

## Preview Exercises

Exercises 87–89 will help you prepare for the material covered in the first section of the next chapter.

87. a. Does  $(4, -1)$  satisfy  $x + 2y = 2$ ?  
 b. Does  $(4, -1)$  satisfy  $x - 2y = 6$ ?
88. Graph  $x + 2y = 2$  and  $x - 2y = 6$  in the same rectangular coordinate system. At what point do the graphs intersect?
89. Solve:  $5(2x - 3) - 4x = 9$ .

# Chapter 6

# Summary, Review, and Test

## Summary

### DEFINITIONS AND CONCEPTS

### EXAMPLES

#### 6.1 and 6.2 The Law of Sines; The Law of Cosines

- a. The Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Ex. 1, p. 645;  
 Ex. 2, p. 646;  
 Ex. 3, p. 647;  
 Ex. 4, p. 648

- b. The Law of Sines is used to solve SAA, ASA, and SSA (the ambiguous case) triangles. The ambiguous case may result in no triangle, one triangle, or two triangles; see the box on page 647.

Ex. 5, p. 649

- c. The area of a triangle equals one-half the product of the lengths of two sides times the sine of their included angle.

Ex. 6, p. 650

- d. The Law of Cosines

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ b^2 &= a^2 + c^2 - 2ac \cos B \\ c^2 &= a^2 + b^2 - 2ab \cos C \end{aligned}$$

- e. The Law of Cosines is used to find the side opposite the given angle in an SAS triangle; see the box on page 657. The Law of Cosines is also used to find the angle opposite the longest side in an SSS triangle; see the box on page 658.

Ex. 1, p. 658;  
 Ex. 2, p. 659

- f. Heron's Formula for the Area of a Triangle

The area of a triangle with sides  $a$ ,  $b$ , and  $c$  is  $\sqrt{s(s-a)(s-b)(s-c)}$ , where  $s$  is one-half its perimeter:  
 $s = \frac{1}{2}(a + b + c)$ .

Ex. 4, p. 660

#### 6.3 and 6.4 Polar Coordinates; Graphs of Polar Equations

- a. A point  $P$  in the polar coordinate system is represented by  $(r, \theta)$ , where  $r$  is the directed distance from the pole to the point and  $\theta$  is the angle from the polar axis to line segment  $OP$ . The elements of the ordered pair  $(r, \theta)$  are called the polar coordinates of  $P$ . See Figure 6.20 on page 664. When  $r$  in  $(r, \theta)$  is negative, a point is located  $|r|$  units along the ray opposite the terminal side of  $\theta$ . Important information about the sign of  $r$  and the location of the point  $(r, \theta)$  is found in the box on page 665.

Ex. 1, p. 665

- b. Multiple Representations of Points

If  $n$  is any integer,  $(r, \theta) = (r, \theta + 2n\pi)$  or  $(r, \theta) = (-r, \theta + \pi + 2n\pi)$ .

Ex. 2, p. 666

- c. Relations between Polar and Rectangular Coordinates

$$x = r \cos \theta, \quad y = r \sin \theta, \quad x^2 + y^2 = r^2, \quad \tan \theta = \frac{y}{x}$$

- d. To convert a point from polar coordinates  $(r, \theta)$  to rectangular coordinates  $(x, y)$ , use  $x = r \cos \theta$  and  $y = r \sin \theta$ .

Ex. 3, p. 667

- e. To convert a point from rectangular coordinates  $(x, y)$  to polar coordinates  $(r, \theta)$ , use the procedure in the box on page 668.

Ex. 4, p. 668;  
 Ex. 5, p. 669

- f. To convert a rectangular equation to a polar equation, replace  $x$  with  $r \cos \theta$  and  $y$  with  $r \sin \theta$ .

Ex. 6, p. 670

## DEFINITIONS AND CONCEPTS

## EXAMPLES

- g.** To convert a polar equation to a rectangular equation, use one or more of

$$r^2 = x^2 + y^2, \quad r \cos \theta = x, \quad r \sin \theta = y, \quad \text{and} \quad \tan \theta = \frac{y}{x}.$$

It is often necessary to do something to the given polar equation before using the preceding expressions.

Ex. 7, p. 671

- h.** A polar equation is an equation whose variables are  $r$  and  $\theta$ . The graph of a polar equation is the set of all points whose polar coordinates satisfy the equation.

Ex. 1, p. 675

- i.** Polar equations can be graphed using point plotting and symmetry (see the box on page 677).

Ex. 2, p. 677

- j.** The graphs of  $r = a \cos \theta$  and  $r = a \sin \theta$  are circles. See the box on page 676. The graphs of  $r = a \pm b \sin \theta$  and  $r = a \pm b \cos \theta$  are called limaçons ( $a > 0$  and  $b > 0$ ), shown in the box on page 679. The graphs of  $r = a \sin n\theta$  and  $r = a \cos n\theta$ ,  $a \neq 0$ , are rose curves with  $2n$  petals if  $n$  is even and  $n$  petals if  $n$  is odd. See the box on page 681. The graphs of  $r^2 = a^2 \sin 2\theta$  and  $r^2 = a^2 \cos 2\theta$ ,  $a \neq 0$ , are called lemniscates and are shown in the box on page 682.

Ex. 3, p. 678;

Ex. 4, p. 680;

Ex. 5, p. 681

## 6.5 Complex Numbers in Polar Form; DeMoivre's Theorem

- a.** The complex number  $z = a + bi$  is represented as a point  $(a, b)$  in the complex plane, shown in Figure 6.38 on page 687.

Ex. 1, p. 687

- b.** The absolute value of  $z = a + bi$  is  $|z| = |a + bi| = \sqrt{a^2 + b^2}$ .

Ex. 2, p. 687

- c.** The polar form of  $z = a + bi$  is  $z = r(\cos \theta + i \sin \theta)$ , where  $a = r \cos \theta$ ,  $b = r \sin \theta$ ,  $r = \sqrt{a^2 + b^2}$ , and  $\tan \theta = \frac{b}{a}$ . We call  $r$  the modulus and  $\theta$  the argument of  $z$ , with  $0 \leq \theta < 2\pi$ .

Ex. 3, p. 688;

Ex. 4, p. 689

- d.** Multiplying Complex Numbers in Polar Form: Multiply moduli and add arguments. See the box on page 690.

Ex. 5, p. 690

- e.** Dividing Complex Numbers in Polar Form: Divide moduli and subtract arguments. See the box on page 691.

Ex. 6, p. 691

- f.** DeMoivre's Theorem is used to find powers of complex numbers in polar form.

$$[r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta)$$

Ex. 7, p. 692;

Ex. 8, p. 692

- g.** DeMoivre's Theorem can be used to find roots of complex numbers in polar form. The  $n$  distinct  $n$ th roots of  $r(\cos \theta + i \sin \theta)$  are

Ex. 9, p. 693;

Ex. 10, p. 694

$$\sqrt[n]{r} \left[ \cos \left( \frac{\theta + 2\pi k}{n} \right) + i \sin \left( \frac{\theta + 2\pi k}{n} \right) \right]$$

or

$$\sqrt[n]{r} \left[ \cos \left( \frac{\theta + 360^\circ k}{n} \right) + i \sin \left( \frac{\theta + 360^\circ k}{n} \right) \right],$$

where  $k = 0, 1, 2, \dots, n - 1$ .

## 6.6 Vectors

- a.** A vector is a directed line segment.

- b.** Equal vectors have the same magnitude and the same direction.

Ex. 1, p. 699

- c.** The vector  $k\mathbf{v}$ , the scalar multiple of the vector  $\mathbf{v}$  and the scalar  $k$ , has magnitude  $|k||\mathbf{v}|$ . The direction of  $k\mathbf{v}$  is the same as that of  $\mathbf{v}$  if  $k > 0$  and opposite  $\mathbf{v}$  if  $k < 0$ .

Figure 6.52,  
p. 700

- d.** The sum  $\mathbf{u} + \mathbf{v}$ , called the resultant vector, can be expressed geometrically. Position  $\mathbf{u}$  and  $\mathbf{v}$  so that the terminal point of  $\mathbf{u}$  coincides with the initial point of  $\mathbf{v}$ . The vector  $\mathbf{u} + \mathbf{v}$  extends from the initial point of  $\mathbf{u}$  to the terminal point of  $\mathbf{v}$ .

Figure 6.53,  
p. 701

- e.** The difference of two vectors,  $\mathbf{u} - \mathbf{v}$ , is defined as  $\mathbf{u} + (-\mathbf{v})$ .

Figure 6.54,  
p. 701

- f.** The vector  $\mathbf{i}$  is the unit vector whose direction is along the positive  $x$ -axis. The vector  $\mathbf{j}$  is the unit vector whose direction is along the positive  $y$ -axis.

DEFINITIONS AND CONCEPTS	EXAMPLES
<p><b>g.</b> Vector <math>\mathbf{v}</math>, from <math>(0, 0)</math> to <math>(a, b)</math>, called a position vector, is represented as <math>\mathbf{v} = a\mathbf{i} + b\mathbf{j}</math>, where <math>a</math> is the horizontal component and <math>b</math> is the vertical component. The magnitude of <math>\mathbf{v}</math> is given by</p> $\ \mathbf{v}\  = \sqrt{a^2 + b^2}.$	Ex. 2, p. 702
<p><b>h.</b> Vector <math>\mathbf{v}</math> from <math>(x_1, y_1)</math> to <math>(x_2, y_2)</math> is equal to the position vector <math>\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}</math>. In rectangular coordinates, the term “vector” refers to the position vector in terms of <math>\mathbf{i}</math> and <math>\mathbf{j}</math> that is equal to it.</p>	Ex. 3, p. 703
<p><b>i.</b> Operations with Vectors in Terms of <math>\mathbf{i}</math> and <math>\mathbf{j}</math> If <math>\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}</math> and <math>\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}</math>, then</p> <ul style="list-style-type: none"> <li>• <math>\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j}</math></li> <li>• <math>\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j}</math></li> <li>• <math>k\mathbf{v} = (ka_1)\mathbf{i} + (kb_1)\mathbf{j}</math></li> </ul>	Ex. 4, p. 704; Ex. 5, p. 704; Ex. 6, p. 705
<p><b>j.</b> The zero vector <math>\mathbf{0}</math> is the vector whose magnitude is 0 and is assigned no direction. Many properties of vector addition and scalar multiplication involve the zero vector. Some of these properties are listed in the box on page 705.</p>	
<p><b>k.</b> The vector <math>\frac{\mathbf{v}}{\ \mathbf{v}\ }</math> is the unit vector that has the same direction as <math>\mathbf{v}</math>.</p>	Ex. 7, p. 706
<p><b>l.</b> A vector with magnitude <math>\ \mathbf{v}\ </math> and direction angle <math>\theta</math>, the angle that <math>\mathbf{v}</math> makes with the positive <math>x</math>-axis, can be expressed in terms of its magnitude and direction angle as</p> $\mathbf{v} = \ \mathbf{v}\  \cos \theta \mathbf{i} + \ \mathbf{v}\  \sin \theta \mathbf{j}.$	Ex. 8, p. 707; Ex. 9, p. 707
<b>6.7 The Dot Product</b>	
<p><b>a.</b> Definition of the Dot Product If <math>\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}</math> and <math>\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}</math>, the dot product of <math>\mathbf{v}</math> and <math>\mathbf{w}</math> is defined by <math>\mathbf{v} \cdot \mathbf{w} = a_1a_2 + b_1b_2</math>.</p>	Ex. 1, p. 713
<p><b>b.</b> Alternative Formula for the Dot Product: <math>\mathbf{v} \cdot \mathbf{w} = \ \mathbf{v}\ \ \mathbf{w}\  \cos \theta</math>, where <math>\theta</math> is the smallest nonnegative angle between <math>\mathbf{v}</math> and <math>\mathbf{w}</math></p>	
<p><b>c.</b> Angle between Two Vectors</p> $\cos \theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\ \mathbf{v}\ \ \mathbf{w}\ } \quad \text{and} \quad \theta = \cos^{-1}\left(\frac{\mathbf{v} \cdot \mathbf{w}}{\ \mathbf{v}\ \ \mathbf{w}\ }\right)$	Ex. 2, p. 715
<p><b>d.</b> Two vectors are orthogonal when the angle between them is <math>90^\circ</math>. To show that two vectors are orthogonal, show that their dot product is zero.</p>	Ex. 3, p. 716
<p><b>e.</b> The vector projection of <math>\mathbf{v}</math> onto <math>\mathbf{w}</math> is given by</p> $\text{proj}_{\mathbf{w}} \mathbf{v} = \frac{\mathbf{v} \cdot \mathbf{w}}{\ \mathbf{w}\ ^2} \mathbf{w}.$	Ex. 4, p. 717
<p><b>f.</b> Expressing a vector as the sum of two orthogonal vectors, called the vector components, is shown in the box on page 717.</p>	Ex. 5, p. 718
<p><b>g.</b> The work, <math>W</math>, done by a force <math>\mathbf{F}</math> moving an object from <math>A</math> to <math>B</math> is <math>W = \mathbf{F} \cdot \overrightarrow{AB}</math>. Thus, <math>W = \ \mathbf{F}\ \ \overrightarrow{AB}\  \cos \theta</math>, where <math>\theta</math> is the angle between the force and the direction of motion.</p>	Ex. 6, p. 719

## Review Exercises

### 6.1 and 6.2

In Exercises 1–12, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree. If no triangle exists, state “no triangle.” If two triangles exist, solve each triangle.

1.  $A = 70^\circ, B = 55^\circ, a = 12$
2.  $B = 107^\circ, C = 30^\circ, c = 126$
3.  $B = 66^\circ, a = 17, c = 12$
4.  $a = 117, b = 66, c = 142$
5.  $A = 35^\circ, B = 25^\circ, c = 68$
6.  $A = 39^\circ, a = 20, b = 26$
7.  $C = 50^\circ, a = 3, c = 1$
8.  $A = 162^\circ, b = 11.2, c = 48.2$

9.  $a = 26.1, b = 40.2, c = 36.5$

10.  $A = 40^\circ, a = 6, b = 4$

11.  $B = 37^\circ, a = 12.4, b = 8.7$

12.  $A = 23^\circ, a = 54.3, b = 22.1$

In Exercises 13–16, find the area of the triangle having the given measurements. Round to the nearest square unit.

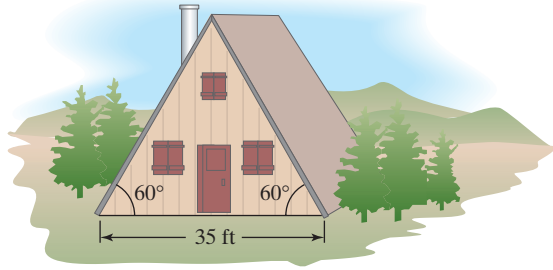
13.  $C = 42^\circ, a = 4$  feet,  $b = 6$  feet

14.  $A = 22^\circ, b = 4$  feet,  $c = 5$  feet

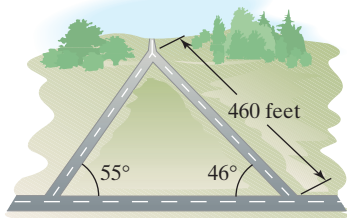
15.  $a = 2$  meters,  $b = 4$  meters,  $c = 5$  meters

16.  $a = 2$  meters,  $b = 2$  meters,  $c = 2$  meters

17. The A-frame cabin shown below is 35 feet wide. The roof of the cabin makes a  $60^\circ$  angle with the cabin's base. Find the length of one side of the roof from its ground level to the peak. Round to the nearest tenth of a foot.



18. Two cars leave a city at the same time and travel along straight highways that differ in direction by  $80^\circ$ . One car averages 60 miles per hour and the other averages 50 miles per hour. How far apart will the cars be after 30 minutes? Round to the nearest tenth of a mile.
19. Two airplanes leave an airport at the same time on different runways. One flies on a bearing of  $N66.5^\circ W$  at 325 miles per hour. The other airplane flies on a bearing of  $S26.5^\circ W$  at 300 miles per hour. How far apart will the airplanes be after two hours?
20. The figure shows three roads that intersect to bound a triangular piece of land. Find the lengths of the other two sides of the land to the nearest foot.



21. A commercial piece of real estate is priced at \$5.25 per square foot. Find the cost, to the nearest dollar, of a triangular lot measuring 260 feet by 320 feet by 450 feet.

**6.3 and 6.4**

In Exercises 22–27, plot each point in polar coordinates and find its rectangular coordinates.

22.  $(4, 60^\circ)$       23.  $(3, 150^\circ)$       24.  $\left(-4, \frac{4\pi}{3}\right)$   
 25.  $\left(-2, \frac{5\pi}{4}\right)$       26.  $\left(-4, -\frac{\pi}{2}\right)$       27.  $\left(-2, -\frac{\pi}{4}\right)$

In Exercises 28–30, plot each point in polar coordinates. Then find another representation  $(r, \theta)$  of this point in which:

- a.  $r > 0, 2\pi < \theta < 4\pi$ .  
 b.  $r < 0, 0 < \theta < 2\pi$ .  
 c.  $r > 0, -2\pi < \theta < 0$ .

28.  $\left(3, \frac{\pi}{6}\right)$       29.  $\left(2, \frac{2\pi}{3}\right)$       30.  $\left(3.5, \frac{\pi}{2}\right)$

In Exercises 31–36, the rectangular coordinates of a point are given. Find polar coordinates of each point.

31.  $(-4, 4)$       32.  $(3, -3)$       33.  $(5, 12)$   
 34.  $(-3, 4)$       35.  $(0, -5)$       36.  $(1, 0)$

In Exercises 37–39, convert each rectangular equation to a polar equation that expresses  $r$  in terms of  $\theta$ .

37.  $2x + 3y = 8$       38.  $x^2 + y^2 = 100$   
 39.  $(x - 6)^2 + y^2 = 36$

In Exercises 40–46, convert each polar equation to a rectangular equation. Then use your knowledge of the rectangular equation to graph the polar equation in a polar coordinate system.

40.  $r = 3$       41.  $\theta = \frac{3\pi}{4}$   
 42.  $r \cos \theta = -1$       43.  $r = 5 \csc \theta$   
 44.  $r = 3 \cos \theta$       45.  $4r \cos \theta + r \sin \theta = 8$   
 46.  $r^2 \sin 2\theta = -2$

In Exercises 47–49, test for symmetry with respect to

- a. the polar axis.      b. the line  $\theta = \frac{\pi}{2}$ .  
 c. the pole.

47.  $r = 5 + 3 \cos \theta$       48.  $r = 3 \sin \theta$   
 49.  $r^2 = 9 \cos 2\theta$

In Exercises 50–56, graph each polar equation. Be sure to test for symmetry.

50.  $r = 3 \cos \theta$       51.  $r = 2 + 2 \sin \theta$   
 52.  $r = \sin 2\theta$       53.  $r = 2 + \cos \theta$   
 54.  $r = 1 + 3 \sin \theta$       55.  $r = 1 - 2 \cos \theta$   
 56.  $r^2 = \cos 2\theta$

**6.5**

In Exercises 57–60, plot each complex number. Then write the complex number in polar form. You may express the argument in degrees or radians.

57.  $1 - i$       58.  $-2\sqrt{3} + 2i$   
 59.  $-3 - 4i$       60.  $-5i$

In Exercises 61–64, write each complex number in rectangular form. If necessary, round to the nearest tenth.

61.  $8(\cos 60^\circ + i \sin 60^\circ)$       62.  $4(\cos 210^\circ + i \sin 210^\circ)$   
 63.  $6\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$       64.  $0.6(\cos 100^\circ + i \sin 100^\circ)$

In Exercises 65–67, find the product of the complex numbers. Leave answers in polar form.

65.  $z_1 = 3(\cos 40^\circ + i \sin 40^\circ)$   
 $z_2 = 5(\cos 70^\circ + i \sin 70^\circ)$   
 66.  $z_1 = \cos 210^\circ + i \sin 210^\circ$   
 $z_2 = \cos 55^\circ + i \sin 55^\circ$   
 67.  $z_1 = 4\left(\cos \frac{3\pi}{7} + i \sin \frac{3\pi}{7}\right)$   
 $z_2 = 10\left(\cos \frac{4\pi}{7} + i \sin \frac{4\pi}{7}\right)$

In Exercises 68–70, find the quotient  $\frac{z_1}{z_2}$  of the complex numbers. Leave answers in polar form.

68.  $z_1 = 10(\cos 10^\circ + i \sin 10^\circ)$   
 $z_2 = 5(\cos 5^\circ + i \sin 5^\circ)$   
 69.  $z_1 = 5\left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$   
 $z_2 = 10\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$   
 70.  $z_1 = 2\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$   
 $z_2 = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$

In Exercises 71–75, use DeMoivre's Theorem to find the indicated power of the complex number. Write answers in rectangular form.

71.  $[2(\cos 20^\circ + i \sin 20^\circ)]^3$       72.  $[4(\cos 50^\circ + i \sin 50^\circ)]^3$

73.  $\left[\frac{1}{2}\left(\cos \frac{\pi}{14} + i \sin \frac{\pi}{14}\right)\right]^7$       74.  $(1 - \sqrt{3}i)^2$

75.  $(-2 - 2i)^5$

In Exercises 76–77, find all the complex roots. Write roots in polar form with  $\theta$  in degrees.

76. The complex square roots of  $49(\cos 50^\circ + i \sin 50^\circ)$

77. The complex cube roots of  $125(\cos 165^\circ + i \sin 165^\circ)$

In Exercises 78–81, find all the complex roots. Write roots in rectangular form.

78. The complex fourth roots of  $16\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

79. The complex cube roots of  $8i$

80. The complex cube roots of  $-1$

81. The complex fifth roots of  $-1 - i$

## 6.6

In Exercises 82–84, sketch each vector as a position vector and find its magnitude.

82.  $\mathbf{v} = -3\mathbf{i} - 4\mathbf{j}$       83.  $\mathbf{v} = 5\mathbf{i} - 2\mathbf{j}$

84.  $\mathbf{v} = -3\mathbf{j}$

In Exercises 85–86, let  $\mathbf{v}$  be the vector from initial point  $P_1$  to terminal point  $P_2$ . Write  $\mathbf{v}$  in terms of  $\mathbf{i}$  and  $\mathbf{j}$ .

85.  $P_1 = (2, -1)$ ,  $P_2 = (5, -3)$

86.  $P_1 = (-3, 0)$ ,  $P_2 = (-2, -2)$

In Exercises 87–90, let

$$\mathbf{v} = \mathbf{i} - 5\mathbf{j} \quad \text{and} \quad \mathbf{w} = -2\mathbf{i} + 7\mathbf{j}.$$

Find each specified vector or scalar.

87.  $\mathbf{v} + \mathbf{w}$       88.  $\mathbf{w} - \mathbf{v}$

89.  $6\mathbf{v} - 3\mathbf{w}$       90.  $\|-2\mathbf{v}\|$

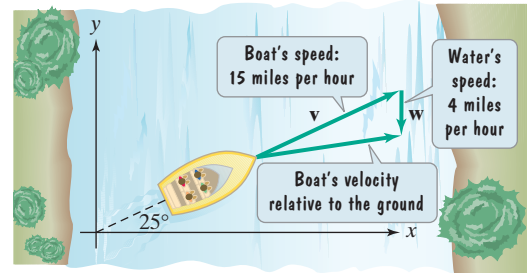
In Exercises 91–92, find the unit vector that has the same direction as the vector  $\mathbf{v}$ .

91.  $\mathbf{v} = 8\mathbf{i} - 6\mathbf{j}$       92.  $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$

93. The magnitude and direction angle of  $\mathbf{v}$  are  $\|\mathbf{v}\| = 12$  and  $\theta = 60^\circ$ . Express  $\mathbf{v}$  in terms of  $\mathbf{i}$  and  $\mathbf{j}$ .

94. The magnitude and direction of two forces acting on an object are 100 pounds,  $N25^\circ E$ , and 200 pounds,  $N80^\circ E$ , respectively. Find the magnitude, to the nearest pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.

95. Your boat is moving at a speed of 15 miles per hour at an angle of  $25^\circ$  upstream on a river flowing at 4 miles per hour. The situation is illustrated in the figure below.



- Find the vector representing your boat's velocity relative to the ground.
- What is the speed of your boat, to the nearest mile per hour, relative to the ground?
- What is the boat's direction angle, to the nearest tenth of a degree, relative to the ground?

## 6.7

96. If  $\mathbf{u} = 5\mathbf{i} + 2\mathbf{j}$ ,  $\mathbf{v} = \mathbf{i} - \mathbf{j}$ , and  $\mathbf{w} = 3\mathbf{i} - 7\mathbf{j}$ , find  $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ .

In Exercises 97–99, find the dot product  $\mathbf{v} \cdot \mathbf{w}$ . Then find the angle between  $\mathbf{v}$  and  $\mathbf{w}$  to the nearest tenth of a degree.

97.  $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j}$ ,  $\mathbf{w} = 7\mathbf{i} - 4\mathbf{j}$

98.  $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j}$ ,  $\mathbf{w} = 6\mathbf{i} - 11\mathbf{j}$

99.  $\mathbf{v} = 2\mathbf{i} + \mathbf{j}$ ,  $\mathbf{w} = \mathbf{i} - \mathbf{j}$

In Exercises 100–101, use the dot product to determine whether  $\mathbf{v}$  and  $\mathbf{w}$  are orthogonal.

100.  $\mathbf{v} = 12\mathbf{i} - 8\mathbf{j}$ ,  $\mathbf{w} = 2\mathbf{i} + 3\mathbf{j}$

101.  $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$ ,  $\mathbf{w} = -3\mathbf{i} - \mathbf{j}$

In Exercises 102–103, find  $\text{proj}_{\mathbf{w}}\mathbf{v}$ . Then decompose  $\mathbf{v}$  into two vectors,  $\mathbf{v}_1$  and  $\mathbf{v}_2$ , where  $\mathbf{v}_1$  is parallel to  $\mathbf{w}$  and  $\mathbf{v}_2$  is orthogonal to  $\mathbf{w}$ .

102.  $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$ ,  $\mathbf{w} = 5\mathbf{i} + 4\mathbf{j}$

103.  $\mathbf{v} = -\mathbf{i} + 2\mathbf{j}$ ,  $\mathbf{w} = 3\mathbf{i} - \mathbf{j}$

104. A heavy crate is dragged 50 feet along a level floor. Find the work done if a force of 30 pounds at an angle of  $42^\circ$  is used.



CHAPTER

**Test Prep**  
VIDEOS

## Chapter 6 Test

- In oblique triangle  $ABC$ ,  $A = 34^\circ$ ,  $B = 68^\circ$ , and  $a = 4.8$ . Find  $b$  to the nearest tenth.
- In oblique triangle  $ABC$ ,  $C = 68^\circ$ ,  $a = 5$ , and  $b = 6$ . Find  $c$  to the nearest tenth.
- In oblique triangle  $ABC$ ,  $a = 17$  inches,  $b = 45$  inches, and  $c = 32$  inches. Find the area of the triangle to the nearest square inch.

- Plot  $\left(4, \frac{5\pi}{4}\right)$  in the polar coordinate system. Then write two other ordered pairs  $(r, \theta)$  that name this point.
- If the rectangular coordinates of a point are  $(1, -1)$ , find polar coordinates of the point.
- Convert  $x^2 + (y + 8)^2 = 64$  to a polar equation that expresses  $r$  in terms of  $\theta$ .

7. Convert to a rectangular equation and then graph:  
 $r = -4 \sec \theta$ .

In Exercises 8–9, graph each polar equation.

8.  $r = 1 + \sin \theta$   
 9.  $r = 1 + 3 \cos \theta$   
 10. Write  $-\sqrt{3} + i$  in polar form.

In Exercises 11–13, perform the indicated operation. Leave answers in polar form.

11.  $5(\cos 15^\circ + i \sin 15^\circ) \cdot 10(\cos 5^\circ + i \sin 5^\circ)$   
 12.  $\frac{2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)}{4\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)}$   
 13.  $[2(\cos 10^\circ + i \sin 10^\circ)]^5$   
 14. Find the three cube roots of 27. Write roots in rectangular form.  
 15. If  $P_1 = (-2, 3)$ ,  $P_2 = (-1, 5)$ , and  $\mathbf{v}$  is the vector from  $P_1$  to  $P_2$ ,  
 a. Write  $\mathbf{v}$  in terms of  $\mathbf{i}$  and  $\mathbf{j}$ .  
 b. Find  $\|\mathbf{v}\|$ .

In Exercises 16–19, let

$$\mathbf{v} = -5\mathbf{i} + 2\mathbf{j} \quad \text{and} \quad \mathbf{w} = 2\mathbf{i} - 4\mathbf{j}.$$

Find the specified vector, scalar, or angle.

16.  $3\mathbf{v} - 4\mathbf{w}$   
 17.  $\mathbf{v} \cdot \mathbf{w}$   
 18. the angle between  $\mathbf{v}$  and  $\mathbf{w}$ , to the nearest degree  
 19.  $\text{proj}_{\mathbf{w}}\mathbf{v}$   
 20. A small fire is sighted from ranger stations A and B. Station B is 1.6 miles due east of station A. The bearing of the fire from station A is  $N40^\circ E$  and the bearing of the fire from station B is  $N50^\circ W$ . How far, to the nearest tenth of a mile, is the fire from station A?  
 21. The magnitude and direction of two forces acting on an object are 250 pounds,  $N60^\circ E$ , and 150 pounds,  $S45^\circ E$ . Find the magnitude, to the nearest pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.  
 22. A child is pulling a wagon with a force of 40 pounds. How much work is done in moving the wagon 60 feet if the handle makes an angle of  $35^\circ$  with the ground? Round to the nearest foot-pound.

## Cumulative Review Exercises (Chapters P–6)

Solve each equation or inequality in Exercises 1–4.

1.  $x^4 - x^3 - x^2 - x - 2 = 0$   
 2.  $2 \sin^2 \theta - 3 \sin \theta + 1 = 0, \quad 0 \leq \theta < 2\pi$   
 3.  $x^2 + 2x + 3 > 11$   
 4.  $\sin \theta \cos \theta = -\frac{1}{2}, \quad 0 \leq \theta < 2\pi$

In Exercises 5–6, graph one complete cycle.

5.  $y = 3 \sin(2x - \pi)$   
 6.  $y = -4 \cos \pi x$

In Exercises 7–8, verify each identity.

7.  $\sin \theta \csc \theta - \cos^2 \theta = \sin^2 \theta$   
 8.  $\cos\left(\theta + \frac{3\pi}{2}\right) = \sin \theta$   
 9. Find the slope and y-intercept of the line whose equation is  $2x + 4y - 8 = 0$ .

In Exercises 10–11, find the exact value of each expression.

10.  $2 \sin \frac{\pi}{3} - 3 \tan \frac{\pi}{6}$   
 11.  $\sin\left(\tan^{-1} \frac{1}{2}\right)$

In Exercises 12–13, find the domain of the function whose equation is given.

12.  $f(x) = \sqrt{5 - x}$   
 13.  $g(x) = \frac{x - 3}{x^2 - 9}$

14. A ball is thrown vertically upward from a height of 8 feet with an initial velocity of 48 feet per second. The ball's height,  $s(t)$ , in feet, after  $t$  seconds is given by

$$s(t) = -16t^2 + 48t + 8.$$

After how many seconds does the ball reach its maximum height? What is the maximum height?

15. An object moves in simple harmonic motion described by  $d = 4 \sin 5t$ , where  $t$  is measured in seconds and  $d$  in meters. Find **a.** the maximum displacement; **b.** the frequency; and **c.** the time required for one cycle.  
 16. Use a half-angle formula to find the exact value of  $\cos 22.5^\circ$ .  
 17. If  $\mathbf{v} = 2\mathbf{i} + 7\mathbf{j}$  and  $\mathbf{w} = \mathbf{i} - 2\mathbf{j}$ , find **a.**  $3\mathbf{v} - \mathbf{w}$  and **b.**  $\mathbf{v} \cdot \mathbf{w}$ .  
 18. Express as a single logarithm with a coefficient of 1:  $\frac{1}{2} \log_b x - \log_b(x^2 + 1)$ .  
 19. Write the slope-intercept form of the line passing through  $(4, -1)$  and  $(-8, 5)$ .  
 20. Psychologists can measure the amount learned,  $L$ , at time  $t$  using the model  $L = A(1 - e^{-kt})$ . The variable  $A$  represents the total amount to be learned and  $k$  is the learning rate. A student preparing for the SAT has 300 new vocabulary words to learn:  $A = 300$ . This particular student can learn 20 vocabulary words after 5 minutes: If  $t = 5$ ,  $L = 20$ .  
 a. Find  $k$ , the learning rate, correct to three decimal places.  
 b. Approximately how many words will the student have learned after 20 minutes?  
 c. How long will it take for the student to learn 260 words?