56. Galileo's telescope brought about revolutionary changes in astronomy. A comparable leap in our ability to observe the universe took place as a result of the Hubble Space Telescope. The space telescope was able to see stars and galaxies whose brightness is $\frac{1}{50}$ of the faintest objects observable using ground-based telescopes. Use the fact that the brightness of a point source, such as a star, varies inversely as the square of its distance from an observer to show that the space telescope was able to see about seven times farther than a ground-based telescope.

Group Exercise

57. Begin by deciding on a product that interests the group because you are now in charge of advertising this product. Members were told that the demand for the product varies directly as the amount spent on advertising and inversely as the price of the product. However, as more money is spent on advertising, the price of your product rises. Under what conditions would members recommend an increased expense in advertising? Once you've determined what your product is, write formulas for the given conditions and experiment with hypothetical numbers. What other factors might you take into consideration in terms of your recommendation? How do these factor affect the demand for your product?

Preview Exercises

Exercises 58–60 will help you prepare for the material covered in the first section of the next chapter.

58. Use point plotting to graph $f(x) = 2^x$. Begin by setting up a partial table of coordinates, selecting integers from -3 to 3, inclusive, for x. Because $y = 0$ is a horizontal asymptote, your graph should approach, but never touch, the negative portion of the x-axis.

In Exercises 59–60, use transformations of your graph from Exercise 58 to graph each function.

59. $g(x) = f(-x) = 2^{-x}$　　　**60.** $h(x) = f(x) + 1 = 2^x + 1$

Chapter 2 Summary, Review, and Test

Summary

DEFINITIONS AND CONCEPTS	EXAMPLES
2.1　Complex Numbers	
a. The imaginary unit i is defined as $$i = \sqrt{-1}, \text{ where } i^2 = -1.$$ The set of numbers in the form $a + bi$ is called the set of complex numbers; a is the real part and b is the imaginary part. If $b = 0$, the complex number is a real number. If $b \neq 0$, the complex number is an imaginary number. Complex numbers in the form bi are called pure imaginary numbers.	Figure 2.1, p. 278
b. Rules for adding and subtracting complex numbers are given in the box on page 279.	Ex. 1, p. 279
c. To multiply complex numbers, multiply as if they are polynomials. After completing the multiplication, replace i^2 with -1 and simplify.	Ex. 2, p. 280
d. The complex conjugate of $a + bi$ is $a - bi$ and vice versa. The multiplication of complex conjugates gives a real number: $$(a + bi)(a - bi) = a^2 + b^2.$$	
e. To divide complex numbers, multiply the numerator and the denominator by the complex conjugate of the denominator.	Ex. 3, p. 281
f. When performing operations with square roots of negative numbers, begin by expressing all square roots in terms of i. The principal square root of $-b$ is defined by $$\sqrt{-b} = i\sqrt{b}.$$	Ex. 4, p. 282
g. Quadratic equations $(ax^2 + bx + c = 0, a \neq 0)$ with negative discriminants $(b^2 - 4ac < 0)$ have imaginary solutions that are complex conjugates.	Ex. 5, p. 283
2.2　Quadratic Functions	
a. A quadratic function is of the form $f(x) = ax^2 + bx + c, a \neq 0$.	
b. The standard form of a quadratic function is $f(x) = a(x - h)^2 + k, a \neq 0$.	

DEFINITIONS AND CONCEPTS	EXAMPLES
c. The graph of a quadratic function is a parabola. The vertex is (h, k) or $\left(-\dfrac{b}{2a}, f\left(-\dfrac{b}{2a}\right)\right)$. A procedure for graphing a quadratic function in standard form is given in the box on page 288. A procedure for graphing a quadratic function in the form $f(x) = ax^2 + bx + c$ is given in the box on page 290.	Ex. 1, p. 288; Ex. 2, p. 289; Ex. 3, p. 291
d. See the box on page 292 for minimum or maximum values of quadratic functions.	Ex. 4, p. 293; Ex. 5, p. 293
e. A strategy for solving problems involving maximizing or minimizing quadratic functions is given in the box on page 295.	Ex. 6, p. 296; Ex. 7, p. 297

2.3 Polynomial Functions and Their Graphs

a. Polynomial Function of Degree n: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0, a_n \neq 0$	
b. The graphs of polynomial functions are smooth and continuous.	Figure 2.13, p. 303
c. The end behavior of the graph of a polynomial function depends on the leading term, given by the Leading Coefficient Test in the box on page 304.	Ex. 1, p. 304; Ex. 2, p. 305; Ex. 3, p. 305; Ex. 4, p. 306
d. The values of x for which $f(x)$ is equal to 0 are the zeros of the polynomial function f. These values are the roots, or solutions, of the polynomial equation $f(x) = 0$.	Ex. 5, p. 306; Ex. 6, p. 307
e. If $x - r$ occurs k times in a polynomial function's factorization, r is a repeated zero with multiplicity k. If k is even, the graph touches the x-axis and turns around at r. If k is odd, the graph crosses the x-axis at r.	Ex. 7, p. 308
f. The Intermediate Value Theorem: If f is a polynomial function and $f(a)$ and $f(b)$ have opposite signs, there is at least one value of c between a and b for which $f(c) = 0$.	Ex. 8, p. 309
g. If f is a polynomial of degree n, the graph of f has at most $n - 1$ turning points.	Figure 2.24, p. 310
h. A strategy for graphing a polynomial function is given in the box on page 310.	Ex. 9, p. 310

2.4 Dividing Polynomials; Remainder and Factor Theorems

a. Long division of polynomials is performed by dividing, multiplying, subtracting, bringing down the next term, and repeating this process until the degree of the remainder is less than the degree of the divisor. The details are given in the box on page 317.	Ex. 1, p. 316; Ex. 2, p. 318; Ex. 3, p. 319
b. The Division Algorithm: $f(x) = d(x)q(x) + r(x)$. The dividend is the product of the divisor and the quotient plus the remainder.	
c. Synthetic division is used to divide a polynomial by $x - c$. The details are given in the box on page 320.	Ex. 4, p. 321
d. The Remainder Theorem: If a polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.	Ex. 5, p. 322
e. The Factor Theorem: If $x - c$ is a factor of a polynomial function $f(x)$, then c is a zero of f and a root of $f(x) = 0$. If c is a zero of f or a root of $f(x) = 0$, then $x - c$ is a factor of $f(x)$.	Ex. 6, p. 323

2.5 Zeros of Polynomial Functions

a. The Rational Zero Theorem states that the possible rational zeros of a polynomial function $= \dfrac{\text{Factors of the constant term}}{\text{Factors of the leading coefficient}}$. The theorem is stated in the box on page 327.	Ex. 1, p. 327; Ex. 2, p. 328; Ex. 3, p. 328; Ex. 4, p. 329; Ex. 5, p. 330
b. Number of roots: If $f(x)$ is a polynomial of degree $n \geq 1$, then, counting multiple roots separately, the equation $f(x) = 0$ has n roots.	
c. If $a + bi$ is a root of $f(x) = 0$ with real coefficients, then $a - bi$ is also a root.	
d. The Linear Factorization Theorem: An nth-degree polynomial can be expressed as the product of n linear factors. Thus, $f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$.	Ex. 6, p. 333
e. Descartes's Rule of Signs: The number of positive real zeros of f equals the number of sign changes of $f(x)$ or is less than that number by an even integer. The number of negative real zeros of f applies a similar statement to $f(-x)$.	Table 2.1, p. 334; Ex. 7, p. 334

DEFINITIONS AND CONCEPTS	EXAMPLES

2.6 Rational Functions and Their Graphs

a. Rational function: $f(x) = \dfrac{p(x)}{q(x)}$; p and q are polynomial functions and $q(x) \neq 0$. The domain of f is the set of all real numbers excluding values of x that make $q(x)$ zero.   Ex. 1, p. 340

b. Arrow notation is summarized in the box on page 342.

c. The line $x = a$ is a vertical asymptote of the graph of f if $f(x)$ increases or decreases without bound as x approaches a. Vertical asymptotes are located using the theorem in the box on page 343.   Ex. 2, p. 343

d. The line $y = b$ is a horizontal asymptote of the graph of f if $f(x)$ approaches b as x increases or decreases without bound. Horizontal asymptotes are located using the theorem in the box on page 345.   Ex. 3, p. 346

e. Table 2.2 on page 346 shows the graphs of $f(x) = \dfrac{1}{x}$ and $f(x) = \dfrac{1}{x^2}$. Some rational functions can be graphed using transformations of these common graphs.   Ex. 4, p. 347

f. A strategy for graphing rational functions is given in the box on page 347.   Ex. 5, p. 348; Ex. 6, p. 349; Ex. 7, p. 350

g. The graph of a rational function has a slant asymptote when the degree of the numerator is one more than the degree of the denominator. The equation of the slant asymptote is found using division and dropping the remainder term.   Ex. 8, p. 352

2.7 Polynomial and Rational Inequalities

a. A polynomial inequality can be expressed as $f(x) < 0$, $f(x) > 0$, $f(x) \leq 0$, or $f(x) \geq 0$, where f is a polynomial function. A procedure for solving polynomial inequalities is given in the box on page 360.   Ex. 1, p. 360; Ex. 2, p. 361

b. A rational inequality can be expressed as $f(x) < 0$, $f(x) > 0$, $f(x) \leq 0$, or $f(x) \geq 0$, where f is a rational function. The procedure for solving such inequalities begins with expressing them so that one side is zero and the other side is a single quotient. Find boundary points by setting the numerator and denominator equal to zero. Then follow a procedure similar to that for solving polynomial inequalities.   Ex. 3, p. 363

2.8 Modeling Using Variation

a. A procedure for solving variation problems is given in the box on page 370.

b.

English Statement	**Equation**	
y varies directly as x. y is directly proportional to x.	$y = kx$	Ex. 1, p. 370
y varies directly as x^n. y is directly proportional to x^n.	$y = kx^n$	Ex. 2, p. 372
y varies inversely as x. y is inversely proportional to x.	$y = \dfrac{k}{x}$	Ex. 3, p. 373; Ex. 4, p. 374
y varies inversely as x^n. y is inversely proportional to x^n.	$y = \dfrac{k}{x^n}$	
y varies jointly as x and z.	$y = kxz$	Ex. 5, p. 376

Review Exercises

2.1

In Exercises 1–10 perform the indicated operations and write the result in standard form.

1. $(8 - 3i) - (17 - 7i)$

2. $4i(3i - 2)$

3. $(7 - i)(2 + 3i)$

4. $(3 - 4i)^2$

5. $(7 + 8i)(7 - 8i)$

6. $\dfrac{6}{5 + i}$

7. $\dfrac{3 + 4i}{4 - 2i}$

8. $\sqrt{-32} - \sqrt{-18}$

9. $\left(-2 + \sqrt{-100}\right)^2$

10. $\dfrac{4 + \sqrt{-8}}{2}$

In Exercises 11–12, solve each quadratic equation using the quadratic formula. Express solutions in standard form.

11. $x^2 - 2x + 4 = 0$

12. $2x^2 - 6x + 5 = 0$

2.2

In Exercises 13–16, use the vertex and intercepts to sketch the graph of each quadratic function. Give the equation for the parabola's axis of symmetry. Use the graph to determine the function's domain and range.

13. $f(x) = -(x + 1)^2 + 4$

14. $f(x) = (x + 4)^2 - 2$

15. $f(x) = -x^2 + 2x + 3$

16. $f(x) = 2x^2 - 4x - 6$

In Exercises 17–18, use the function's equation, and not its graph, to find

   a. *the minimum or maximum value and where it occurs.*

   b. *the function's domain and its range.*

17. $f(x) = -x^2 + 14x - 106$ **18.** $f(x) = 2x^2 + 12x + 703$

19. A quarterback tosses a football to a receiver 40 yards downfield. The height of the football, $f(x)$, in feet, can be modeled by

$$f(x) = -0.025x^2 + x + 6,$$

where x is the ball's horizontal distance, in yards, from the quarterback.

   a. What is the ball's maximum height and how far from the quarterback does this occur?

   b. From what height did the quarterback toss the football?

   c. If the football is not blocked by a defensive player nor caught by the receiver, how far down the field will it go before hitting the ground? Round to the nearest tenth of a yard.

   d. Graph the function that models the football's parabolic path.

20. A field bordering a straight stream is to be enclosed. The side bordering the stream is not to be fenced. If 1000 yards of fencing material is to be used, what are the dimensions of the largest rectangular field that can be fenced? What is the maximum area?

21. Among all pairs of numbers whose difference is 14, find a pair whose product is as small as possible. What is the minimum product?

22. You have 1000 feet of fencing to construct six corrals, as shown in the figure. Find the dimensions that maximize the enclosed area. What is the maximum area?
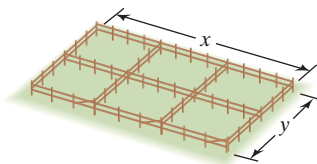
23. The annual yield per fruit tree is fairly constant at 150 pounds per tree when the number of trees per acre is 35 or fewer. For each additional tree over 35, the annual yield per tree for all trees on the acre decreases by 4 pounds due to overcrowding. How many fruit trees should be planted per acre to maximize the annual yield for the acre? What is the maximum number of pounds of fruit per acre?
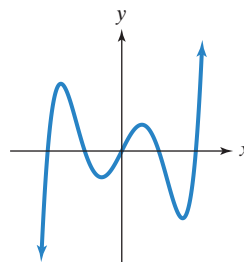
2.3

In Exercises 24–27, use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function. Then use this end behavior to match the polynomial function with its graph. [The graphs are labeled (a) through (d).]
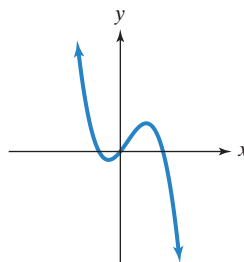
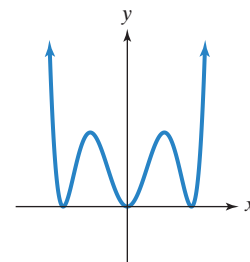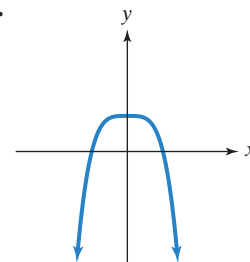**24.** $f(x) = -x^3 + x^2 + 2x$ **25.** $f(x) = x^6 - 6x^4 + 9x^2$

26. $f(x) = x^5 - 5x^3 + 4x$ **27.** $f(x) = -x^4 + 1$

a.

b.

c.

d.

28. The polynomial function

$$f(x) = -0.87x^3 + 0.35x^2 + 81.62x + 7684.94$$

models the number of thefts, $f(x)$, in thousands, in the United States x years after 1987. Will this function be useful in modeling the number of thefts over an extended period of time? Explain your answer.

29. A herd of 100 elk is introduced to a small island. The number of elk, $f(x)$, after x years is modeled by the polynomial function

$$f(x) = -x^4 + 21x^2 + 100.$$

Use the Leading Coefficient Test to determine the graph's end behavior to the right. What does this mean about what will eventually happen to the elk population?

In Exercises 30–31, find the zeros for each polynomial function and give the multiplicity of each zero. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each zero.

30. $f(x) = -2(x - 1)(x + 2)^2(x + 5)^3$

31. $f(x) = x^3 - 5x^2 - 25x + 125$

32. Show that $f(x) = x^3 - 2x - 1$ has a real zero between 1 and 2.

In Exercises 33–38,

   a. *Use the Leading Coefficient Test to determine the graph's end behavior.*

   b. *Determine whether the graph has y-axis symmetry, origin symmetry, or neither.*

   c. *Graph the function.*

33. $f(x) = x^3 - x^2 - 9x + 9$ **34.** $f(x) = 4x - x^3$

35. $f(x) = 2x^3 + 3x^2 - 8x - 12$ **36.** $f(x) = -x^4 + 25x^2$

37. $f(x) = -x^4 + 6x^3 - 9x^2$ **38.** $f(x) = 3x^4 - 15x^3$

In Exercises 39–40, graph each polynomial function.

39. $f(x) = 2x^2(x - 1)^3(x + 2)$

40. $f(x) = -x^3(x + 4)^2(x - 1)$

2.4

In Exercises 41–43, divide using long division.

41. $(4x^3 - 3x^2 - 2x + 1) \div (x + 1)$

42. $(10x^3 - 26x^2 + 17x - 13) \div (5x - 3)$

43. $(4x^4 + 6x^3 + 3x - 1) \div (2x^2 + 1)$

In Exercises 44–45, divide using synthetic division.

44. $(3x^4 + 11x^3 - 20x^2 + 7x + 35) \div (x + 5)$

45. $(3x^4 - 2x^2 - 10x) \div (x - 2)$

46. Given $f(x) = 2x^3 - 7x^2 + 9x - 3$, use the Remainder Theorem to find $f(-13)$.

47. Use synthetic division to divide $f(x) = 2x^3 + x^2 - 13x + 6$ by $x - 2$. Use the result to find all zeros of f.

48. Solve the equation $x^3 - 17x + 4 = 0$ given that 4 is a root.

2.5

In Exercises 49–50, use the Rational Zero Theorem to list all possible rational zeros for each given function.

49. $f(x) = x^4 - 6x^3 + 14x^2 - 14x + 5$

50. $f(x) = 3x^5 - 2x^4 - 15x^3 + 10x^2 + 12x - 8$

In Exercises 51–52, use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for each given function.

51. $f(x) = 3x^4 - 2x^3 - 8x + 5$

52. $f(x) = 2x^5 - 3x^3 - 5x^2 + 3x - 1$

53. Use Descartes's Rule of Signs to explain why $2x^4 + 6x^2 + 8 = 0$ has no real roots.

For Exercises 54–60,

   a. *List all possible rational roots or rational zeros.*
   b. *Use Descartes's Rule of Signs to determine the possible number of positive and negative real roots or real zeros.*
   c. *Use synthetic division to test the possible rational roots or zeros and find an actual root or zero.*
   d. *Use the quotient from part (c) to find all the remaining roots or zeros.*

54. $f(x) = x^3 + 3x^2 - 4$ **55.** $f(x) = 6x^3 + x^2 - 4x + 1$

56. $8x^3 - 36x^2 + 46x - 15 = 0$ **57.** $2x^3 + 9x^2 - 7x + 1 = 0$

58. $x^4 - x^3 - 7x^2 + x + 6 = 0$ **59.** $4x^4 + 7x^2 - 2 = 0$

60. $f(x) = 2x^4 + x^3 - 9x^2 - 4x + 4$

In Exercises 61–62, find an nth-degree polynomial function with real coefficients satisfying the given conditions. If you are using a graphing utility, graph the function and verify the real zeros and the given function value.

61. $n = 3$; 2 and $2 - 3i$ are zeros; $f(1) = -10$

62. $n = 4$; i is a zero; -3 is a zero of multiplicity 2; $f(-1) = 16$

In Exercises 63–64, find all the zeros of each polynomial function and write the polynomial as a product of linear factors.

63. $f(x) = 2x^4 + 3x^3 + 3x - 2$

64. $g(x) = x^4 - 6x^3 + x^2 + 24x + 16$

In Exercises 65–68, graphs of fifth-degree polynomial functions are shown. In each case, specify the number of real zeros and the number of imaginary zeros. Indicate whether there are any real zeros with multiplicity other than 1.

65.

66.

67.

68.
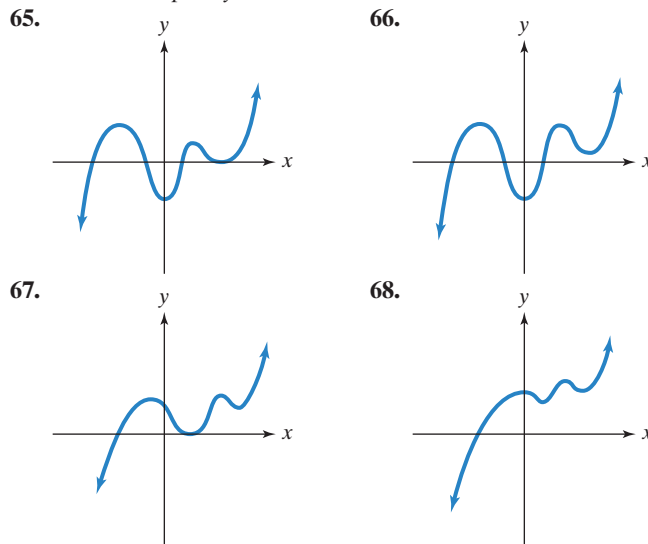
2.6

In Exercises 69–70, use transformations of $f(x) = \dfrac{1}{x}$ or $f(x) = \dfrac{1}{x^2}$ to graph each rational function.

69. $g(x) = \dfrac{1}{(x + 2)^2} - 1$ **70.** $h(x) = \dfrac{1}{x - 1} + 3$

In Exercises 71–78, find the vertical asymptotes, if any, the horizontal asymptote, if one exists, and the slant asymptote, if there is one, of the graph of each rational function. Then graph the rational function.

71. $f(x) = \dfrac{2x}{x^2 - 9}$ **72.** $g(x) = \dfrac{2x - 4}{x + 3}$

73. $h(x) = \dfrac{x^2 - 3x - 4}{x^2 - x - 6}$ **74.** $r(x) = \dfrac{x^2 + 4x + 3}{(x + 2)^2}$

75. $y = \dfrac{x^2}{x + 1}$ **76.** $y = \dfrac{x^2 + 2x - 3}{x - 3}$

77. $f(x) = \dfrac{-2x^3}{x^2 + 1}$ **78.** $g(x) = \dfrac{4x^2 - 16x + 16}{2x - 3}$

79. A company is planning to manufacture affordable graphing calculators. The fixed monthly cost will be $50,000 and it will cost $25 to produce each calculator.

   a. Write the cost function, C, of producing x graphing calculators.
   b. Write the average cost function, $\overline{C}$, of producing x graphing calculators.
   c. Find and interpret $\overline{C}(50)$, $\overline{C}(100)$, $\overline{C}(1000)$, and $\overline{C}(100,000)$.
   d. What is the horizontal asymptote for the graph of this function and what does it represent?

Exercises 80–81 involve rational functions that model the given situations. In each case, find the horizontal asymptote as $x \to \infty$ and then describe what this means in practical terms.

80. $f(x) = \dfrac{150x + 120}{0.05x + 1}$; the number of bass, $f(x)$, after x months in a lake that was stocked with 120 bass

81. $P(x) = \dfrac{72,900}{100x^2 + 729}$; the percentage, $P(x)$, of people in the United States with x years of education who are unemployed

82. The bar graph shows the population of the United States, in millions, for five selected years.

Population of the United States

■ Male   ■ Female
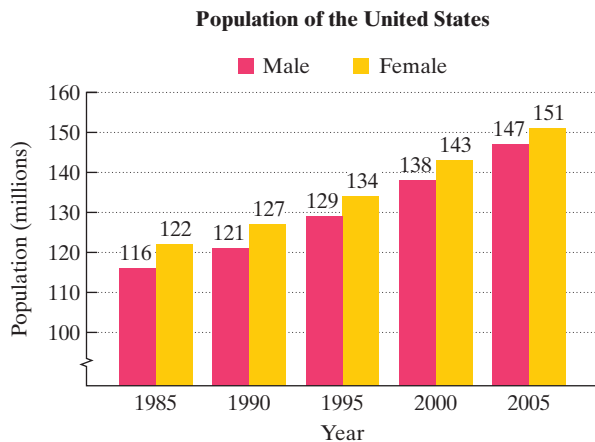
Source: U.S. Census Bureau

Here are two functions that model the data:

$$M(x) = 1.58x + 114.4$$

Male U.S. population, $M(x)$, in millions, x years after 1985

$$F(x) = 1.48x + 120.6.$$

Female U.S. population, $F(x)$, in millions, x years after 1985

a. Write a function that models the total U.S. population, $P(x)$, in millions, x years after 1985.

b. Write a rational function that models the fraction of men in the U.S. population, $R(x)$, x years after 1985.

c. What is the equation of the horizontal asymptote associated with the function in part (b)? Round to two decimal places. What does this mean about the percentage of men in the U.S. population over time?

83. A jogger ran 4 miles and then walked 2 miles. The average velocity running was 3 miles per hour faster than the average velocity walking. Express the total time for running and walking, T, as a function of the average velocity walking, x.

84. The area of a rectangular floor is 1000 square feet. Express the perimeter of the floor, P, as a function of the width of the rectangle, x.

2.7

In Exercises 85–90, solve each inequality and graph the solution set on a real number line.

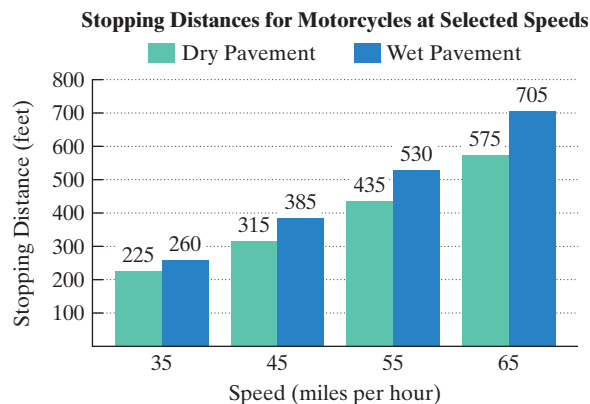**85.** $2x^2 + 5x - 3 < 0$

86. $2x^2 + 9x + 4 \geq 0$

87. $x^3 + 2x^2 > 3x$

88. $\dfrac{x - 6}{x + 2} > 0$

89. $\dfrac{(x + 1)(x - 2)}{x - 1} \geq 0$

90. $\dfrac{x + 3}{x - 4} \leq 5$

91. The graph shows stopping distances for motorcycles at various speeds on dry roads and on wet roads.

Stopping Distances for Motorcycles at Selected Speeds

■ Dry Pavement   ■ Wet Pavement

Source: National Highway Traffic Safety Administration

The functions

$$f(x) = 0.125x^2 - 0.8x + 99$$

Dry pavement

and   Wet pavement

$$g(x) = 0.125x^2 + 2.3x + 27$$

model a motorcycle's stopping distance, $f(x)$ or $g(x)$, in feet, traveling at x miles per hour. Function f models stopping distance on dry pavement and function g models stopping distance on wet pavement.

a. Use function g to find the stopping distance on wet pavement for a motorcycle traveling at 35 miles per hour. Round to the nearest foot. Does your rounded answer overestimate or underestimate the stopping distance shown by the graph? By how many feet?

b. Use function f to determine speeds on dry pavement requiring stopping distances that exceed 267 feet.

92. Use the position function

$$s(t) = -16t^2 + v_0 t + s_0$$

to solve this problem. A projectile is fired vertically upward from ground level with an initial velocity of 48 feet per second. During which time period will the projectile's height exceed 32 feet?

2.8

Solve the variation problems in Exercises 93–98.

93. Many areas of Northern California depend on the snowpack of the Sierra Nevada mountain range for their water supply. The volume of water produced from melting snow varies directly as the volume of snow. Meteorologists have determined that 250 cubic centimeters of snow will melt to 28 cubic centimeters of water. How much water does 1200 cubic centimeters of melting snow produce?

94. The distance that a body falls from rest is directly proportional to the square of the time of the fall. If skydivers fall 144 feet in 3 seconds, how far will they fall in 10 seconds?

95. The pitch of a musical tone varies inversely as its wavelength. A tone has a pitch of 660 vibrations per second and a wavelength of 1.6 feet. What is the pitch of a tone that has a wavelength of 2.4 feet?

96. The loudness of a stereo speaker, measured in decibels, varies inversely as the square of your distance from the speaker. When you are 8 feet from the speaker, the loudness is 28 decibels. What is the loudness when you are 4 feet from the speaker?

97. The time required to assemble computers varies directly as the number of computers assembled and inversely as the number of workers. If 30 computers can be assembled by 6 workers in 10 hours, how long would it take 5 workers to assemble 40 computers?

98. The volume of a pyramid varies jointly as its height and the area of its base. A pyramid with a height of 15 feet and a base with an area of 35 square feet has a volume of 175 cubic feet. Find the volume of a pyramid with a height of 20 feet and a base with an area of 120 square feet.

99. Heart rates and life spans of most mammals can be modeled using inverse variation. The bar graph shows the average heart rate and the average life span of five mammals.

Heart Rate and Life Span
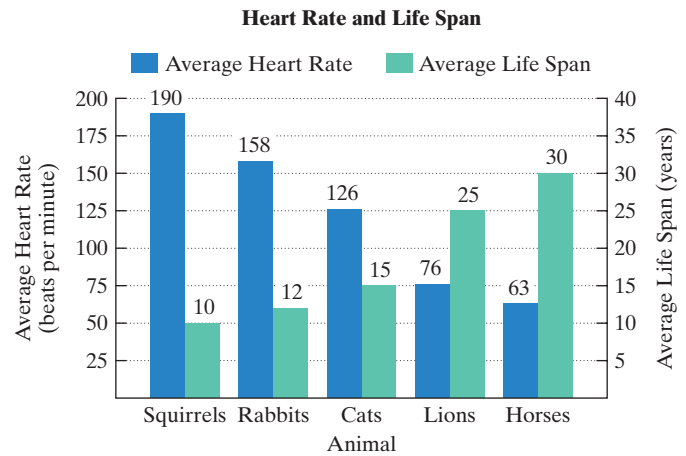
Source: *The Handy Science Answer Book,* Visible Ink Press, 2003

a. A mammal's average life span, L, in years, varies inversely as its average heart rate, R, in beats per minute. Use the data shown for horses to write the equation that models this relationship.

b. Is the inverse variation equation in part (a) an exact model or an approximate model for the data shown for lions?

c. Elephants have an average heart rate of 27 beats per minute. Determine their average life span.

CHAPTER Test Prep VIDEOS

Chapter 2   Test

In Exercises 1–3, perform the indicated operations and write the result in standard form.

1. $(6 - 7i)(2 + 5i)$

2. $\dfrac{5}{2 - i}$

3. $2\sqrt{-49} + 3\sqrt{-64}$

4. Solve and express solutions in standard form: $x^2 = 4x - 8$.

In Exercises 5–6, use the vertex and intercepts to sketch the graph of each quadratic function. Give the equation for the parabola's axis of symmetry. Use the graph to determine the function's domain and range.

5. $f(x) = (x + 1)^2 + 4$

6. $f(x) = x^2 - 2x - 3$

7. Determine, without graphing, whether the quadratic function $f(x) = -2x^2 + 12x - 16$ has a minimum value or a maximum value. Then find
 a. the minimum or maximum value and where it occurs.
 b. the function's domain and its range.

8. The function $f(x) = -x^2 + 46x - 360$ models the daily profit, $f(x)$, in hundreds of dollars, for a company that manufactures x computers daily. How many computers should be manufactured each day to maximize profit? What is the maximum daily profit?

9. Among all pairs of numbers whose sum is 14, find a pair whose product is as large as possible. What is the maximum product?

**10.** Consider the function $f(x) = x^3 - 5x^2 - 4x + 20$.
 a. Use factoring to find all zeros of f.
 b. Use the Leading Coefficient Test and the zeros of f to graph the function.

11. Use end behavior to explain why the graph at the top of the next column cannot be the graph of $f(x) = x^5 - x$. Then use

intercepts to explain why the graph cannot represent $f(x) = x^5 - x$.
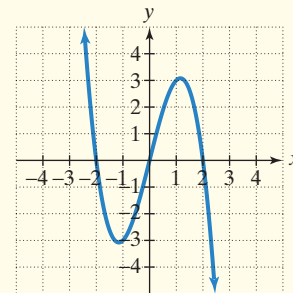
**12.** The graph of $f(x) = 6x^3 - 19x^2 + 16x - 4$ is shown in the figure.
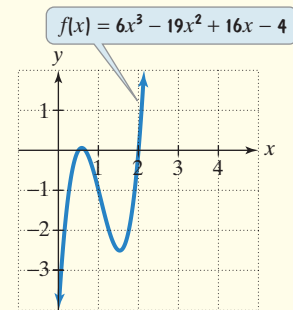
$f(x) = 6x^3 - 19x^2 + 16x - 4$

a. Based on the graph of f, find the root of the equation $6x^3 - 19x^2 + 16x - 4 = 0$ that is an integer.

b. Use synthetic division to find the other two roots of $6x^3 - 19x^2 + 16x - 4 = 0$.

13. Use the Rational Zero Theorem to list all possible rational zeros of $f(x) = 2x^3 + 11x^2 - 7x - 6$.

14. Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros of
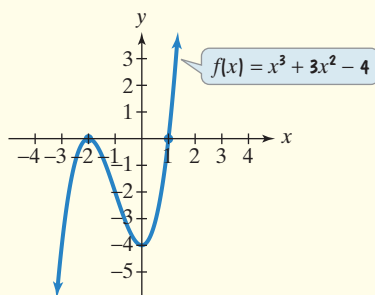$$f(x) = 3x^5 - 2x^4 - 2x^2 + x - 1.$$

15. Solve: $x^3 + 9x^2 + 16x - 6 = 0$.

16. Consider the function whose equation is given by $f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15$.

   a. List all possible rational zeros.

   b. Use the graph of f in the figure shown and synthetic division to find all zeros of the function.

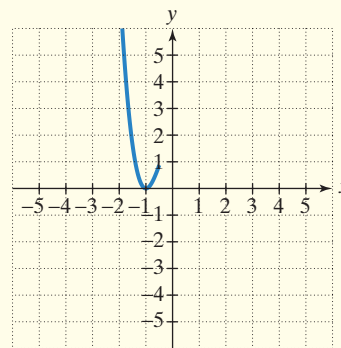$f(x) = 2x^4 - x^3 - 13x^2 + 5x + 15$
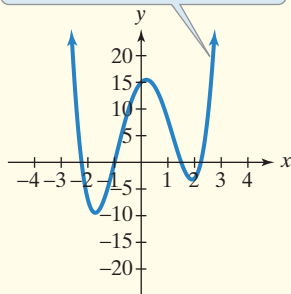
17. Use the graph of $f(x) = x^3 + 3x^2 - 4$ in the figure shown to factor $x^3 + 3x^2 - 4$.

$f(x) = x^3 + 3x^2 - 4$

18. Find a fourth-degree polynomial function $f(x)$ with real coefficients that has $-1, 1$, and i as zeros and such that $f(3) = 160$.

19. The figure shows an incomplete graph of $f(x) = -3x^3 - 4x^2 + x + 2$. Find all the zeros of the function. Then draw a complete graph.

In Exercises 20–25, find the domain of each rational function and graph the function.

20. $f(x) = \dfrac{1}{(x + 3)^2}$

21. $f(x) = \dfrac{1}{x - 1} + 2$

22. $f(x) = \dfrac{x}{x^2 - 16}$

23. $f(x) = \dfrac{x^2 - 9}{x - 2}$

24. $f(x) = \dfrac{x + 1}{x^2 + 2x - 3}$

25. $f(x) = \dfrac{4x^2}{x^2 + 3}$

26. A company is planning to manufacture portable satellite radio players. The fixed monthly cost will be $300,000 and it will cost $10 to produce each player.

   a. Write the average cost function, $\overline{C}$, of producing x players.

   **b.** What is the horizontal asymptote for the graph of this function and what does it represent?

Solve each inequality in Exercises 27–28 and graph the solution set on a real number line. Express each solution set in interval notation.

27. $x^2 < x + 12$

28. $\dfrac{2x + 1}{x - 3} \le 3$

29. The intensity of light received at a source varies inversely as the square of the distance from the source. A particular light has an intensity of 20 foot-candles at 15 feet. What is the light's intensity at 10 feet?

Cumulative Review Exercises (Chapters P–2)

Use the graph of $y = f(x)$ to solve Exercises 1–6.

$y = f(x)$
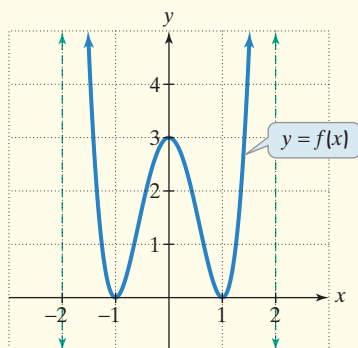
1. Find the domain and the range of f.

2. Find the zeros and the least possible multiplicity of each zero.

3. Where does the relative maximum occur?

4. Find $(f \circ f)(-1)$.

5. Use arrow notation to complete this statement: $f(x) \to \infty$ as _____ or as _____.

6. Graph $g(x) = f(x + 2) + 1$.

In Exercises 7–12, solve each equation or inequality.

7. $|2x - 1| = 3$

8. $3x^2 - 5x + 1 = 0$

9. $9 + \dfrac{3}{x} = \dfrac{2}{x^2}$

10. $x^3 + 2x^2 - 5x - 6 = 0$

11. $|2x - 5| > 3$

12. $3x^2 > 2x + 5$

In Exercises 13–18, graph each equation in a rectangular coordinate system. If two functions are given, graph both in the same system.

13. $f(x) = x^3 - 4x^2 - x + 4$

14. $f(x) = x^2 + 2x - 8$

15. $f(x) = x^2(x - 3)$

16. $f(x) = \dfrac{x - 1}{x - 2}$

17. $f(x) = |x|$ and $g(x) = -|x| - 1$

18. $x^2 + y^2 - 2x + 4y - 4 = 0$

In Exercises 19–20, let $f(x) = 2x^2 - x - 1$ and $g(x) = 4x - 1$.

19. Find $(f \circ g)(x)$.

20. Find $\dfrac{f(x + h) - f(x)}{h}$.