Writing in Mathematics

105. If you are given a point on the terminal side of angle θ, explain how to find $\sin\theta$.

106. Explain why $\tan 90°$ is undefined.

107. If $\cos\theta > 0$ and $\tan\theta < 0$, explain how to find the quadrant in which θ lies.

108. What is a reference angle? Give an example with your description.

109. Explain how reference angles are used to evaluate trigonometric functions. Give an example with your description.

Critical Thinking Exercises

Make Sense? *In Exercises 110–113, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

110. I'm working with a quadrantal angle θ for which $\sin\theta$ is undefined.

111. This angle θ is in a quadrant in which $\sin\theta < 0$ and $\csc\theta > 0$.

112. I am given that $\tan\theta = \frac{3}{5}$, so I can conclude that $y = 3$ and $x = 5$.

113. When I found the exact value of $\cos\frac{14\pi}{3}$, I used a number of concepts, including coterminal angles, reference angles, finding the cosine of a special angle, and knowing the cosine's sign in various quadrants.

Preview Exercises

Exercises 114–116 will help you prepare for the material covered in the next section. In each exercise, complete the table of coordinates. Do not use a calculator.

114. $y = \frac{1}{2}\cos(4x + \pi)$

x	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
y					

115. $y = 4\sin\left(2x - \frac{2\pi}{3}\right)$

x	$\frac{\pi}{3}$	$\frac{7\pi}{12}$	$\frac{5\pi}{6}$	$\frac{13\pi}{12}$	$\frac{4\pi}{3}$
y					

116. $y = 3\sin\frac{\pi}{2}x$

x	0	$\frac{1}{3}$	1	$\frac{5}{3}$	2	$\frac{7}{3}$	3	$\frac{11}{3}$	4
y									

After completing this table of coordinates, plot the nine ordered pairs as points in a rectangular coordinate system. Then connect the points with a smooth curve.

Chapter 4 Mid-Chapter Check Point

What You Know: We learned to use radians to measure angles: One radian (approximately 57°) is the measure of the central angle that intercepts an arc equal in length to the radius of the circle. Using $180° = \pi$ radians, we converted degrees to radians $\left(\text{multiply by } \frac{\pi}{180°}\right)$ and radians to degrees $\left(\text{multiply by } \frac{180°}{\pi}\right)$. We defined the six trigonometric functions using coordinates of points along the unit circle, right triangles, and angles in standard position. Evaluating trigonometric functions using reference angles involved connecting a number of concepts, including finding coterminal and reference angles, locating special angles, determining the signs of the trigonometric functions in specific quadrants, and finding the function values at special angles. Use the important Study Tip on page 512 as a reference sheet to help connect these concepts.

In Exercises 1–2, convert each angle in degrees to radians. Express your answer as a multiple of π.

1. $10°$ **2.** $-105°$

In Exercises 3–4, convert each angle in radians to degrees.

3. $\frac{5\pi}{12}$ **4.** $-\frac{13\pi}{20}$

In Exercises 5–7,

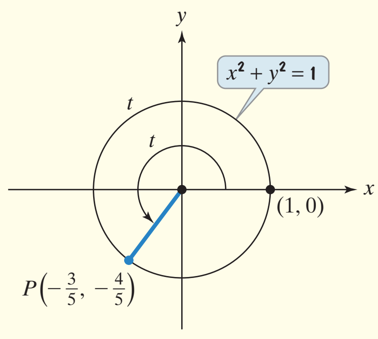
a. *Find a positive angle less than 360° or 2π that is coterminal with the given angle.*

b. *Draw the given angle in standard position.*

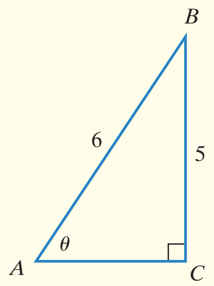
c. *Find the reference angle for the given angle.*

5. $\frac{11\pi}{3}$ **6.** $-\frac{19\pi}{4}$ **7.** $510°$

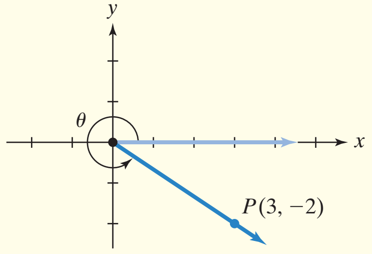
8. Use the point shown on the unit circle to find each of the six trigonometric functions at t.



9. Use the triangle to find each of the six trigonometric functions of θ.



10. Use the point on the terminal side of θ to find each of the six trigonometric functions of θ.



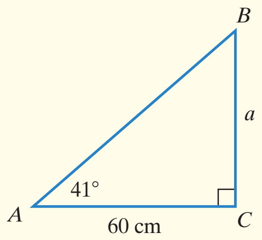
In Exercises 11–12, find the exact value of the remaining trigonometric functions of θ.

11. $\tan\theta = -\frac{3}{4}, \cos\theta < 0$

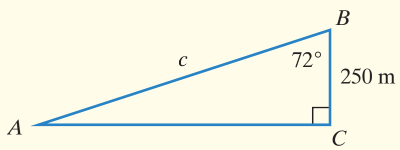
12. $\cos\theta = \frac{3}{7}, \sin\theta < 0$

In Exercises 13–14, find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round the answer to the nearest whole number.

13.



14.



15. If $\cos\theta = \frac{1}{6}$ and θ is acute, find $\cot\left(\frac{\pi}{2} - \theta\right)$.

In Exercises 16–26, find the exact value of each expression. Do not use a calculator.

16. $\tan 30°$

17. $\cot 120°$

18. $\cos 240°$

19. $\sec \frac{11\pi}{6}$

20. $\sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7}$

21. $\sin\left(-\frac{2\pi}{3}\right)$

22. $\csc\left(\frac{22\pi}{3}\right)$

23. $\cos 495°$

24. $\tan\left(-\frac{17\pi}{6}\right)$

25. $\sin^2 \frac{\pi}{2} - \cos\pi$

26. $\cos\left(\frac{5\pi}{6} + 2\pi n\right) + \tan\left(\frac{5\pi}{6} + n\pi\right)$, n is an integer.

27. A circle has a radius of 40 centimeters. Find the length of the arc intercepted by a central angle of 36°. Express the answer in terms of π. Then round to two decimal places.

28. A merry-go-round makes 8 revolutions per minute. Find the linear speed, in feet per minute, of a horse 10 feet from the center. Express the answer in terms of π. Then round to one decimal place.

29. A plane takes off at an angle of 6°. After traveling for one mile, or 5280 feet, along this flight path, find the plane's height, to the nearest tenth of a foot, above the ground.

30. A tree that is 50 feet tall casts a shadow that is 60 feet long. Find the angle of elevation, to the nearest degree, of the sun.

Section 4.5 Graphs of Sine and Cosine Functions

Objectives

1. Understand the graph of $y = \sin x$.
2. Graph variations of $y = \sin x$.
3. Understand the graph of $y = \cos x$.
4. Graph variations of $y = \cos x$.
5. Use vertical shifts of sine and cosine curves.
6. Model periodic behavior.

Take a deep breath and relax. Many relaxation exercises involve slowing down our breathing. Some people suggest that the way we breathe affects every part of our lives. Did you know that graphs of trigonometric functions can be used to analyze the breathing cycle, which is our closest link to both life and death?

In this section, we use graphs of sine and cosine functions to visualize their properties. We use the traditional symbol x, rather than θ or t, to represent the independent variable. We use the symbol y for the dependent variable, or the function's value at x. Thus, we will be graphing $y = \sin x$ and $y = \cos x$ in rectangular coordinates. In all graphs of trigonometric functions, the independent variable, x, is measured in radians.