

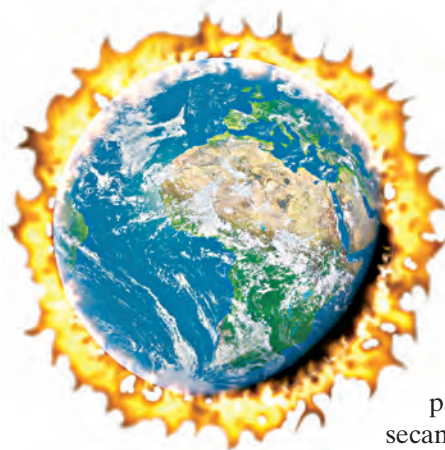
Section 4.6

Graphs of Other Trigonometric Functions

Objectives

- 1 Understand the graph of $y = \tan x$.
- 2 Graph variations of $y = \tan x$.
- 3 Understand the graph of $y = \cot x$.
- 4 Graph variations of $y = \cot x$.
- 5 Understand the graphs of $y = \csc x$ and $y = \sec x$.
- 6 Graph variations of $y = \csc x$ and $y = \sec x$.

- 1 Understand the graph of $y = \tan x$.



The debate over whether Earth is warming up is over: Humankind's reliance on fossil fuels—coal, fuel oil, and natural gas—is to blame for global warming. In an earlier chapter, we developed a linear function that modeled average global temperature in terms of atmospheric carbon dioxide. In this section's exercise set, you will see how trigonometric graphs reveal interesting patterns in carbon dioxide concentration from 1990 through 2008. In this section, trigonometric graphs will reveal patterns involving the tangent, cotangent, secant, and cosecant functions.

The Graph of $y = \tan x$

The properties of the tangent function discussed in Section 4.2 will help us determine its graph. Because the tangent function has properties that are different from sinusoidal functions, its graph differs significantly from those of sine and cosine. Properties of the tangent function include the following:

- The period is π . It is only necessary to graph $y = \tan x$ over an interval of length π . The remainder of the graph consists of repetitions of that graph at intervals of π .
- The tangent function is an odd function: $\tan(-x) = -\tan x$. The graph is symmetric with respect to the origin.
- The tangent function is undefined at $\frac{\pi}{2}$. The graph of $y = \tan x$ has a vertical asymptote at $x = \frac{\pi}{2}$.

We obtain the graph of $y = \tan x$ using some points on the graph and origin symmetry. **Table 4.5** lists some values of (x, y) on the graph of $y = \tan x$ on the interval $\left[0, \frac{\pi}{2}\right)$.

Table 4.5 Values of (x, y) on the graph of $y = \tan x$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$ (75°)	$\frac{17\pi}{36}$ (85°)	$\frac{89\pi}{180}$ (89°)	1.57	$\frac{\pi}{2}$
$y = \tan x$	0	$\frac{\sqrt{3}}{3} \approx 0.6$	1	$\sqrt{3} \approx 1.7$	3.7	11.4	57.3	1255.8	undefined

As x increases from 0 toward $\frac{\pi}{2}$, y increases slowly at first, then more and more rapidly.

The graph in **Figure 4.78(a)** is based on our observation that as x increases from 0 toward $\frac{\pi}{2}$, y increases slowly at first, then more and more rapidly. Notice that y increases without bound as x approaches $\frac{\pi}{2}$. As the figure shows, the graph of $y = \tan x$ has a vertical asymptote at $x = \frac{\pi}{2}$.

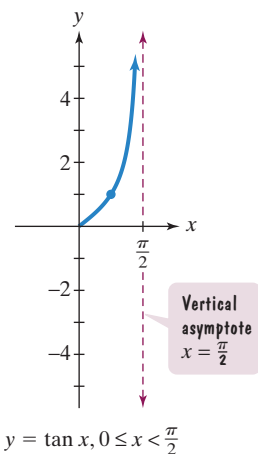


Figure 4.78(a)

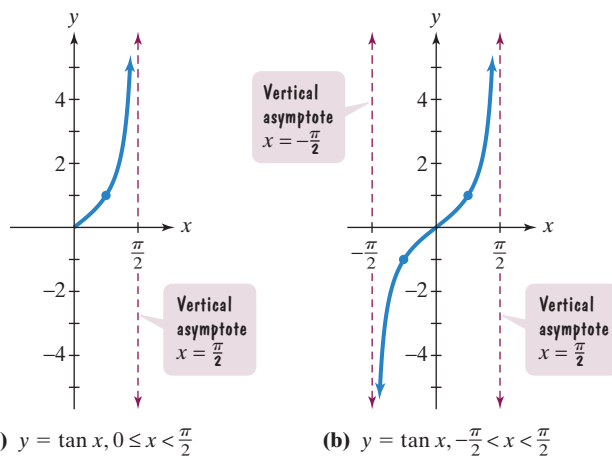


Figure 4.78 Graphing the tangent function

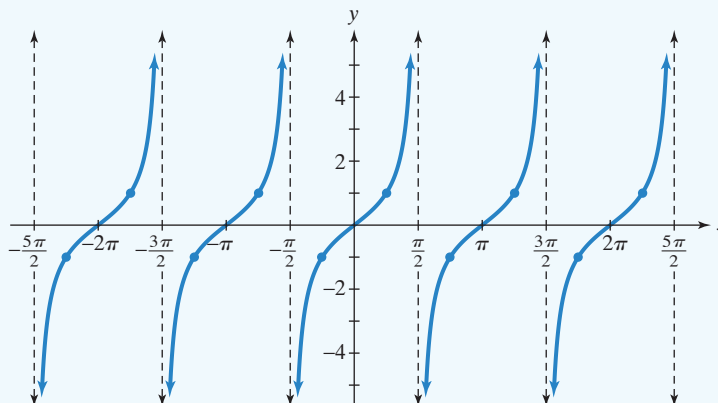
The graph of $y = \tan x$ can be completed on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ by using origin symmetry. **Figure 4.78(b)** shows the result of reflecting the graph in **Figure 4.78(a)** about the origin. The graph of $y = \tan x$ has another vertical asymptote at $x = -\frac{\pi}{2}$. Notice that y decreases without bound as x approaches $-\frac{\pi}{2}$.

Because the period of the tangent function is π , the graph in **Figure 4.78(b)** shows one complete period of $y = \tan x$. We obtain the complete graph of $y = \tan x$ by repeating the graph in **Figure 4.78(b)** to the left and right over intervals of π . The resulting graph and its main characteristics are shown in the following box:

The Tangent Curve: The Graph of $y = \tan x$ and Its Characteristics

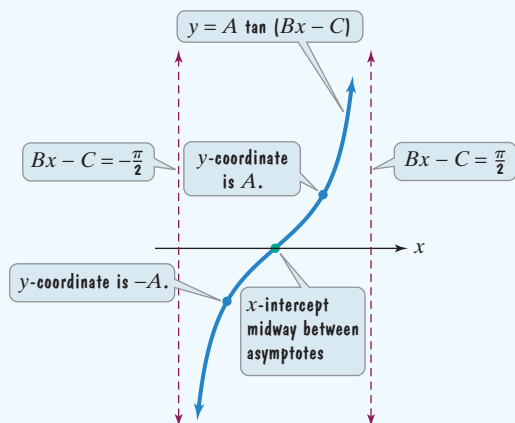
Characteristics

- **Period:** π
- **Domain:** All real numbers except odd multiples of $\frac{\pi}{2}$
- **Range:** All real numbers
- **Vertical asymptotes** at odd multiples of $\frac{\pi}{2}$
- **An x -intercept** occurs midway between each pair of consecutive asymptotes.
- **Odd function** with origin symmetry
- Points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between consecutive asymptotes have y -coordinates of -1 and 1 , respectively.



2 Graph variations of $y = \tan x$.Graphing Variations of $y = \tan x$

We use the characteristics of the tangent curve to graph tangent functions of the form $y = A \tan(Bx - C)$.

Graphing $y = A \tan(Bx - C)$ 

1. Find two consecutive asymptotes by finding an interval containing one period:

$$-\frac{\pi}{2} < Bx - C < \frac{\pi}{2}.$$

A pair of consecutive asymptotes occur at

$$Bx - C = -\frac{\pi}{2} \text{ and } Bx - C = \frac{\pi}{2}.$$

2. Identify an x -intercept, midway between the consecutive asymptotes.
3. Find the points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes. These points have y -coordinates of $-A$ and A , respectively.
4. Use steps 1–3 to graph one full period of the function. Add additional cycles to the left or right as needed.

EXAMPLE 1 Graphing a Tangent Function

Graph $y = 2 \tan \frac{x}{2}$ for $-\pi < x < 3\pi$.

Solution Refer to **Figure 4.79** as you read each step.

Step 1 Find two consecutive asymptotes. We do this by finding an interval containing one period.

$$-\frac{\pi}{2} < \frac{x}{2} < \frac{\pi}{2} \quad \text{Set up the inequality } -\frac{\pi}{2} < \text{variable expression in tangent} < \frac{\pi}{2}.$$

$$-\pi < x < \pi \quad \text{Multiply all parts by 2 and solve for } x.$$

An interval containing one period is $(-\pi, \pi)$. Thus, two consecutive asymptotes occur at $x = -\pi$ and $x = \pi$.

Step 2 Identify an x -intercept, midway between the consecutive asymptotes. Midway between $x = -\pi$ and $x = \pi$ is $x = 0$. An x -intercept is 0 and the graph passes through $(0, 0)$.

Step 3 Find points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes. These points have y -coordinates of $-A$ and A . Because A , the coefficient of the tangent in $y = 2 \tan \frac{x}{2}$ is 2, these points have y -coordinates of -2 and 2 .

The graph passes through $(-\frac{\pi}{2}, -2)$ and $(\frac{\pi}{2}, 2)$.

Step 4 Use steps 1–3 to graph one full period of the function. We use the two consecutive asymptotes, $x = -\pi$ and $x = \pi$, an x -intercept of 0, and points midway between the x -intercept and asymptotes with y -coordinates of -2 and 2 . We graph one period of $y = 2 \tan \frac{x}{2}$ from $-\pi$ to π . In order to graph for $-\pi < x < 3\pi$, we continue the pattern and extend the graph another full period to the right. The graph is shown in **Figure 4.79**.

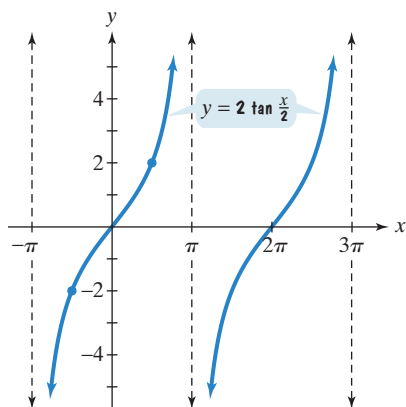



Figure 4.79 The graph is shown for two full periods.

 **Check Point 1** Graph $y = 3 \tan 2x$ for $-\frac{\pi}{4} < x < \frac{3\pi}{4}$.

EXAMPLE 2 Graphing a Tangent Function

Graph two full periods of $y = \tan\left(x + \frac{\pi}{4}\right)$.

Solution The graph of $y = \tan\left(x + \frac{\pi}{4}\right)$ is the graph of $y = \tan x$ shifted horizontally to the left $\frac{\pi}{4}$ units. Refer to **Figure 4.80** as you read each step.

Step 1 Find two consecutive asymptotes. We do this by finding an interval containing one period.

$$-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$$

Set up the inequality $-\frac{\pi}{2} < \text{variable expression in tangent} < \frac{\pi}{2}$.

$$-\frac{\pi}{2} - \frac{\pi}{4} < x < \frac{\pi}{2} - \frac{\pi}{4}$$

Subtract $\frac{\pi}{4}$ from all parts and solve for x .

$$-\frac{3\pi}{4} < x < \frac{\pi}{4}$$

Simplify: $-\frac{\pi}{2} - \frac{\pi}{4} = -\frac{2\pi}{4} - \frac{\pi}{4} = -\frac{3\pi}{4}$
and $\frac{\pi}{2} - \frac{\pi}{4} = \frac{2\pi}{4} - \frac{\pi}{4} = \frac{\pi}{4}$.


An interval containing one period is $\left(-\frac{3\pi}{4}, \frac{\pi}{4}\right)$. Thus, two consecutive asymptotes occur at $x = -\frac{3\pi}{4}$ and $x = \frac{\pi}{4}$.


Step 2 Identify an x -intercept, midway between the consecutive asymptotes.

$$x\text{-intercept} = \frac{-\frac{3\pi}{4} + \frac{\pi}{4}}{2} = \frac{-\frac{2\pi}{4}}{2} = \frac{-2\pi}{8} = -\frac{\pi}{4}$$

An x -intercept is $-\frac{\pi}{4}$ and the graph passes through $\left(-\frac{\pi}{4}, 0\right)$.

Step 3 Find points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes. These points have y -coordinates of $-A$ and A . Because A , the coefficient of the tangent in $y = \tan\left(x + \frac{\pi}{4}\right)$ is 1, these points have y -coordinates of -1 and 1 . They are shown as blue dots in **Figure 4.80**.

Step 4 Use steps 1–3 to graph one full period of the function. We use the two consecutive asymptotes, $x = -\frac{3\pi}{4}$ and $x = \frac{\pi}{4}$, to graph one full period of $y = \tan\left(x + \frac{\pi}{4}\right)$ from $-\frac{3\pi}{4}$ to $\frac{\pi}{4}$. We graph two full periods by continuing the pattern and extending the graph another full period to the right. The graph is shown in **Figure 4.80**. 

 **Check Point 2** Graph two full periods of $y = \tan\left(x - \frac{\pi}{2}\right)$.

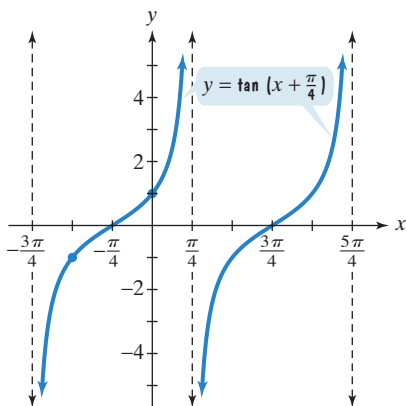
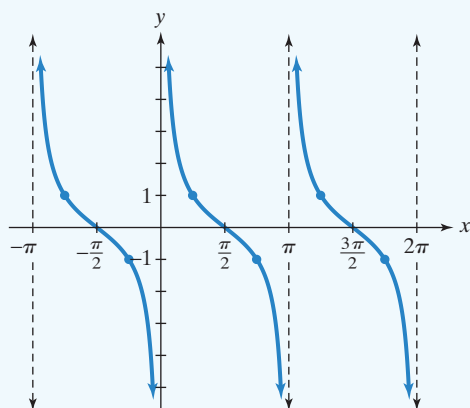


Figure 4.80 The graph is shown for two full periods.

3 Understand the graph of $y = \cot x$.

The Graph of $y = \cot x$

Like the tangent function, the cotangent function, $y = \cot x$, has a period of π . The graph and its main characteristics are shown in the box at the top of the next page.

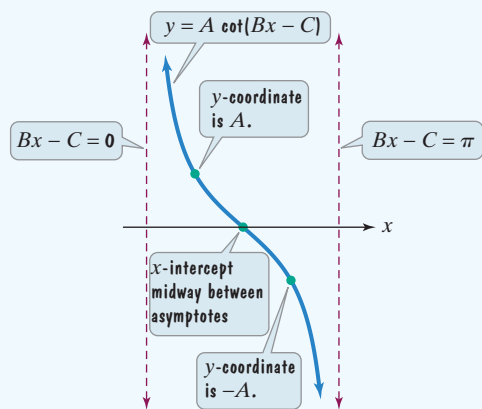
The Cotangent Curve: The Graph of $y = \cot x$ and Its Characteristics

Characteristics

- **Period:** π
- **Domain:** All real numbers except integral multiples of π
- **Range:** All real numbers
- **Vertical asymptotes** at integral multiples of π
- **An x -intercept** occurs midway between each pair of consecutive asymptotes.
- **Odd function** with origin symmetry
- Points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between consecutive asymptotes have y -coordinates of 1 and -1 , respectively.

4 Graph variations of $y = \cot x$.Graphing Variations of $y = \cot x$

We use the characteristics of the cotangent curve to graph cotangent functions of the form $y = A \cot(Bx - C)$.

Graphing $y = A \cot(Bx - C)$ 

1. Find two consecutive asymptotes by finding an interval containing one full period:

$$0 < Bx - C < \pi.$$

A pair of consecutive asymptotes occur at

$$Bx - C = 0 \text{ and } Bx - C = \pi.$$

2. Identify an x -intercept, midway between the consecutive asymptotes.
3. Find the points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between the consecutive asymptotes. These points have y -coordinates of A and $-A$, respectively.
4. Use steps 1–3 to graph one full period of the function. Add additional cycles to the left or right as needed.

EXAMPLE 3 Graphing a Cotangent Function

Graph $y = 3 \cot 2x$.

Solution

Step 1 Find two consecutive asymptotes. We do this by finding an interval containing one period.

$$0 < 2x < \pi \quad \text{Set up the inequality } 0 < \text{variable expression in cotangent} < \pi.$$

$$0 < x < \frac{\pi}{2} \quad \text{Divide all parts by 2 and solve for } x.$$

Because $0 < x < \frac{\pi}{2}$, an interval containing one period is $\left(0, \frac{\pi}{2}\right)$. Thus, two consecutive asymptotes occur at $x = 0$ and $x = \frac{\pi}{2}$. This is shown in **Figure 4.81**.

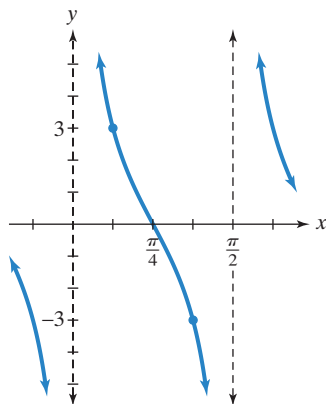


Figure 4.81 The graph of $y = 3 \cot 2x$

Step 2 Identify an x -intercept, midway between the consecutive asymptotes.

Midway between $x = 0$ and $x = \frac{\pi}{2}$ is $x = \frac{\pi}{4}$. An x -intercept is $\frac{\pi}{4}$ and the graph passes through $\left(\frac{\pi}{4}, 0\right)$.

Step 3 Find points on the graph $\frac{1}{4}$ and $\frac{3}{4}$ of the way between consecutive asymptotes.

These points have y -coordinates of A and $-A$. Because A , the coefficient of the cotangent in $y = 3 \cot 2x$ is 3, these points have y -coordinates of 3 and -3 . They are shown as blue dots in **Figure 4.81**.

Step 4 Use steps 1–3 to graph one full period of the function. We use the two consecutive asymptotes, $x = 0$ and $x = \frac{\pi}{2}$, to graph one full period of $y = 3 \cot 2x$.

This curve is repeated to the left and right, as shown in **Figure 4.81**.

 **Check Point 3** Graph $y = \frac{1}{2} \cot \frac{\pi}{2}x$.

- 5** Understand the graphs of $y = \csc x$ and $y = \sec x$.

The Graphs of $y = \csc x$ and $y = \sec x$

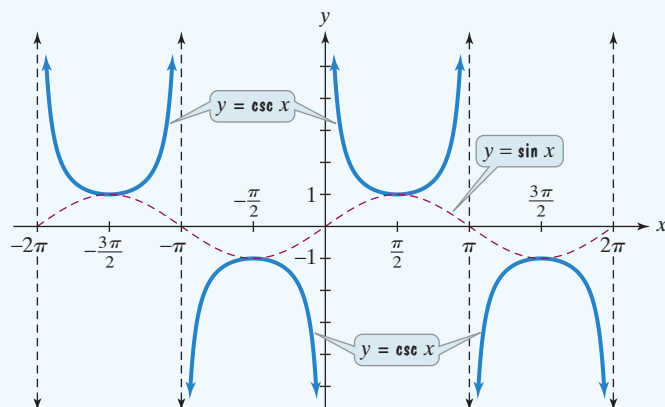
We obtain the graphs of the cosecant and secant curves by using the reciprocal identities

$$\csc x = \frac{1}{\sin x} \quad \text{and} \quad \sec x = \frac{1}{\cos x}.$$

The identity $\csc x = \frac{1}{\sin x}$ tells us that the value of the cosecant function $y = \csc x$ at a given value of x equals the reciprocal of the corresponding value of the sine function, provided that the value of the sine function is not 0. If the value of $\sin x$ is 0, then at each of these values of x , the cosecant function is not defined. A vertical asymptote is associated with each of these values on the graph of $y = \csc x$.

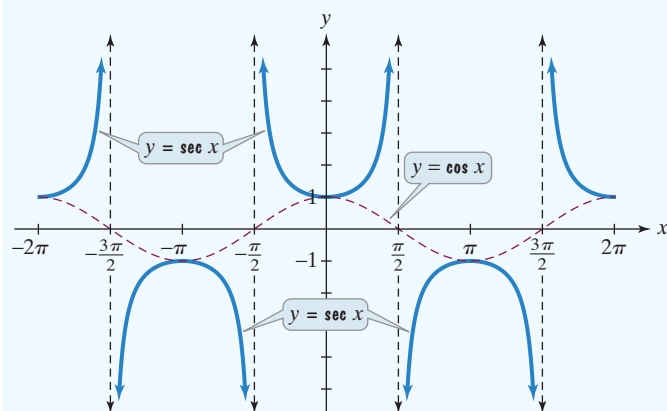
We obtain the graph of $y = \csc x$ by taking reciprocals of the y -values in the graph of $y = \sin x$. Vertical asymptotes of $y = \csc x$ occur at the x -intercepts of $y = \sin x$. Likewise, we obtain the graph of $y = \sec x$ by taking the reciprocal of $y = \cos x$. Vertical asymptotes of $y = \sec x$ occur at the x -intercepts of $y = \cos x$. The graphs of $y = \csc x$ and $y = \sec x$ and their key characteristics are shown in the following boxes. We have used dashed red lines to graph $y = \sin x$ and $y = \cos x$ first, drawing vertical asymptotes through the x -intercepts.

The Cosecant Curve: The Graph of $y = \csc x$ and Its Characteristics



Characteristics

- **Period:** 2π
- **Domain:** All real numbers except integral multiples of π
- **Range:** All real numbers y such that $y \leq -1$ or $y \geq 1$: $(-\infty, -1] \cup [1, \infty)$
- **Vertical asymptotes** at integral multiples of π
- **Odd function**, $\csc(-x) = -\csc x$, with origin symmetry

The Secant Curve: The Graph of $y = \sec x$ and Its Characteristics

Characteristics

- **Period:** 2π
- **Domain:** All real numbers except odd multiples of $\frac{\pi}{2}$
- **Range:** All real numbers y such that $y \leq -1$ or $y \geq 1$: $(-\infty, -1] \cup [1, \infty)$
- **Vertical asymptotes** at odd multiples of $\frac{\pi}{2}$
- **Even function,** $\sec(-x) = \sec x$, with y -axis symmetry

- 6 Graph variations of $y = \csc x$ and $y = \sec x$.

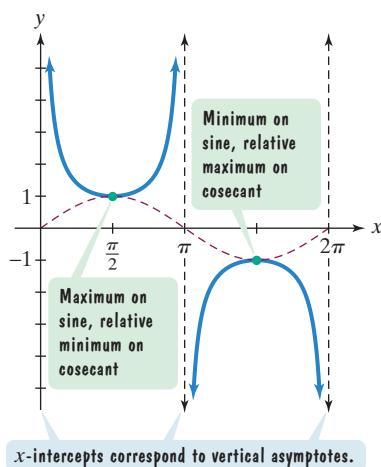


Figure 4.82

Graphing Variations of $y = \csc x$ and $y = \sec x$

We use graphs of functions involving the corresponding reciprocal functions to obtain graphs of cosecant and secant functions. To graph a cosecant or secant curve, begin by graphing the function where cosecant or secant is replaced by its reciprocal function. For example, to graph $y = 2 \csc 2x$, we use the graph of $y = 2 \sin 2x$.

Likewise, to graph $y = -3 \sec \frac{x}{2}$, we use the graph of $y = -3 \cos \frac{x}{2}$.

Figure 4.82 illustrates how we use a sine curve to obtain a cosecant curve. Notice that

- x -intercepts on the red sine curve correspond to vertical asymptotes of the blue cosecant curve.
- A maximum point on the red sine curve corresponds to a minimum point on a continuous portion of the blue cosecant curve.
- A minimum point on the red sine curve corresponds to a maximum point on a continuous portion of the blue cosecant curve.

EXAMPLE 4 Using a Sine Curve to Obtain a Cosecant Curve

Use the graph of $y = 2 \sin 2x$ in **Figure 4.83** to obtain the graph of $y = 2 \csc 2x$.

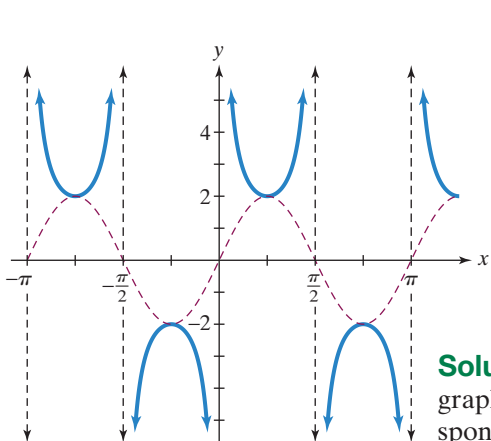
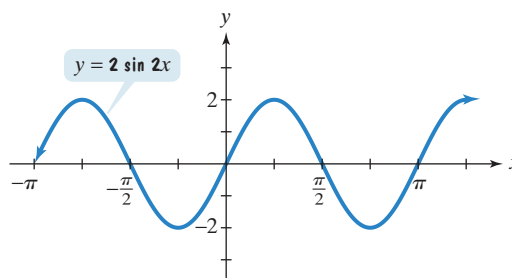

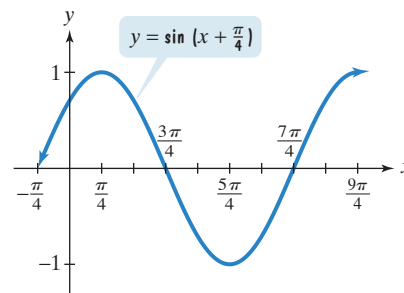
Figure 4.84 Using a sine curve to graph $y = 2 \csc 2x$ 

Figure 4.83

Solution We begin our work in **Figure 4.84** by showing the given graph, the graph of $y = 2 \sin 2x$, using dashed red lines. The x -intercepts of $y = 2 \sin 2x$ correspond to the vertical asymptotes of $y = 2 \csc 2x$. Thus, we draw vertical asymptotes through the x -intercepts, shown in **Figure 4.84**. Using the asymptotes as guides, we sketch the graph of $y = 2 \csc 2x$ in **Figure 4.84**.

 **Check Point 4** Use the graph of $y = \sin\left(x + \frac{\pi}{4}\right)$, shown on the right, to obtain the graph of $y = \csc\left(x + \frac{\pi}{4}\right)$.



We use a cosine curve to obtain a secant curve in exactly the same way we used a sine curve to obtain a cosecant curve. Thus,

- x -intercepts on the cosine curve correspond to vertical asymptotes on the secant curve.
- A maximum point on the cosine curve corresponds to a minimum point on a continuous portion of the secant curve.
- A minimum point on the cosine curve corresponds to a maximum point on a continuous portion of the secant curve.

EXAMPLE 5 Graphing a Secant Function

Graph $y = -3 \sec \frac{x}{2}$ for $-\pi < x < 5\pi$.

Solution We begin by graphing the function $y = -3 \cos \frac{x}{2}$, where secant has been replaced by cosine, its reciprocal function. This equation is of the form $y = A \cos Bx$ with $A = -3$ and $B = \frac{1}{2}$.

amplitude: $|A| = |-3| = 3$

The maximum y is 3 and the minimum is -3 .

period: $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

Each cycle is of length 4π .

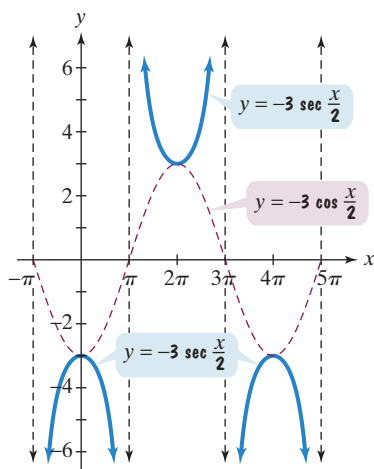



Figure 4.85 Using a cosine curve to graph $y = -3 \sec \frac{x}{2}$

We use quarter-periods, $\frac{4\pi}{4}$, or π , to find the x -values for the five key points. Starting with $x = 0$, the x -values are $0, \pi, 2\pi, 3\pi$, and 4π . Evaluating the function $y = -3 \cos \frac{x}{2}$ at each of these values of x , the key points are

$$(0, -3), (\pi, 0), (2\pi, 3), (3\pi, 0), \text{ and } (4\pi, -3).$$

We use these key points to graph $y = -3 \cos \frac{x}{2}$ from 0 to 4π , shown using a dashed

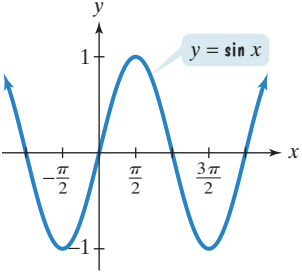
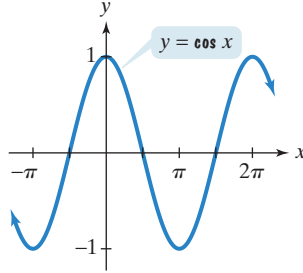
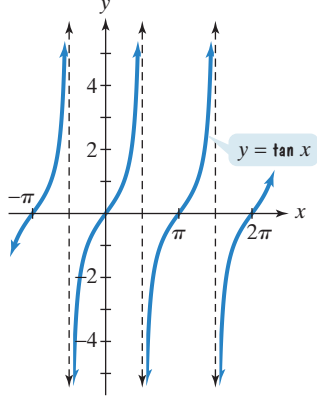
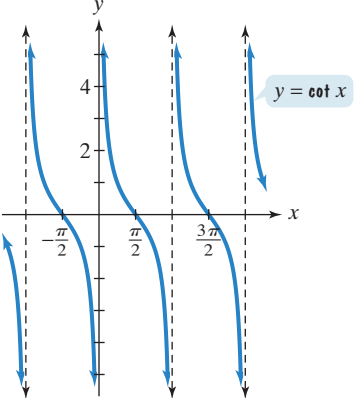
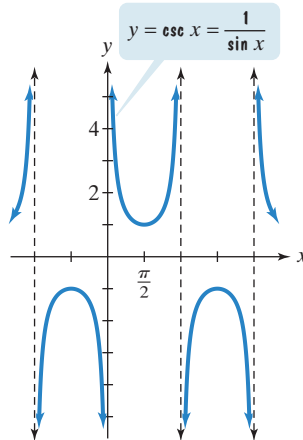
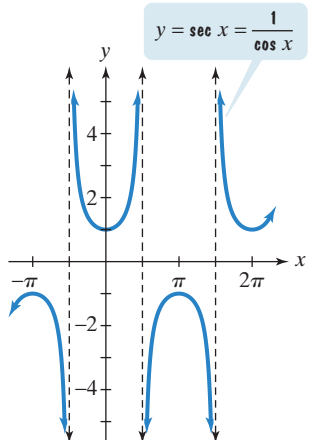
red line in **Figure 4.85**. In order to graph $y = -3 \sec \frac{x}{2}$ for $-\pi < x < 5\pi$, extend the dashed red graph of the cosine function π units to the left and π units to the right. Now use this dashed red graph to obtain the graph of the corresponding secant function, its reciprocal function. Draw vertical asymptotes through the x -intercepts. Using these asymptotes as guides, the graph of $y = -3 \sec \frac{x}{2}$ is shown in blue in **Figure 4.85**.

 **Check Point 5** Graph $y = 2 \sec 2x$ for $-\frac{3\pi}{4} < x < \frac{3\pi}{4}$.

The Six Curves of Trigonometry

Table 4.6 summarizes the graphs of the six trigonometric functions. Below each of the graphs is a description of the domain, range, and period of the function.

Table 4.6 Graphs of the Six Trigonometric Functions

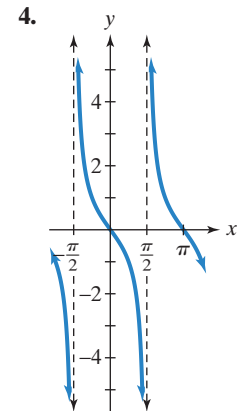
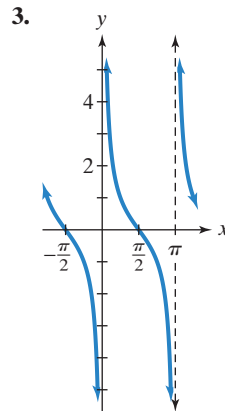
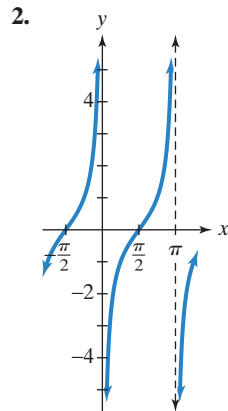
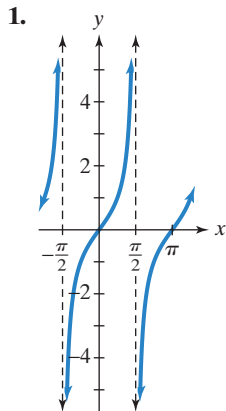
 <p>The graph shows the sine function $y = \sin x$ on a coordinate plane. The x-axis is labeled with $-\frac{\pi}{2}$, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$. The y-axis is labeled with 1 and -1. The curve passes through the origin $(0,0)$, reaches a maximum of 1 at $x = \frac{\pi}{2}$, and a minimum of -1 at $x = \frac{3\pi}{2}$.</p> <p>Domain: all real numbers: $(-\infty, \infty)$ Range: $[-1, 1]$ Period: 2π</p>	 <p>The graph shows the cosine function $y = \cos x$ on a coordinate plane. The x-axis is labeled with $-\pi$, π, and 2π. The y-axis is labeled with 1 and -1. The curve has a maximum of 1 at $x = 0$ and $x = 2\pi$, and a minimum of -1 at $x = \pi$.</p> <p>Domain: all real numbers: $(-\infty, \infty)$ Range: $[-1, 1]$ Period: 2π</p>	 <p>The graph shows the tangent function $y = \tan x$ on a coordinate plane. The x-axis is labeled with $-\pi$, π, and 2π. The y-axis is labeled with 4, 2, -2, and -4. Vertical dashed lines represent asymptotes at $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$, and $x = \frac{3\pi}{2}$. The curve passes through the origin $(0,0)$ and has a period of π.</p> <p>Domain: all real numbers except odd multiples of $\frac{\pi}{2}$ Range: all real numbers Period: π</p>
 <p>The graph shows the cotangent function $y = \cot x$ on a coordinate plane. The x-axis is labeled with $-\frac{\pi}{2}$, $\frac{\pi}{2}$, and $\frac{3\pi}{2}$. The y-axis is labeled with 4, 2, and -2. Vertical dashed lines represent asymptotes at $x = -\pi$, $x = 0$, and $x = \pi$. The curve has a period of π.</p> <p>Domain: all real numbers except integral multiples of π Range: all real numbers Period: π</p>	 <p>The graph shows the cosecant function $y = \csc x = \frac{1}{\sin x}$ on a coordinate plane. The x-axis is labeled with $\frac{\pi}{2}$. The y-axis is labeled with 4, 2, and -2. Vertical dashed lines represent asymptotes at $x = 0$, $x = \pi$, and $x = 2\pi$. The curve has a period of 2π.</p> <p>Domain: all real numbers except integral multiples of π Range: $(-\infty, -1] \cup [1, \infty)$ Period: 2π</p>	 <p>The graph shows the secant function $y = \sec x = \frac{1}{\cos x}$ on a coordinate plane. The x-axis is labeled with $-\pi$, π, and 2π. The y-axis is labeled with 4, 2, -2, and -4. Vertical dashed lines represent asymptotes at $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$, and $x = \frac{3\pi}{2}$. The curve has a period of 2π.</p> <p>Domain: all real numbers except odd multiples of $\frac{\pi}{2}$ Range: $(-\infty, -1] \cup [1, \infty)$ Period: 2π</p>

Exercise Set 4.6

Practice Exercises

In Exercises 1–4, the graph of a tangent function is given. Select the equation for each graph from the following options:

$$y = \tan\left(x + \frac{\pi}{2}\right), \quad y = \tan(x + \pi), \quad y = -\tan x, \quad y = -\tan\left(x - \frac{\pi}{2}\right).$$



In Exercises 5–12, graph two periods of the given tangent function.

5. $y = 3 \tan \frac{x}{4}$

6. $y = 2 \tan \frac{x}{4}$

7. $y = \frac{1}{2} \tan 2x$

8. $y = 2 \tan 2x$

9. $y = -2 \tan \frac{1}{2}x$

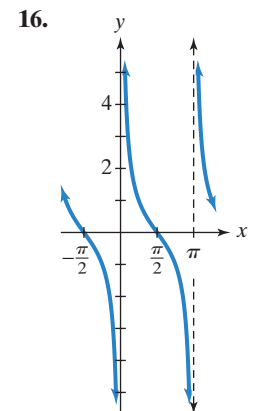
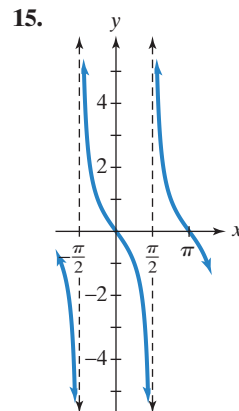
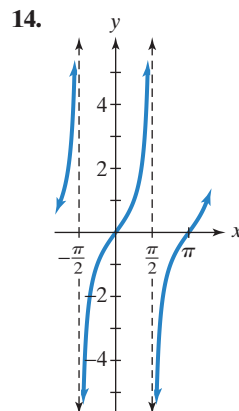
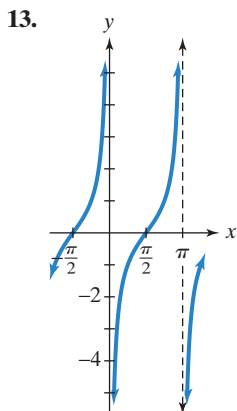
10. $y = -3 \tan \frac{1}{2}x$

11. $y = \tan(x - \pi)$

12. $y = \tan\left(x - \frac{\pi}{4}\right)$

In Exercises 13–16, the graph of a cotangent function is given. Select the equation for each graph from the following options:

$$y = \cot\left(x + \frac{\pi}{2}\right), \quad y = \cot(x + \pi), \quad y = -\cot x, \quad y = -\cot\left(x - \frac{\pi}{2}\right).$$



In Exercises 17–24, graph two periods of the given cotangent function.

17. $y = 2 \cot x$

18. $y = \frac{1}{2} \cot x$

19. $y = \frac{1}{2} \cot 2x$

20. $y = 2 \cot 2x$

21. $y = -3 \cot \frac{\pi}{2}x$

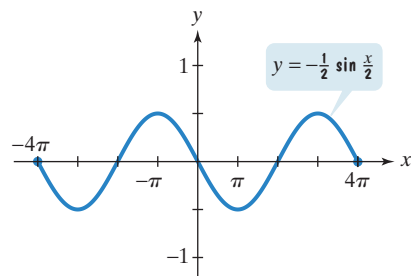
22. $y = -2 \cot \frac{\pi}{4}x$

23. $y = 3 \cot\left(x + \frac{\pi}{2}\right)$

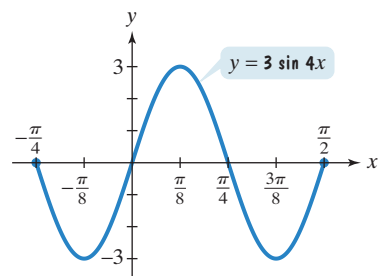
24. $y = 3 \cot\left(x + \frac{\pi}{4}\right)$

In Exercises 25–28, use each graph to obtain the graph of the corresponding reciprocal function, cosecant or secant. Give the equation of the function for the graph that you obtain.

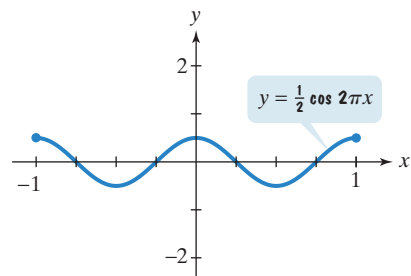
25.



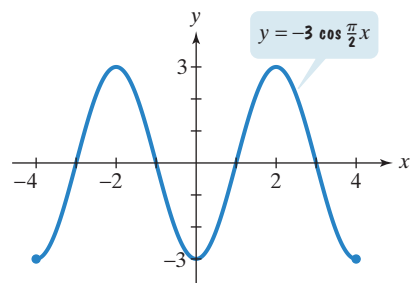
26.



27.



28.



In Exercises 29–44, graph two periods of the given cosecant or secant function.

29. $y = 3 \csc x$

31. $y = \frac{1}{2} \csc \frac{x}{2}$

33. $y = 2 \sec x$

35. $y = \sec \frac{x}{3}$

37. $y = -2 \csc \pi x$

39. $y = -\frac{1}{2} \sec \pi x$

30. $y = 2 \csc x$

32. $y = \frac{3}{2} \csc \frac{x}{4}$

34. $y = 3 \sec x$

36. $y = \sec \frac{x}{2}$

38. $y = -\frac{1}{2} \csc \pi x$

40. $y = -\frac{3}{2} \sec \pi x$

41. $y = \csc(x - \pi)$

42. $y = \csc\left(x - \frac{\pi}{2}\right)$

43. $y = 2 \sec(x + \pi)$

44. $y = 2 \sec\left(x + \frac{\pi}{2}\right)$

Practice Plus

In Exercises 45–52, graph two periods of each function.

45. $y = 2 \tan\left(x - \frac{\pi}{6}\right) + 1$

46. $y = 2 \cot\left(x + \frac{\pi}{6}\right) - 1$

47. $y = \sec\left(2x + \frac{\pi}{2}\right) - 1$

48. $y = \csc\left(2x - \frac{\pi}{2}\right) + 1$

49. $y = \csc|x|$

50. $y = \sec|x|$

51. $y = |\cot \frac{1}{2}x|$

52. $y = |\tan \frac{1}{2}x|$

In Exercises 53–54, let $f(x) = 2 \sec x$, $g(x) = -2 \tan x$, and $h(x) = 2x - \frac{\pi}{2}$.

53. Graph two periods of

$$y = (f \circ h)(x).$$

54. Graph two periods of

$$y = (g \circ h)(x).$$

In Exercises 55–58, use a graph to solve each equation for $-2\pi \leq x \leq 2\pi$.

55. $\tan x = -1$

56. $\cot x = -1$

57. $\csc x = 1$

58. $\sec x = 1$

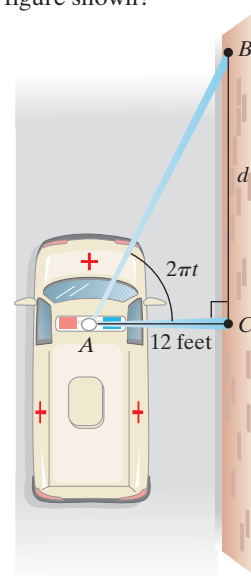
Application Exercises

59. An ambulance with a rotating beam of light is parked 12 feet from a building. The function

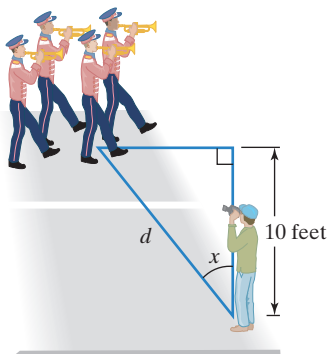
$$d = 12 \tan 2\pi t$$

describes the distance, d , in feet, of the rotating beam of light from point C after t seconds.

- Graph the function on the interval $[0, 2]$.
- For what values of t in $[0, 2]$ is the function undefined? What does this mean in terms of the rotating beam of light in the figure shown?



60. The angle of elevation from the top of a house to a jet flying 2 miles above the house is x radians. If d represents the horizontal distance, in miles, of the jet from the house, express d in terms of a trigonometric function of x . Then graph the function for $0 < x < \pi$.
61. Your best friend is marching with a band and has asked you to film him. The figure below shows that you have set yourself up 10 feet from the street where your friend will be passing from left to right. If d represents your distance, in feet, from your friend and x is the radian measure of the angle shown, express d in terms of a trigonometric function of x . Then graph the function for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Negative angles indicate that your marching buddy is on your left.



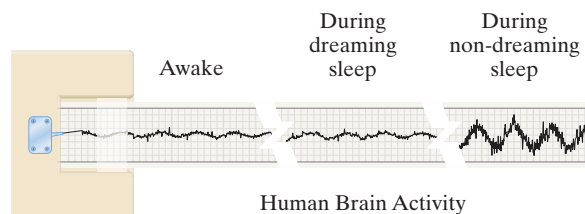
In Exercises 62–64, sketch a reasonable graph that models the given situation.

62. The number of hours of daylight per day in your hometown over a two-year period
63. The motion of a diving board vibrating 10 inches in each direction per second just after someone has dived off
64. The distance of a rotating beam of light from a point on a wall (See the figure for Exercise 59.)

Writing in Mathematics

65. Without drawing a graph, describe the behavior of the basic tangent curve.
66. If you are given the equation of a tangent function, how do you find a pair of consecutive asymptotes?
67. If you are given the equation of a tangent function, how do you identify an x -intercept?
68. Without drawing a graph, describe the behavior of the basic cotangent curve.
69. If you are given the equation of a cotangent function, how do you find a pair of consecutive asymptotes?
70. Explain how to determine the range of $y = \csc x$ from the graph. What is the range?
71. Explain how to use a sine curve to obtain a cosecant curve. Why can the same procedure be used to obtain a secant curve from a cosine curve?

72. Scientists record brain activity by attaching electrodes to the scalp and then connecting these electrodes to a machine. The brain activity recorded with this machine is shown in the three graphs. Which trigonometric functions would be most appropriate for describing the oscillations in brain activity? Describe similarities and differences among these functions when modeling brain activity when awake, during dreaming sleep, and during non-dreaming sleep.



Technology Exercises

In working Exercises 73–76, describe what happens at the asymptotes on the graphing utility. Compare the graphs in the connected and dot modes.

73. Use a graphing utility to verify any two of the tangent curves that you drew by hand in Exercises 5–12.
74. Use a graphing utility to verify any two of the cotangent curves that you drew by hand in Exercises 17–24.
75. Use a graphing utility to verify any two of the cosecant curves that you drew by hand in Exercises 29–44.
76. Use a graphing utility to verify any two of the secant curves that you drew by hand in Exercises 29–44.

In Exercises 77–82, use a graphing utility to graph each function. Use a range setting so that the graph is shown for at least two periods.

77. $y = \tan \frac{x}{4}$
78. $y = \tan 4x$
79. $y = \cot 2x$
80. $y = \cot \frac{x}{2}$
81. $y = \frac{1}{2} \tan \pi x$
82. $y = \frac{1}{2} \tan(\pi x + 1)$

In Exercises 83–86, use a graphing utility to graph each pair of functions in the same viewing rectangle. Use a viewing rectangle so that the graphs are shown for at least two periods.

83. $y = 0.8 \sin \frac{x}{2}$ and $y = 0.8 \csc \frac{x}{2}$
84. $y = -2.5 \sin \frac{\pi}{3} x$ and $y = -2.5 \csc \frac{\pi}{3} x$
85. $y = 4 \cos\left(2x - \frac{\pi}{6}\right)$ and $y = 4 \sec\left(2x - \frac{\pi}{6}\right)$
86. $y = -3.5 \cos\left(\pi x - \frac{\pi}{6}\right)$ and $y = -3.5 \sec\left(\pi x - \frac{\pi}{6}\right)$

87. Carbon dioxide particles in our atmosphere trap heat and raise the planet's temperature. Even if all greenhouse-gas emissions miraculously ended today, the planet would continue to warm through the rest of the century because of the amount of carbon we have already added to the atmosphere. Carbon dioxide accounts for about half of global warming. The function

$$y = 2.5 \sin 2\pi x + 0.0216x^2 + 0.654x + 316$$

models carbon dioxide concentration, y , in parts per million, where $x = 0$ represents January 1960; $x = \frac{1}{12}$, February 1960; $x = \frac{2}{12}$, March 1960; \dots , $x = 1$, January 1961; $x = \frac{13}{12}$, February 1961; and so on. Use a graphing utility to graph the function in a $[30, 48, 5]$ by $[310, 420, 5]$ viewing rectangle. Describe what the graph reveals about carbon dioxide concentration from 1990 through 2008.

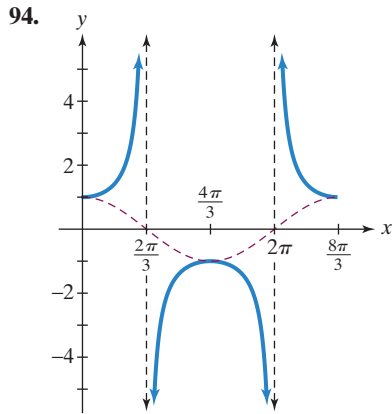
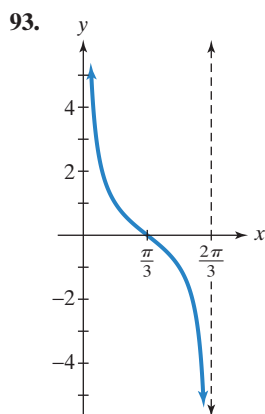
88. Graph $y = \sin \frac{1}{x}$ in a $[-0.2, 0.2, 0.01]$ by $[-1.2, 1.2, 0.01]$ viewing rectangle. What is happening as x approaches 0 from the left or the right? Explain this behavior.

Critical Thinking Exercises

Make Sense? In Exercises 89–92, determine whether each statement makes sense or does not make sense, and explain your reasoning.

89. I use the pattern asymptote, $-A$, x -intercept, A , asymptote to graph one full period of $y = A \tan(Bx - C)$.
90. After using the four-step procedure to graph $y = -\cot\left(x + \frac{\pi}{4}\right)$, I checked my graph by verifying it was the graph of $y = \cot x$ shifted left $\frac{\pi}{4}$ unit and reflected about the x -axis.
91. I used the graph of $y = 3 \cos 2x$ to obtain the graph of $y = 3 \csc 2x$.
92. I used a tangent function to model the average monthly temperature of New York City, where $x = 1$ represents January, $x = 2$ represents February, and so on.

In Exercises 93–94, write an equation for each blue graph.



In Exercises 95–96, write the equation for a cosecant function satisfying the given conditions.

95. period: 3π ; range: $(-\infty, -2] \cup [2, \infty)$

96. period: 2; range: $(-\infty, -\pi] \cup [\pi, \infty)$

97. Determine the range of the following functions. Then give a viewing rectangle, or window, that shows two periods of the function's graph.

a. $f(x) = \sec\left(3x + \frac{\pi}{2}\right)$

b. $g(x) = 3 \sec \pi\left(x + \frac{1}{2}\right)$

98. For $x > 0$, what effect does 2^{-x} in $y = 2^{-x} \sin x$ have on the graph of $y = \sin x$? What kind of behavior can be modeled by a function such as $y = 2^{-x} \sin x$?

Preview Exercises

Exercises 99–101 will help you prepare for the material covered in the next section.

99. a. Graph $y = \sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

- b. Based on your graph in part (a), does $y = \sin x$ have an inverse function if the domain is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$? Explain your answer.

- c. Determine the angle in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is $-\frac{1}{2}$. Identify this information as a point on your graph in part (a).

100. a. Graph $y = \cos x$ for $0 \leq x \leq \pi$.

- b. Based on your graph in part (a), does $y = \cos x$ have an inverse function if the domain is restricted to $[0, \pi]$? Explain your answer.

- c. Determine the angle in the interval $[0, \pi]$ whose cosine is $\frac{\sqrt{3}}{2}$. Identify this information as a point on your graph in part (a).

101. a. Graph $y = \tan x$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

- b. Based on your graph in part (a), does $y = \tan x$ have an inverse function if the domain is restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$? Explain your answer.

- c. Determine the angle in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $-\sqrt{3}$. Identify this information as a point on your graph in part (a).