the directed line segment $\overrightarrow{PQ}$, we write

$$\mathbf{v} = \overrightarrow{PQ}.$$

Because it is difficult to write boldface on paper, use an arrow over a single letter, such as $\vec{v}$, to denote **v**, the vector **v**.

Figure 6.49 shows four possible relationships between vectors **v** and **w**. In **Figure 6.49(a)**, the vectors have the same magnitude and the same direction, and are said to be *equal*. In general, vectors **v** and **w** are **equal** if they have the *same magnitude* and the *same direction*. We write this as $\mathbf{v} = \mathbf{w}$.



(a) $\mathbf{v} = \mathbf{w}$ because the vectors have the same magnitude and same direction.

(b) Vectors **v** and **w** have the same magnitude, but different directions.

(c) Vectors **v** and **w** have the same magnitude, but opposite directions.

(d) Vectors **v** and **w** have the same direction, but different magnitudes.

Figure 6.49 Relationships between vectors

1 Use magnitude and direction to show vectors are equal.

EXAMPLE 1 Showing That Two Vectors Are Equal

Use **Figure 6.50** to show that $\mathbf{u} = \mathbf{v}$.

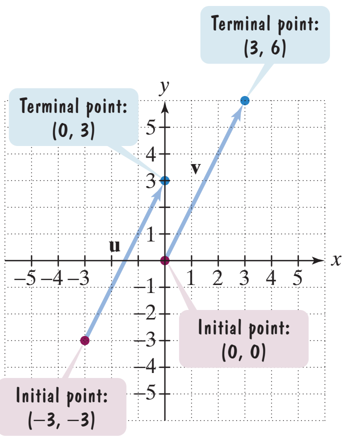


Figure 6.50

Solution Equal vectors have the same magnitude and the same direction. Use the distance formula to show that **u** and **v** have the same magnitude.

Magnitude of **u**

$$\|\mathbf{u}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[0 - (-3)]^2 + [3 - (-3)]^2}$$
$$= \sqrt{3^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45} \quad (\text{or } 3\sqrt{5})$$

Magnitude of **v**

$$\|\mathbf{v}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 0)^2 + (6 - 0)^2}$$
$$= \sqrt{3^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45} \quad (\text{or } 3\sqrt{5})$$

Thus, **u** and **v** have the same magnitude: $\|\mathbf{u}\| = \|\mathbf{v}\|$.

One way to show that **u** and **v** have the same direction is to find the slopes of the lines on which they lie. We continue to use **Figure 6.50** on the previous page.

Line on which **u** lies

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{0 - (-3)} = \frac{6}{3} = 2$$

u lies on a line passing through $(-3, -3)$ and $(0, 3)$.

Line on which **v** lies

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{3 - 0} = \frac{6}{3} = 2$$

v lies on a line passing through $(0, 0)$ and $(3, 6)$.

Because **u** and **v** are both directed toward the upper right on lines having the same slope, 2, they have the same direction.

Thus, **u** and **v** have the same magnitude and direction, and $\mathbf{u} = \mathbf{v}$.

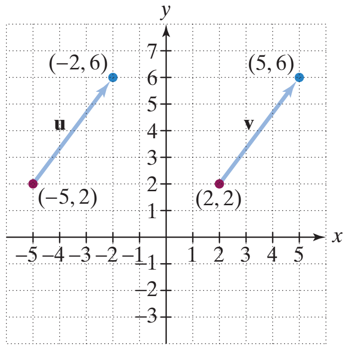


Figure 6.51

Check Point 1 Use **Figure 6.51** to show that $\mathbf{u} = \mathbf{v}$.

2 Visualize scalar multiplication, vector addition, and vector subtraction as geometric vectors.

A vector can be multiplied by a real number. **Figure 6.52** shows three such multiplications: $2\mathbf{v}$, $\frac{1}{2}\mathbf{v}$, and $-\frac{3}{2}\mathbf{v}$. **Multiplying a vector by any positive real number (except for 1) changes the magnitude of the vector, but not its direction.** This can be seen by the blue and green vectors in **Figure 6.52**. Compare the black and blue vectors. Can you see that $2\mathbf{v}$ has the same direction as **v** but is twice the magnitude of **v**? Now, compare the black and green vectors: $\frac{1}{2}\mathbf{v}$ has the same direction as **v** but is half the magnitude of **v**.

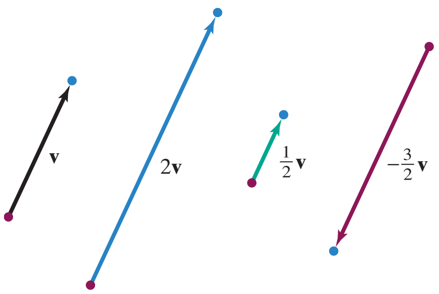


Figure 6.52 Multiplying vector **v** by real numbers

Now compare the black and red vectors in **Figure 6.52**. **Multiplying a vector by a negative number reverses the direction of the vector.** Notice that $-\frac{3}{2}\mathbf{v}$ has the opposite direction as **v** and is $\frac{3}{2}$ the magnitude of **v**.

The multiplication of a real number k and a vector **v** is called **scalar multiplication**. We write this product as $k\mathbf{v}$.

Scalar Multiplication

If k is a real number and **v** a vector, the vector $k\mathbf{v}$ is called a **scalar multiple** of the vector **v**. The magnitude and direction of $k\mathbf{v}$ are given as follows:

The vector $k\mathbf{v}$ has a *magnitude* of $|k|\|\mathbf{v}\|$. We describe this as the absolute value of k times the magnitude of vector **v**.

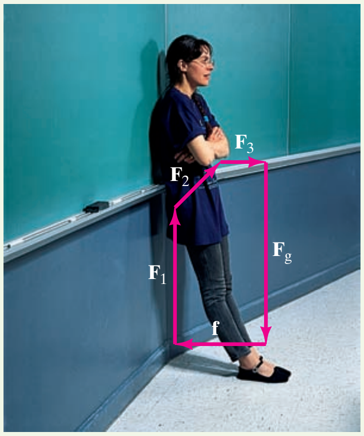
The vector $k\mathbf{v}$ has a *direction* that is

- the same as the direction of **v** if $k > 0$, and
- opposite the direction of **v** if $k < 0$.

A geometric method for adding two vectors is shown in **Figure 6.53** at the top of the next page. The sum of **u** and **v**, denoted by $\mathbf{u} + \mathbf{v}$ is called the **resultant vector**. Here is how we find this vector:

1. Position **u** and **v**, so that the terminal point of **u** coincides with the initial point of **v**.
2. The resultant vector, $\mathbf{u} + \mathbf{v}$, extends from the initial point of **u** to the terminal point of **v**.

Wiped Out, But Not Sliding Down the Wall



The figure shows the sum of five vectors:

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_g + \mathbf{f}.$$

Notice how the terminal point of each vector coincides with the initial point of the vector that's being added to it. The vector sum, from the initial point of $\mathbf{F}_1$ to the terminal point of $\mathbf{f}$, is a single point. The magnitude of a single point is zero. These forces add up to a net force of zero, allowing the professor to be motionless.

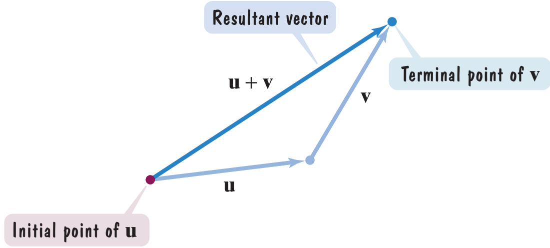


Figure 6.53 Vector addition $\mathbf{u} + \mathbf{v}$; the terminal point of $\mathbf{u}$ coincides with the initial point of $\mathbf{v}$.

The **difference of two vectors**, $\mathbf{v} - \mathbf{u}$, is defined as $\mathbf{v} - \mathbf{u} = \mathbf{v} + (-\mathbf{u})$, where $-\mathbf{u}$ is the scalar multiplication of $\mathbf{u}$ and -1: $-1\mathbf{u}$. The difference $\mathbf{v} - \mathbf{u}$ is shown geometrically in **Figure 6.54**.

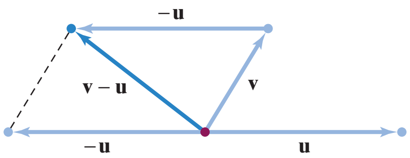


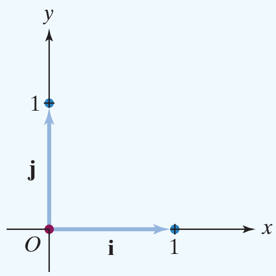
Figure 6.54 Vector subtraction $\mathbf{v} - \mathbf{u}$; the terminal point of $\mathbf{v}$ coincides with the initial point of $-\mathbf{u}$.

Vectors in the Rectangular Coordinate System

As you saw in Example 1, vectors can be shown in the rectangular coordinate system. Now let's see how we can use the rectangular coordinate system to represent vectors. We begin with two vectors that both have a magnitude of 1. Such vectors are called **unit vectors**.

The i and j Unit Vectors

Vector $\mathbf{i}$ is the unit vector whose direction is along the positive x-axis. Vector $\mathbf{j}$ is the unit vector whose direction is along the positive y-axis.



3 Represent vectors in the rectangular coordinate system.

Why are the unit vectors $\mathbf{i}$ and $\mathbf{j}$ important? Vectors in the rectangular coordinate system can be represented in terms of $\mathbf{i}$ and $\mathbf{j}$. For example, consider vector $\mathbf{v}$ with initial point at the origin, $(0, 0)$, and terminal point at $P = (a, b)$. The vector $\mathbf{v}$ is shown in **Figure 6.55**. We can represent $\mathbf{v}$ using $\mathbf{i}$ and $\mathbf{j}$ as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$.

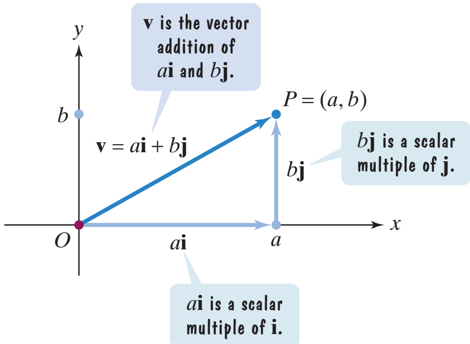


Figure 6.55 Using vector addition, vector $\mathbf{v}$ is represented as $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$.

Representing Vectors in Rectangular Coordinates

Vector **v**, from $(0, 0)$ to (a, b), is represented as

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j}.$$

The real numbers a and b are called the **scalar components of v**. Note that

- a is the **horizontal component** of **v**, and
- b is the **vertical component** of **v**.

The vector sum $a\mathbf{i} + b\mathbf{j}$ is called a **linear combination** of the vectors **i** and **j**. The magnitude of $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ is given by

$$\|\mathbf{v}\| = \sqrt{a^2 + b^2}.$$

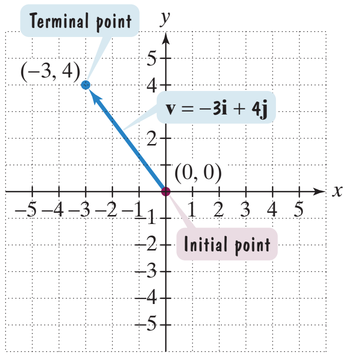


Figure 6.56 Sketching $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$ in the rectangular coordinate system

EXAMPLE 2 Representing a Vector in Rectangular Coordinates and Finding Its Magnitude

Sketch the vector $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$ and find its magnitude.

Solution For the given vector $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$, $a = -3$ and $b = 4$. The vector can be represented with its initial point at the origin, $(0, 0)$, as shown in **Figure 6.56**. The vector's terminal point is then $(a, b) = (-3, 4)$. We sketch the vector by drawing an arrow from $(0, 0)$ to $(-3, 4)$. We determine the magnitude of the vector by using the distance formula. Thus, the magnitude is

$$\|\mathbf{v}\| = \sqrt{a^2 + b^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Check Point 2 Sketch the vector $\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$ and find its magnitude.

The vector in Example 2 was represented with its initial point at the origin. A vector whose initial point is at the origin is called a **position vector**. Any vector in rectangular coordinates whose initial point is not at the origin can be shown to be equal to a position vector. As shown in the following box, this gives us a way to represent vectors between any two points.

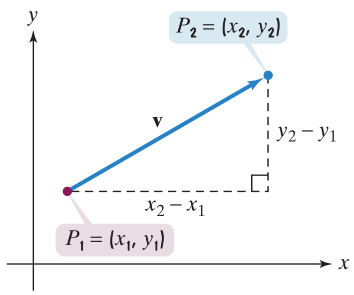


Figure 6.57(a)

Representing Vectors in Rectangular Coordinates

Vector **v** with initial point $P_1 = (x_1, y_1)$ and terminal point $P_2 = (x_2, y_2)$ is equal to the position vector

$$\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}.$$

We can use congruent triangles, triangles with the same size and shape, to derive this formula. Begin with the right triangle in **Figure 6.57(a)**. This triangle shows vector **v** from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$. In **Figure 6.57(b)**, we move vector **v**, without changing its magnitude or its direction, so that its initial point is at the origin. Using this position vector in **Figure 6.57(b)**, we see that

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j},$$

where a and b are the components of **v**. The equal vectors and the right angles in the right triangles in **Figures 6.57(a)** and **(b)** result in congruent triangles. The corresponding sides of these congruent triangles are equal, so that $a = x_2 - x_1$ and $b = y_2 - y_1$. This means that **v** may be expressed as

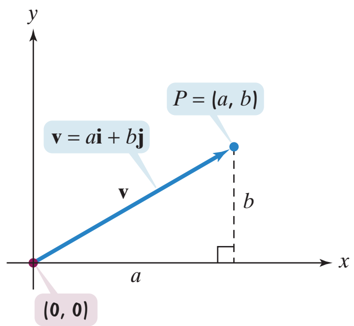


Figure 6.57(b)

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}.$$

Horizontal component: x-coordinate of terminal point minus x-coordinate of initial point

Vertical component: y-coordinate of terminal point minus y-coordinate of initial point

Thus, any vector between two points in rectangular coordinates can be expressed in terms of **i** and **j**. In rectangular coordinates, the term *vector* refers to the position vector expressed in terms of **i** and **j** that is equal to it.

EXAMPLE 3 Representing a Vector in Rectangular Coordinates

Let **v** be the vector from initial point $P_1 = (3, -1)$ to terminal point $P_2 = (-2, 5)$. Write **v** in terms of **i** and **j**.

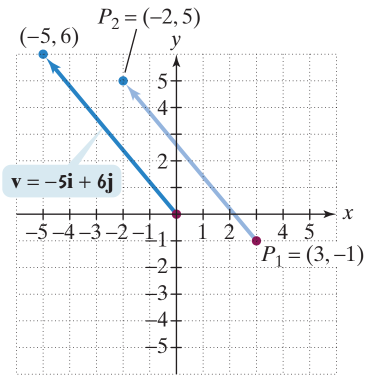


Figure 6.58 Representing the vector from $(3, -1)$ to $(-2, 5)$ as a position vector

Solution We identify the values for the variables in the formula.

$$P_1 = (\underset{x_1}{3}, \underset{y_1}{-1}) \qquad P_2 = (\underset{x_2}{-2}, \underset{y_2}{5})$$

Using these values, we write **v** in terms of **i** and **j** as follows:

$$\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} = (-2 - 3)\mathbf{i} + [5 - (-1)]\mathbf{j} = -5\mathbf{i} + 6\mathbf{j}.$$

Figure 6.58 shows the vector from $P_1 = (3, -1)$ to $P_2 = (-2, 5)$ represented in terms of **i** and **j** and as a position vector.

Study Tip

When finding the distance from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$, the order in which the subtractions are performed makes no difference:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{or} \quad d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

When writing the vector from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$, P_2 must be the terminal point and the order in the subtractions is important:

$$\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}.$$

(x_2, y_2), the terminal point, is used first in each subtraction.

Check Point 3 Let **v** be the vector from initial point $P_1 = (-1, 3)$ to terminal point $P_2 = (2, 7)$. Write **v** in terms of **i** and **j**.

4 Perform operations with vectors in terms of **i** and **j**.

Operations with Vectors in Terms of i and j

If vectors are expressed in terms of **i** and **j**, we can easily carry out operations such as vector addition, vector subtraction, and scalar multiplication. Recall the geometric definitions of these operations given earlier. Based on these ideas, we can add and subtract vectors using the following procedure:

Adding and Subtracting Vectors in Terms of i and j

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, then

$$\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j}$$
$$\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j}.$$

EXAMPLE 4 Adding and Subtracting Vectors

If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$ and $\mathbf{w} = 6\mathbf{i} - 9\mathbf{j}$, find each of the following vectors:

a. $\mathbf{v} + \mathbf{w}$ **b.** $\mathbf{v} - \mathbf{w}$.

Solution

a.

$$\mathbf{v} + \mathbf{w} = (5\mathbf{i} + 4\mathbf{j}) + (6\mathbf{i} - 9\mathbf{j})$$ These are the given vectors.

$$= (5 + 6)\mathbf{i} + [4 + (-9)]\mathbf{j}$$ Add the horizontal components. Add the vertical components.

$$= 11\mathbf{i} - 5\mathbf{j}$$ Simplify.

b.

$$\mathbf{v} - \mathbf{w} = (5\mathbf{i} + 4\mathbf{j}) - (6\mathbf{i} - 9\mathbf{j})$$ These are the given vectors.

$$= (5 - 6)\mathbf{i} + [4 - (-9)]\mathbf{j}$$ Subtract the horizontal components. Subtract the vertical components.

$$= -\mathbf{i} + 13\mathbf{j}$$ Simplify.

Check Point 4 If $\mathbf{v} = 7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} - 5\mathbf{j}$, find each of the following vectors:

a. $\mathbf{v} + \mathbf{w}$ **b.** $\mathbf{v} - \mathbf{w}$.

How do we perform scalar multiplication if vectors are expressed in terms of **i** and **j**? We use the following procedure to multiply the vector **v** by the scalar k:

Scalar Multiplication with a Vector in Terms of i and j

If $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ and k is a real number, then the scalar multiplication of the vector **v** and the scalar k is

$$k\mathbf{v} = (ka)\mathbf{i} + (kb)\mathbf{j}.$$

EXAMPLE 5 Scalar Multiplication

If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$, find each of the following vectors:

a. $6\mathbf{v}$ **b.** $-3\mathbf{v}$.

Solution

a.

$$6\mathbf{v} = 6(5\mathbf{i} + 4\mathbf{j})$$ The scalar multiplication is expressed with the given vector.

$$= (6 \cdot 5)\mathbf{i} + (6 \cdot 4)\mathbf{j}$$ Multiply each component by 6.

$$= 30\mathbf{i} + 24\mathbf{j}$$ Simplify.

b.

$$-3\mathbf{v} = -3(5\mathbf{i} + 4\mathbf{j})$$ The scalar multiplication is expressed with the given vector.

$$= (-3 \cdot 5)\mathbf{i} + (-3 \cdot 4)\mathbf{j}$$ Multiply each component by -3.

$$= -15\mathbf{i} - 12\mathbf{j}$$ Simplify.

Check Point 5 If $\mathbf{v} = 7\mathbf{i} + 10\mathbf{j}$, find each of the following vectors:

a. $8\mathbf{v}$ **b.** $-5\mathbf{v}$.

EXAMPLE 6 Vector Operations

If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$ and $\mathbf{w} = 6\mathbf{i} - 9\mathbf{j}$, find $4\mathbf{v} - 2\mathbf{w}$.

Solution

$$
\begin{aligned}
4\mathbf{v} - 2\mathbf{w} &= 4(5\mathbf{i} + 4\mathbf{j}) - 2(6\mathbf{i} - 9\mathbf{j}) && \text{Operations are expressed with the given vectors.}\\
&= 20\mathbf{i} + 16\mathbf{j} - 12\mathbf{i} + 18\mathbf{j} && \text{Perform each scalar multiplication.}\\
&= (20 - 12)\mathbf{i} + (16 + 18)\mathbf{j} && \text{Add horizontal and vertical components to perform the vector addition.}\\
&= 8\mathbf{i} + 34\mathbf{j} && \text{Simplify.}
\end{aligned}
$$

Check Point 6 If $\mathbf{v} = 7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} - 5\mathbf{j}$, find $6\mathbf{v} - 3\mathbf{w}$.

Properties involving vector operations resemble familiar properties of real numbers. For example, the order in which vectors are added makes no difference:

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}.$$

Does this remind you of the commutative property $a + b = b + a$?

Just as 0 plays an important role in the properties of real numbers, the **zero vector 0** plays exactly the same role in the properties of vectors.

The Zero Vector

The vector whose magnitude is 0 is called the **zero vector, 0**. The zero vector is assigned no direction. It can be expressed in terms of **i** and **j** using

$$\mathbf{0} = 0\mathbf{i} + 0\mathbf{j}.$$

Properties of Vector Addition and Scalar Multiplication are given as follows:

Properties of Vector Addition and Scalar Multiplication

If **u**, **v**, and **w** are vectors, and c and d are scalars, then the following properties are true.

Vector Addition Properties

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ — Commutative property
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ — Associative property
3. $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$ — Additive identity
4. $\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$ — Additive inverse

Scalar Multiplication Properties

1. $(cd)\mathbf{u} = c(d\mathbf{u})$ — Associative property
2. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$ — Distributive property
3. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$ — Distributive property
4. $1\mathbf{u} = \mathbf{u}$ — Multiplicative identity
5. $0\mathbf{u} = \mathbf{0}$ — Multiplication property of zero
6. $\|c\mathbf{v}\| = |c|\|\mathbf{v}\|$ — Magnitude property

5 Find the unit vector in the direction of **v**.

Unit Vectors

A **unit vector** is defined to be a vector whose magnitude is one. In many applications of vectors, it is helpful to find the unit vector that has the same direction as a given vector.

Discovery

To find out why the procedure in the box produces a unit vector, work Exercise 112 in Exercise Set 6.6.

Finding the Unit Vector that Has the Same Direction as a Given Nonzero Vector v

For any nonzero vector **v**, the vector

$$\frac{\mathbf{v}}{\|\mathbf{v}\|}$$

is the unit vector that has the same direction as **v**. To find this vector, divide **v** by its magnitude.

EXAMPLE 7 Finding a Unit Vector

Find the unit vector in the same direction as $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$. Then verify that the vector has magnitude 1.

Solution We find the unit vector in the same direction as **v** by dividing **v** by its magnitude. We first find the magnitude of **v**.

$$\|\mathbf{v}\| = \sqrt{a^2 + b^2} = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

The unit vector in the same direction as **v** is

$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{5\mathbf{i} - 12\mathbf{j}}{13} = \frac{5}{13}\mathbf{i} - \frac{12}{13}\mathbf{j}.$$ This is the scalar multiplication of v and $\frac{1}{13}$.

Now we must verify that the magnitude of this vector is 1. Recall that the magnitude of $a\mathbf{i} + b\mathbf{j}$ is $\sqrt{a^2 + b^2}$. Thus, the magnitude of $\frac{5}{13}\mathbf{i} - \frac{12}{13}\mathbf{j}$ is

$$\sqrt{\left(\frac{5}{13}\right)^2 + \left(-\frac{12}{13}\right)^2} = \sqrt{\frac{25}{169} + \frac{144}{169}} = \sqrt{\frac{169}{169}} = \sqrt{1} = 1.$$

Check Point 7 Find the unit vector in the same direction as $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j}$. Then verify that the vector has magnitude 1.

6 Write a vector in terms of its magnitude and direction.

Writing a Vector in Terms of Its Magnitude and Direction

Consider the vector $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$. The components a and b can be expressed in terms of the magnitude of **v** and the angle θ that **v** makes with the positive x-axis. This angle is called the **direction angle** of **v** and is shown in **Figure 6.59**. By the definitions of sine and cosine, we have

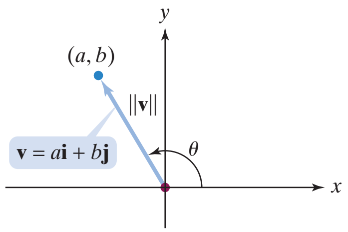


Figure 6.59 Expressing a vector in terms of its magnitude, $\|\mathbf{v}\|$, and its direction angle, θ

$$\cos\theta = \frac{a}{\|\mathbf{v}\|} \quad \text{and} \quad \sin\theta = \frac{b}{\|\mathbf{v}\|}$$

$$a = \|\mathbf{v}\|\cos\theta \qquad b = \|\mathbf{v}\|\sin\theta.$$

Thus,

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}.$$

Writing a Vector in Terms of Its Magnitude and Direction

Let **v** be a nonzero vector. If θ is the direction angle measured from the positive x-axis to **v**, then the vector can be expressed in terms of its magnitude and direction angle as

$$\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}.$$

A vector that represents the direction and speed of an object in motion is called a **velocity vector**. In Example 8, we express a wind's velocity vector in terms of the wind's magnitude and direction.

EXAMPLE 8 Writing a Vector Whose Magnitude and Direction Are Given

The wind is blowing at 20 miles per hour in the direction N30°W. Express its velocity as a vector $\mathbf{v}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

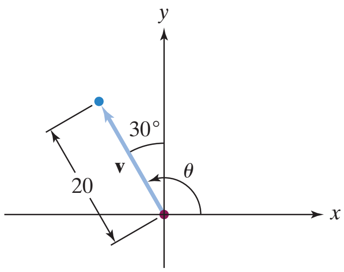


Figure 6.60 Vector $\mathbf{v}$ represents a wind blowing at 20 miles per hour in the direction N30°W.

Solution The vector $\mathbf{v}$ is shown in **Figure 6.60**. The vector's direction angle, from the positive x-axis to $\mathbf{v}$, is

$$\theta = 90° + 30° = 120°.$$

Because the wind is blowing at 20 miles per hour, the magnitude of $\mathbf{v}$ is 20 miles per hour: $\|\mathbf{v}\| = 20$. Thus,

$$\mathbf{v} = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j} \quad \text{Use the formula for a vector in terms of magnitude and direction.}$$

$$= 20\cos 120°\mathbf{i} + 20\sin 120°\mathbf{j} \quad \|\mathbf{v}\| = 20 \text{ and } \theta = 120°.$$

$$= 20\left(-\frac{1}{2}\right)\mathbf{i} + 20\left(\frac{\sqrt{3}}{2}\right)\mathbf{j} \quad \cos 120° = -\frac{1}{2} \text{ and } \sin 120° = \frac{\sqrt{3}}{2}.$$

$$= -10\mathbf{i} + 10\sqrt{3}\mathbf{j} \quad \text{Simplify.}$$

The wind's velocity can be expressed in terms of $\mathbf{i}$ and $\mathbf{j}$ as $\mathbf{v} = -10\mathbf{i} + 10\sqrt{3}\mathbf{j}$.

Check Point 8 The jet stream is blowing at 60 miles per hour in the direction N45°E. Express its velocity as a vector $\mathbf{v}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

7 Solve applied problems involving vectors.

Application

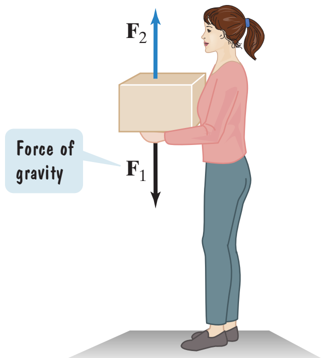


Figure 6.61 Force vectors

Many physical concepts can be represented by vectors. A vector that represents a pull or push of some type is called a **force vector**. If you are holding a 10-pound package, two force vectors are involved. The force of gravity is exerting a force of magnitude 10 pounds directly downward. This force is shown by vector $\mathbf{F}_1$ in **Figure 6.61**. Assuming there is no upward or downward movement of the package, you are exerting a force of magnitude 10 pounds directly upward. This force is shown by vector $\mathbf{F}_2$ in **Figure 6.61**. It has the same magnitude as the force exerted on your package by gravity, but it acts in the opposite direction.

If $\mathbf{F}_1$ and $\mathbf{F}_2$ are two forces acting on an object, the net effect is the same as if just the resultant force, $\mathbf{F}_1 + \mathbf{F}_2$, acted on the object. If the object is not moving, as is the case with your 10-pound package, the vector sum of all forces is the zero vector.

EXAMPLE 9 Finding the Resultant Force

Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, of magnitude 10 and 30 pounds, respectively, act on an object. The direction of $\mathbf{F}_1$ is N20°E and the direction of $\mathbf{F}_2$ is N65°E. Find the magnitude and the direction of the resultant force. Express the magnitude to the nearest hundredth of a pound and the direction angle to the nearest tenth of a degree.

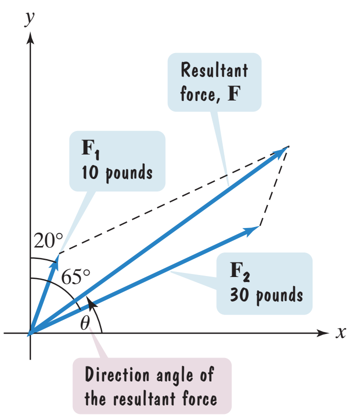


Figure 6.62

Solution The vectors $\mathbf{F}_1$ and $\mathbf{F}_2$ are shown in **Figure 6.62**. The direction angle for $\mathbf{F}_1$, from the positive x-axis to the vector, is $90° - 20°$, or 70°. We express $\mathbf{F}_1$ using the formula for a vector in terms of its magnitude and direction.

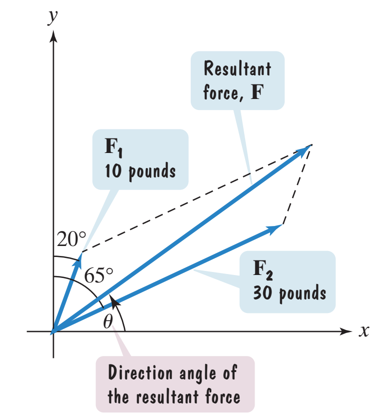


Figure 6.62 (repeated)

$$\mathbf{F}_1 = \|\mathbf{F}_1\|\cos\theta\mathbf{i} + \|\mathbf{F}_1\|\sin\theta\mathbf{j}$$

$$= 10\cos 70°\mathbf{i} + 10\sin 70°\mathbf{j} \qquad \|\mathbf{F}_1\| = 10 \text{ and } \theta = 70°.$$

$$\approx 3.42\mathbf{i} + 9.40\mathbf{j} \qquad \text{Use a calculator.}$$

Figure 6.62 illustrates that the direction angle for $\mathbf{F}_2$, from the positive x-axis to the vector, is $90° - 65°$, or $25°$. We express $\mathbf{F}_2$ using the formula for a vector in terms of its magnitude and direction.

$$\mathbf{F}_2 = \|\mathbf{F}_2\|\cos\theta\mathbf{i} + \|\mathbf{F}_2\|\sin\theta\mathbf{j}$$

$$= 30\cos 25°\mathbf{i} + 30\sin 25°\mathbf{j} \qquad \|\mathbf{F}_2\| = 30 \text{ and } \theta = 25°.$$

$$\approx 27.19\mathbf{i} + 12.68\mathbf{j} \qquad \text{Use a calculator.}$$

The resultant force, $\mathbf{F}$, is $\mathbf{F}_1 + \mathbf{F}_2$. Thus,

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

$$\approx (3.42\mathbf{i} + 9.40\mathbf{j}) + (27.19\mathbf{i} + 12.68\mathbf{j}) \qquad \text{Use } F_1 \text{ and } F_2\text{, found above.}$$

$$= (3.42 + 27.19)\mathbf{i} + (9.40 + 12.68)\mathbf{j} \qquad \text{Add the horizontal components. Add the vertical components.}$$

$$= 30.61\mathbf{i} + 22.08\mathbf{j}. \qquad \text{Simplify.}$$

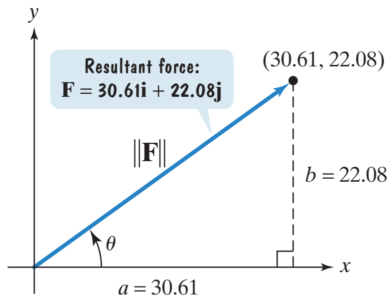


Figure 6.63

Figure 6.63 shows the resultant force, $\mathbf{F}$, without showing $\mathbf{F}_1$ and $\mathbf{F}_2$.

Now that we have the resultant force vector, $\mathbf{F}$, we can find its magnitude.

$$\|\mathbf{F}\| = \sqrt{a^2 + b^2} = \sqrt{(30.61)^2 + (22.08)^2} \approx 37.74$$

The magnitude of the resultant force is approximately 37.74 pounds.

To find θ, the direction angle of the resultant force, we can use

$$\cos\theta = \frac{a}{\|\mathbf{F}\|} \quad \text{or} \quad \sin\theta = \frac{b}{\|\mathbf{F}\|}.$$

These ratios are illustrated for the right triangle in **Figure 6.63**.

Using the first formula, we obtain

$$\cos\theta = \frac{a}{\|\mathbf{F}\|} \approx \frac{30.61}{37.74}.$$

Thus,

$$\theta = \cos^{-1}\left(\frac{30.61}{37.74}\right) \approx 35.8°. \qquad \text{Use a calculator.}$$

The direction angle of the resultant force is approximately 35.8°.

In summary, the two given forces are equivalent to a single force of approximately 37.74 pounds with a direction angle of approximately 35.8°.

Study Tip

If $\mathbf{F} = a\mathbf{i} + b\mathbf{j}$, the direction angle, θ, of $\mathbf{F}$ can also be found using

$$\tan\theta = \frac{b}{a}.$$

Check Point 9 Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, of magnitude 30 and 60 pounds, respectively, act on an object. The direction of $\mathbf{F}_1$ is N10°E and the direction of $\mathbf{F}_2$ is N60°E. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

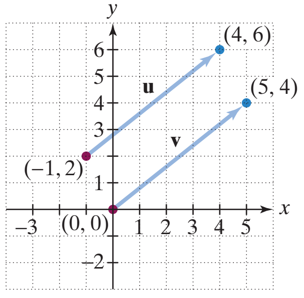
We have seen that velocity vectors represent the direction and speed of moving objects. Boats moving in currents and airplanes flying in winds are situations in which two velocity vectors act simultaneously. For example, suppose $\mathbf{v}$ represents the velocity of a plane in still air. Further suppose that $\mathbf{w}$ represents the velocity of the wind. The actual speed and direction of the plane is given by the vector $\mathbf{v} + \mathbf{w}$. This resultant vector describes the plane's speed and direction relative to the ground. Problems involving the resultant velocity of a boat or plane are solved using the same method that we used in Example 9 to find a single resultant force equivalent to two given forces.

Exercise Set 6.6

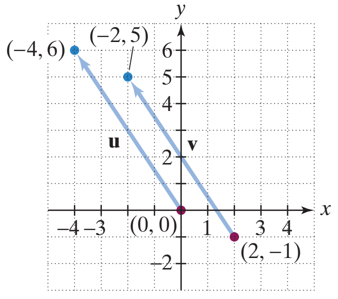
Practice Exercises

In Exercises 1–4, **u** *and* **v** *have the same direction. In each exercise:* **a.** *Find* $\|\mathbf{u}\|$. **b.** *Find* $\|\mathbf{v}\|$. **c.** *Is* $\mathbf{u} = \mathbf{v}$? *Explain.*

1.



2.



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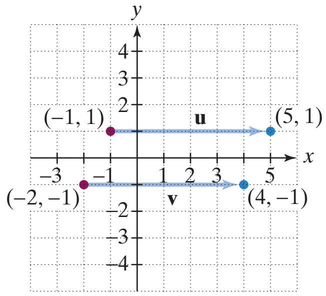

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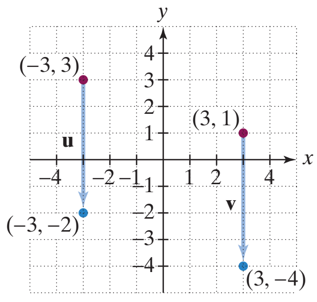


In Exercises 5–12, sketch each vector as a position vector and find its magnitude.

5. $\mathbf{v} = 3\mathbf{i} + \mathbf{j}$

6. $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$

7. $\mathbf{v} = \mathbf{i} - \mathbf{j}$

8. $\mathbf{v} = -\mathbf{i} - \mathbf{j}$

9. $\mathbf{v} = -6\mathbf{i} - 2\mathbf{j}$

10. $\mathbf{v} = 5\mathbf{i} - 2\mathbf{j}$

11. $\mathbf{v} = -4\mathbf{i}$

12. $\mathbf{v} = -5\mathbf{j}$

In Exercises 13–20, let **v** *be the vector from initial point* P_1 *to terminal point* P_2. *Write* **v** *in terms of* **i** *and* **j**.

13. $P_1 = (-4, -4)$, $P_2 = (6, 2)$

14. $P_1 = (2, -5)$, $P_2 = (-6, 6)$

15. $P_1 = (-8, 6)$, $P_2 = (-2, 3)$

16. $P_1 = (-7, -4)$, $P_2 = (0, -2)$

17. $P_1 = (-1, 7)$, $P_2 = (-7, -7)$

18. $P_1 = (-1, 6)$, $P_2 = (7, -5)$

19. $P_1 = (-3, 4)$, $P_2 = (6, 4)$

20. $P_1 = (4, -5)$, $P_2 = (4, 3)$

In Exercises 21–38, let

$$\mathbf{u} = 2\mathbf{i} - 5\mathbf{j},\ \mathbf{v} = -3\mathbf{i} + 7\mathbf{j},\ \text{and}\ \mathbf{w} = -\mathbf{i} - 6\mathbf{j}.$$

Find each specified vector or scalar.

21. $\mathbf{u} + \mathbf{v}$

22. $\mathbf{v} + \mathbf{w}$

23. $\mathbf{u} - \mathbf{v}$

24. $\mathbf{v} - \mathbf{w}$

25. $\mathbf{v} - \mathbf{u}$

26. $\mathbf{w} - \mathbf{v}$

27. $5\mathbf{v}$

28. $6\mathbf{v}$

29. $-4\mathbf{w}$

30. $-7\mathbf{w}$

31. $3\mathbf{w} + 2\mathbf{v}$

32. $3\mathbf{u} + 4\mathbf{v}$

33. $3\mathbf{v} - 4\mathbf{w}$

34. $4\mathbf{w} - 3\mathbf{v}$

35. $\|2\mathbf{u}\|$

36. $\|-2\mathbf{u}\|$

37. $\|\mathbf{w} - \mathbf{u}\|$

38. $\|\mathbf{u} - \mathbf{w}\|$

In Exercises 39–46, find the unit vector that has the same direction as the vector **v**.

39. $\mathbf{v} = 6\mathbf{i}$

40. $\mathbf{v} = -5\mathbf{j}$

41. $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$

42. $\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$

43. $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$

44. $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j}$

45. $\mathbf{v} = \mathbf{i} + \mathbf{j}$

46. $\mathbf{v} = \mathbf{i} - \mathbf{j}$

In Exercises 47–52, write the vector **v** *in terms of* **i** *and* **j** *whose magnitude* $\|\mathbf{v}\|$ *and direction angle* θ *are given.*

47. $\|\mathbf{v}\| = 6, \theta = 30°$

48. $\|\mathbf{v}\| = 8, \theta = 45°$

49. $\|\mathbf{v}\| = 12, \theta = 225°$

50. $\|\mathbf{v}\| = 10, \theta = 330°$

51. $\|\mathbf{v}\| = \frac{1}{2}, \theta = 113°$

52. $\|\mathbf{v}\| = \frac{1}{4}, \theta = 200°$

Practice Plus

In Exercises 53–56, let

$$\mathbf{u} = -2\mathbf{i} + 3\mathbf{j},\ \mathbf{v} = 6\mathbf{i} - \mathbf{j},\ \mathbf{w} = -3\mathbf{i}.$$

Find each specified vector or scalar.

53. $4\mathbf{u} - (2\mathbf{v} - \mathbf{w})$

54. $3\mathbf{u} - (4\mathbf{v} - \mathbf{w})$

55. $\|\mathbf{u} + \mathbf{v}\|^2 - \|\mathbf{u} - \mathbf{v}\|^2$

56. $\|\mathbf{v} + \mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2$

In Exercises 57–60, let

$$\mathbf{u} = a_1\mathbf{i} + b_1\mathbf{j}$$
$$\mathbf{v} = a_2\mathbf{i} + b_2\mathbf{j}$$
$$\mathbf{w} = a_3\mathbf{i} + b_3\mathbf{j}.$$

Prove each property by obtaining the vector on each side of the equation. Have you proved a distributive, associative, or commutative property of vectors?

57. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

58. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

59. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

60. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

In Exercises 61–64, find the magnitude $\|\mathbf{v}\|$*, to the nearest hundredth, and the direction angle* θ*, to the nearest tenth of a degree, for each given vector* $\mathbf{v}$.

61. $\mathbf{v} = -10\mathbf{i} + 15\mathbf{j}$

62. $\mathbf{v} = 2\mathbf{i} - 8\mathbf{j}$

63. $\mathbf{v} = (4\mathbf{i} - 2\mathbf{j}) - (4\mathbf{i} - 8\mathbf{j})$

64. $\mathbf{v} = (7\mathbf{i} - 3\mathbf{j}) - (10\mathbf{i} - 3\mathbf{j})$

Application Exercises

In Exercises 65–68, a vector is described. Express the vector in terms of **i** *and* **j**. *If exact values are not possible, round components to the nearest tenth.*

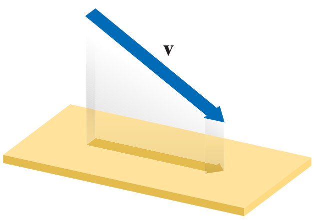
65. A quarterback releases a football with a speed of 44 feet per second at an angle of 30° with the horizontal.

66. A child pulls a sled along level ground by exerting a force of 30 pounds on a handle that makes an angle of 45° with the ground.

67. A plane approaches a runway at 150 miles per hour at an angle of 8° with the runway.

68. A plane with an airspeed of 450 miles per hour is flying in the direction N35°W.

Vectors are used in computer graphics to determine lengths of shadows over flat surfaces. The length of the shadow for **v** *in the figure shown is the absolute value of the vector's horizontal component. In Exercises 69–70, the magnitude and direction angle of* **v** *are given. Write* **v** *in terms of* **i** *and* **j**. *Then find the length of the shadow to the nearest tenth of an inch.*



69. $\|\mathbf{v}\| = 1.5$ inches, $\theta = 25°$

70. $\|\mathbf{v}\| = 1.8$ inches, $\theta = 40°$

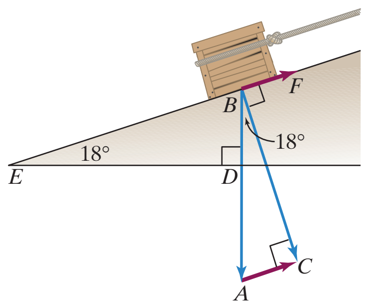
71. The magnitude and direction of two forces acting on an object are 70 pounds, S56°E, and 50 pounds, N72°E, respectively. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

72. The magnitude and direction exerted by two tugboats towing a ship are 4200 pounds, N65°E, and 3000 pounds, S58°E, respectively. Find the magnitude, to the nearest pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

73. The magnitude and direction exerted by two tugboats towing a ship are 1610 kilograms, N35°W, and 1250 kilograms, S55°W, respectively. Find the magnitude, to the nearest kilogram, and the direction angle, to the nearest tenth of a degree, of the resultant force.

74. The magnitude and direction of two forces acting on an object are 64 kilograms, N39°W, and 48 kilograms, S59°W, respectively. Find the magnitude, to the nearest hundredth of a kilogram, and the direction angle, to the nearest tenth of a degree, of the resultant force.

The figure shows a box being pulled up a ramp inclined at 18° from the horizontal.



Use the following information to solve Exercises 75–76.

$\overrightarrow{BA}$ = force of gravity

$\|\overrightarrow{BA}\|$ = weight of the box

$\|\overrightarrow{AC}\|$ = magnitude of the force needed to pull the box up the ramp

$\|\overrightarrow{BC}\|$ = magnitude of the force of the box against the ramp

75. If the box weighs 100 pounds, find the magnitude of the force needed to pull it up the ramp.

76. If a force of 30 pounds is needed to pull the box up the ramp, find the weight of the box.

In Exercises 77–78, round answers to the nearest pound.

77. **a.** Find the magnitude of the force required to keep a 3500-pound car from sliding down a hill inclined at 5.5° from the horizontal.

b. Find the magnitude of the force of the car against the hill.

78. **a.** Find the magnitude of the force required to keep a 280-pound barrel from sliding down a ramp inclined at 12.5° from the horizontal.

b. Find the magnitude of the force of the barrel against the ramp.

The forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \dots, \mathbf{F}_n$ *acting on an object are in* **equilibrium** *if the resultant force is the zero vector:*

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots + \mathbf{F}_n = \mathbf{0}.$$

In Exercises 79–82, the given forces are acting on an object.

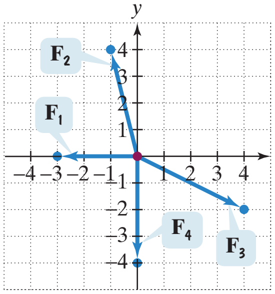
a. *Find the resultant force.*

b. *What additional force is required for the given forces to be in equilibrium?*

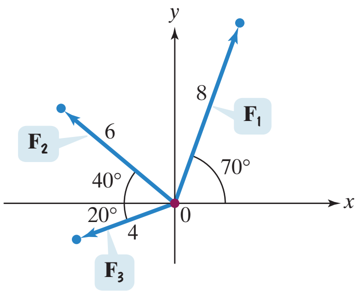
79. $\mathbf{F}_1 = 3\mathbf{i} - 5\mathbf{j}, \quad \mathbf{F}_2 = 6\mathbf{i} + 2\mathbf{j}$

80. $\mathbf{F}_1 = -2\mathbf{i} + 3\mathbf{j}, \quad \mathbf{F}_2 = \mathbf{i} - \mathbf{j}, \quad \mathbf{F}_3 = 5\mathbf{i} - 12\mathbf{j}$

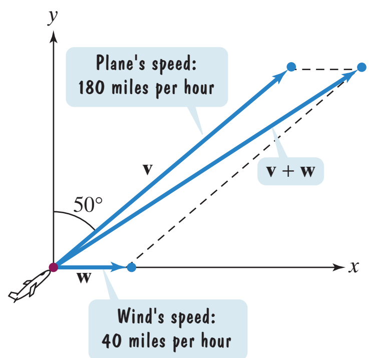
81.



82.



83. The figure shows a small plane flying at a speed of 180 miles per hour on a bearing of N50°E. The wind is blowing from west to east at 40 miles per hour. The figure indicates that **v** represents the velocity of the plane in still air and **w** represents the velocity of the wind.



a. Express **v** and **w** in terms of their magnitudes and direction angles.

b. Find the resultant vector, $\mathbf{v} + \mathbf{w}$.

c. The magnitude of $\mathbf{v} + \mathbf{w}$, called the **ground speed** of the plane, gives its speed relative to the ground. Approximate the ground speed to the nearest mile per hour.

d. The direction angle of $\mathbf{v} + \mathbf{w}$ gives the plane's true course relative to the ground. Approximate the true course to the nearest tenth of a degree. What is the plane's true bearing?

84. Use the procedure outlined in Exercise 83 to solve this exercise. A plane is flying at a speed of 400 miles per hour on a bearing of N50°W. The wind is blowing at 30 miles per hour on a bearing of N25°E.

a. Approximate the plane's ground speed to the nearest mile per hour.

b. Approximate the plane's true course to the nearest tenth of a degree. What is its true bearing?

85. A plane is flying at a speed of 320 miles per hour on a bearing of N70°E. Its ground speed is 370 miles per hour and its true course is 30°. Find the speed, to the nearest mile per hour, and the direction angle, to the nearest tenth of a degree, of the wind.

86. A plane is flying at a speed of 540 miles per hour on a bearing of S36°E. Its ground speed is 500 miles per hour and its true bearing is S44°E. Find the speed, to the nearest mile per hour, and the direction angle, to the nearest tenth of a degree, of the wind.

Writing in Mathematics

87. What is a directed line segment?

88. What are equal vectors?

89. If vector **v** is represented by an arrow, how is $-3\mathbf{v}$ represented?

90. If vectors **u** and **v** are represented by arrows, describe how the vector sum $\mathbf{u} + \mathbf{v}$ is represented.

91. What is the vector **i**?

92. What is the vector **j**?

93. What is a position vector? How is a position vector represented using **i** and **j**?

94. If **v** is a vector between any two points in the rectangular coordinate system, explain how to write **v** in terms of **i** and **j**.

95. If two vectors are expressed in terms of **i** and **j**, explain how to find their sum.

96. If two vectors are expressed in terms of **i** and **j**, explain how to find their difference.

97. If a vector is expressed in terms of **i** and **j**, explain how to find the scalar multiplication of the vector and a given scalar k.

98. What is the zero vector?

99. Describe one similarity between the zero vector and the number 0.

100. Explain how to find the unit vector in the direction of any given vector **v**.

101. Explain how to write a vector in terms of its magnitude and direction.

102. You are on an airplane. The pilot announces the plane's speed over the intercom. Which speed do you think is being reported: the speed of the plane in still air or the speed after the effect of the wind has been accounted for? Explain your answer.

103. Use vectors to explain why it is difficult to hold a heavy stack of books perfectly still for a long period of time. As you become exhausted, what eventually happens? What does this mean in terms of the forces acting on the books?

Critical Thinking Exercises

Make Sense? *In Exercises 104–107, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

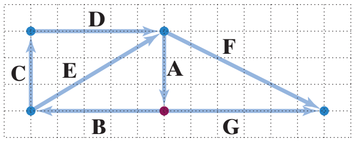
104. I used a vector to represent a wind velocity of 13 miles per hour from the west.

105. I used a vector to represent the average yearly rate of change in a man's height between ages 13 and 18.

106. Once I've found a unit vector $\mathbf{u}$, the vector $-\mathbf{u}$ must also be a unit vector.

107. The resultant force of two forces that each have a magnitude of one pound is a vector whose magnitude is two pounds.

In Exercises 108–111, use the figure shown to determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.



108. $\mathbf{A} + \mathbf{B} = \mathbf{E}$

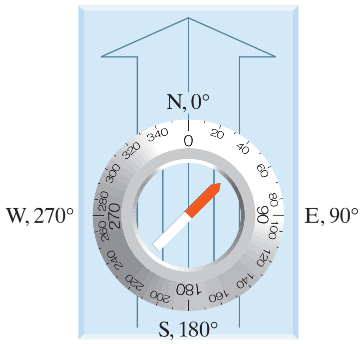
109. $\mathbf{D} + \mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$

110. $\mathbf{B} - \mathbf{E} = \mathbf{G} - \mathbf{F}$

111. $\|\mathbf{A}\| = \|\mathbf{C}\|$

112. Let $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$. Show that $\dfrac{\mathbf{v}}{\|\mathbf{v}\|}$ is a unit vector in the direction of $\mathbf{v}$.

In Exercises 113–114, refer to the navigational compass shown in the figure. The compass is marked clockwise in degrees that start at north 0°.



113. An airplane has an air speed of 240 miles per hour and a compass heading of 280°. A steady wind of 30 miles per hour is blowing in the direction of 265°. What is the plane's true speed relative to the ground? What is its compass heading relative to the ground?

114. Two tugboats are pulling on a large ship that has gone aground. One tug pulls with a force of 2500 pounds in a compass direction of 55°. The second tug pulls with a force of 2000 pounds in a compass direction of 95°. Find the magnitude and the compass direction of the resultant force.

115. You want to fly your small plane due north, but there is a 75 kilometer wind blowing from west to east.

a. Find the direction angle for where you should head the plane if your speed relative to the ground is 310 kilometers per hour.

b. If you increase your air speed, should the direction angle in part (a) increase or decrease? Explain your answer.

Preview Exercises

Exercises 116–118 will help you prepare for the material covered in the next section.

116. Find the obtuse angle θ, rounded to the nearest tenth of a degree, satisfying

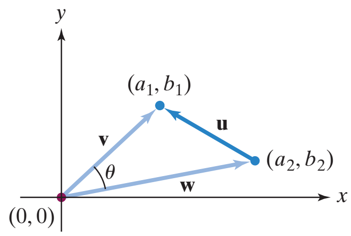
$$\cos\theta = \frac{3(-1) + (-2)(4)}{\|\mathbf{v}\|\|\mathbf{w}\|},$$

where $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{w} = -\mathbf{i} + 4\mathbf{j}$.

117. If $\mathbf{w} = -2\mathbf{i} + 6\mathbf{j}$, find the following vector:

$$\frac{2(-2) + 4(-6)}{\|\mathbf{w}\|^2}\mathbf{w}.$$

118. Consider the triangle formed by vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.



a. Use the magnitudes of the three vectors to write the Law of Cosines for the triangle shown in the figure: $\|\mathbf{u}\|^2 = ?$.

b. Use the coordinates of the points shown in the figure to write algebraic expressions for $\|\mathbf{u}\|$, $\|\mathbf{u}\|^2$, $\|\mathbf{v}\|$, $\|\mathbf{v}\|^2$, $\|\mathbf{w}\|$, and $\|\mathbf{w}\|^2$.