115. This work by artist Scott Kim (1955–) has the same kind of symmetry as an even function.

"DYSLEXIA," 1981

116. I graphed

$$f(x) = \begin{cases} 2 & \text{if} \quad x \neq 4 \\ 3 & \text{if} \quad x = 4 \end{cases}$$

and one piece of my graph is a single point.

117. I noticed that the difference quotient is always zero if $f(x) = c$, where c is any constant.

118. Sketch the graph of f using the following properties. (More than one correct graph is possible.) f is a piecewise function that is decreasing on $(-\infty, 2)$, $f(2) = 0$, f is increasing on $(2, \infty)$, and the range of f is $[0, \infty)$.

119. Define a piecewise function on the intervals $(-\infty, 2]$, $(2, 5)$, and $[5, \infty)$ that does not "jump" at 2 or 5 such that one piece is a constant function, another piece is an increasing function, and the third piece is a decreasing function.

120. Suppose that $h(x) = \frac{f(x)}{g(x)}$. The function f can be even, odd, or neither. The same is true for the function g.

a. Under what conditions is h definitely an even function?

b. Under what conditions is h definitely an odd function?

Group Exercise

121. (For assistance with this exercise, refer to the discussion of piecewise functions beginning on page 169, as well as to Exercises 79–80.) Group members who have cellular phone plans should describe the total monthly cost of the plan as follows:

> $\$$_____ per month buys _____ minutes. Additional time costs $\$$_____ per minute.

(For simplicity, ignore other charges.) The group should select any three plans, from "basic" to "premier." For each plan selected, write a piecewise function that describes the plan and graph the function. Graph the three functions in the same rectangular coordinate system. Now examine the graphs. For any given number of calling minutes, the best plan is the one whose graph is lowest at that point. Compare the three calling plans. Is one plan always a better deal than the other two? If not, determine the interval of calling minutes for which each plan is the best deal. (You can check out cellular phone plans by visiting www.point.com.)

Preview Exercises

Exercises 122–124 will help you prepare for the material covered in the next section.

122. If $(x_1, y_1) = (-3, 1)$ and $(x_2, y_2) = (-2, 4)$, find $\frac{y_2 - y_1}{x_2 - x_1}$.

123. Find the ordered pairs (_____, 0) and (0, _____) satisfying $4x - 3y - 6 = 0$.

124. Solve for y: $3x + 2y - 4 = 0$.

Section 1.4 Linear Functions and Slope

Objectives

1. Calculate a line's slope.
2. Write the point-slope form of the equation of a line.
3. Write and graph the slope-intercept form of the equation of a line.
4. Graph horizontal or vertical lines.
5. Recognize and use the general form of a line's equation.
6. Use intercepts to graph the general form of a line's equation.
7. Model data with linear functions and make predictions.

Is there a relationship between literacy and child mortality? As the percentage of adult females who are literate increases, does the mortality of children under five decrease? **Figure 1.35** on the next page indicates that this is, indeed, the case. Each point in the figure represents one country.

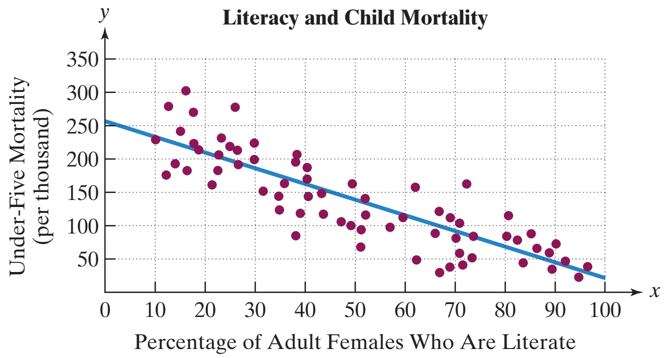


Figure 1.35
Source: United Nations

Data presented in a visual form as a set of points is called a **scatter plot**. Also shown in **Figure 1.35** is a line that passes through or near the points. A line that best fits the data points in a scatter plot is called a **regression line**. By writing the equation of this line, we can obtain a model for the data and make predictions about child mortality based on the percentage of literate adult females in a country.

Data often fall on or near a line. In this section, we will use functions to model such data and make predictions. We begin with a discussion of a line's steepness.

1 Calculate a line's slope.

The Slope of a Line

Mathematicians have developed a useful measure of the steepness of a line, called the *slope* of the line. Slope compares the vertical change (the **rise**) to the horizontal change (the **run**) when moving from one fixed point to another along the line. To calculate the slope of a line, we use a ratio that compares the change in y (the rise) to the corresponding change in x (the run).

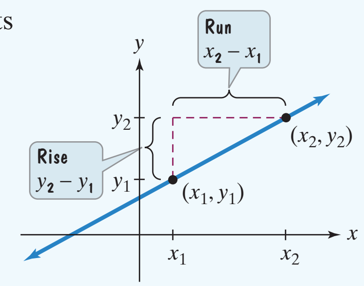
Definition of Slope

The **slope** of the line through the distinct points (x_1, y_1) and (x_2, y_2) is

$$\frac{\text{Change in } y}{\text{Change in } x} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1},$$

(Rise: Vertical change; Run: Horizontal change)

where $x_2 - x_1 \neq 0$.



It is common notation to let the letter m represent the slope of a line. The letter m is used because it is the first letter of the French verb *monter*, meaning "to rise" or "to ascend."

EXAMPLE 1 Using the Definition of Slope

Find the slope of the line passing through each pair of points:

a. $(-3, -1)$ and $(-2, 4)$ **b.** $(-3, 4)$ and $(2, -2)$.

Solution

a. Let $(x_1, y_1) = (-3, -1)$ and $(x_2, y_2) = (-2, 4)$. We obtain the slope as follows:

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{-2 - (-3)} = \frac{5}{1} = 5.$$

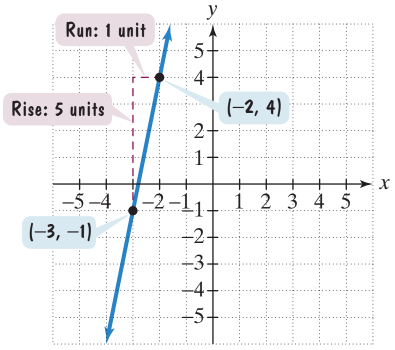


Figure 1.36 Visualizing a slope of 5

The situation is illustrated in **Figure 1.36**. The slope of the line is 5. For every vertical change, or rise, of 5 units, there is a corresponding horizontal change, or run, of 1 unit. The slope is positive and the line rises from left to right.

Study Tip

When computing slope, it makes no difference which point you call (x_1, y_1) and which point you call (x_2, y_2). If we let $(x_1, y_1) = (-2, 4)$ and $(x_2, y_2) = (-3, -1)$, the slope is still 5:

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{-3 - (-2)} = \frac{-5}{-1} = 5.$$

However, you should not subtract in one order in the numerator $(y_2 - y_1)$ and then in a different order in the denominator $(x_1 - x_2)$.

$$\frac{-1 - 4}{-2 - (-3)} = \frac{-5}{1} = -5.$$ Incorrect! The slope is not −5.

b. To find the slope of the line passing through $(-3, 4)$ and $(2, -2)$, we can let $(x_1, y_1) = (-3, 4)$ and $(x_2, y_2) = (2, -2)$. The slope of the line is computed as follows:

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 4}{2 - (-3)} = \frac{-6}{5} = -\frac{6}{5}.$$

The situation is illustrated in **Figure 1.37**. The slope of the line is $-\frac{6}{5}$. For every vertical change of −6 units (6 units down), there is a corresponding horizontal change of 5 units. The slope is negative and the line falls from left to right.

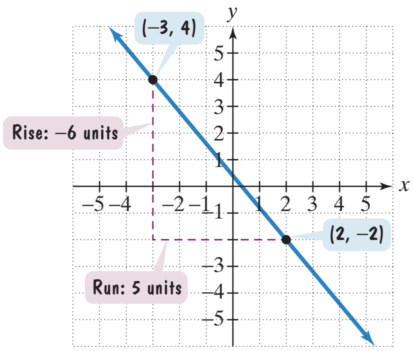


Figure 1.37 Visualizing a slope of $-\frac{6}{5}$

Check Point 1 Find the slope of the line passing through each pair of points:

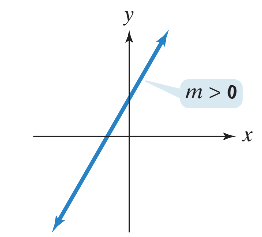
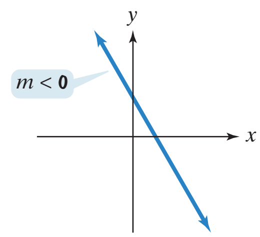
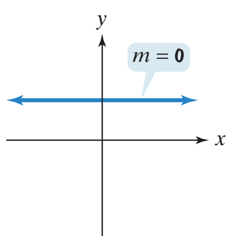
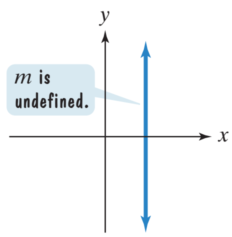
a. $(-3, 4)$ and $(-4, -2)$ **b.** $(4, -2)$ and $(-1, 5)$.

Study Tip

Always be clear in the way you use language, especially in mathematics. For example, it's not a good idea to say that a line has "no slope." This could mean that the slope is zero or that the slope is undefined.

Example 1 illustrates that a line with a positive slope is increasing and a line with a negative slope is decreasing. By contrast, a horizontal line is a constant function and has a slope of zero. A vertical line has no horizontal change, so $x_2 - x_1 = 0$ in the formula for slope. Because we cannot divide by zero, the slope of a vertical line is undefined. This discussion is summarized in **Table 1.2**.

Table 1.2 Possibilities for a Line's Slope

Positive Slope	Negative Slope	Zero Slope	Undefined Slope
 Line rises from left to right.	 Line falls from left to right.	 Line is horizontal.	 Line is vertical.

2 Write the point-slope form of the equation of a line.

The Point-Slope Form of the Equation of a Line

We can use the slope of a line to obtain various forms of the line's equation. For example, consider a nonvertical line that has slope m and that contains the point (x_1, y_1).

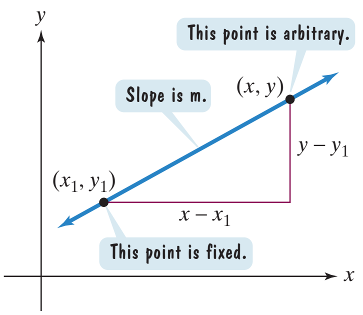


Figure 1.38 A line passing through (x_1, y_1) with slope m

The line in **Figure 1.38** has slope m and contains the point (x_1, y_1). Let (x, y) represent any other point on the line.

Regardless of where the point (x, y) is located, the steepness of the line in **Figure 1.38** remains the same. Thus, the ratio for the slope stays a constant m. This means that for all points (x, y) along the line

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{y - y_1}{x - x_1}.$$

We can clear the fraction by multiplying both sides by $x - x_1$, the least common denominator.

$$m = \frac{y - y_1}{x - x_1}$$ This is the slope of the line in Figure 1.38.

$$m(x - x_1) = \frac{y - y_1}{x - x_1} \cdot (x - x_1)$$ Multiply both sides by $x - x_1$.

$$m(x - x_1) = y - y_1$$ Simplify: $\frac{y - y_1}{\cancel{x - x_1}} \cdot (\cancel{x - x_1}) = y - y_1$.

Now, if we reverse the two sides, we obtain the *point-slope form* of the equation of a line.

Study Tip

When writing the point-slope form of a line's equation, you will never substitute numbers for x and y. You will substitute values for x_1, y_1, and m.

Point-Slope Form of the Equation of a Line

The **point-slope form of the equation** of a nonvertical line with slope m that passes through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1).$$

For example, the point-slope form of the equation of the line passing through $(1, 5)$ with slope 2 $(m = 2)$ is

$$y - 5 = 2(x - 1).$$

We will soon be expressing the equation of a nonvertical line in function notation. To do so, we need to solve the point-slope form of a line's equation for y. Example 2 illustrates how to isolate y on one side of the equal sign.

EXAMPLE 2 Writing an Equation in Point-Slope Form for a Line

Write an equation in point-slope form for the line with slope 4 that passes through the point $(-1, 3)$. Then solve the equation for y.

Solution We use the point-slope form of the equation of a line with $m = 4$, $x_1 = -1$, and $y_1 = 3$.

$$y - y_1 = m(x - x_1)$$ This is the point-slope form of the equation.

$$y - 3 = 4[x - (-1)]$$ Substitute the given values: $m = 4$ and $(x_1, y_1) = (-1, 3)$.

$$y - 3 = 4(x + 1)$$ We now have an equation in point-slope form for the given line.

Now we solve this equation for y and write an equivalent equation that can be expressed in function notation.

We need to isolate y.

$$y - 3 = 4(x + 1)$$ This is the point-slope form of the equation.

$$y - 3 = 4x + 4$$ Use the distributive property.

$$y = 4x + 7$$ Add 3 to both sides.

Check Point 2 Write an equation in point-slope form for the line with slope 6 that passes through the point $(2, -5)$. Then solve the equation for y.

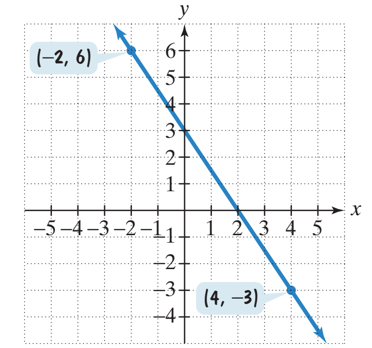


Figure 1.39 Write an equation in point-slope form for this line.

EXAMPLE 3 Writing an Equation in Point-Slope Form for a Line

Write an equation in point-slope form for the line passing through the points $(4, -3)$ and $(-2, 6)$. (See **Figure 1.39**.) Then solve the equation for y.

Solution To use the point-slope form, we need to find the slope. The slope is the change in the y-coordinates divided by the corresponding change in the x-coordinates.

$$m = \frac{6-(-3)}{-2-4} = \frac{9}{-6} = -\frac{3}{2}$$ This is the definition of slope using $(4, -3)$ and $(-2, 6)$.

We can take either point on the line to be (x_1, y_1). Let's use $(x_1, y_1) = (4, -3)$. Now, we are ready to write the point-slope form of the equation.

$$y - y_1 = m(x - x_1)$$ This is the point-slope form of the equation.

$$y - (-3) = -\tfrac{3}{2}(x - 4)$$ Substitute: $(x_1, y_1) = (4, -3)$ and $m = -\frac{3}{2}$.

$$y + 3 = -\tfrac{3}{2}(x - 4)$$ Simplify.

We now have an equation in point-slope form for the line shown in **Figure 1.39**. Now, we solve this equation for y.

We need to isolate y.

$$y + 3 = -\tfrac{3}{2}(x - 4)$$ This is the point-slope form of the equation.

$$y + 3 = -\tfrac{3}{2}x + 6$$ Use the distributive property.

$$y = -\tfrac{3}{2}x + 3$$ Subtract 3 from both sides.

Discovery

You can use either point for (x_1, y_1) when you write a point-slope equation for a line. Rework Example 3 using $(-2, 6)$ for (x_1, y_1). Once you solve for y, you should still obtain

$$y = -\tfrac{3}{2}x + 3.$$

Check Point 3 Write an equation in point-slope form for the line passing through the points $(-2, -1)$ and $(-1, -6)$. Then solve the equation for y.

3 Write and graph the slope-intercept form of the equation of a line.

The Slope-Intercept Form of the Equation of a Line

Let's write the point-slope form of the equation of a nonvertical line with slope m and y-intercept b. The line is shown in **Figure 1.40**. Because the y-intercept is b, the line passes through $(0, b)$. We use the point-slope form with $x_1 = 0$ and $y_1 = b$.

$$y - y_1 = m(x - x_1)$$

Let $y_1 = b$. Let $x_1 = 0$.

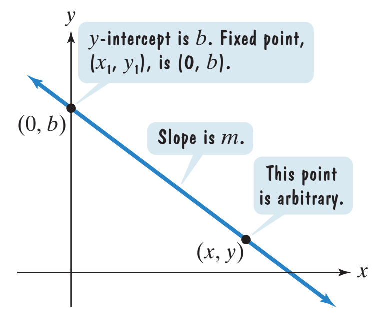


Figure 1.40 A line with slope m and y-intercept b

We obtain

$$y - b = m(x - 0).$$

Simplifying on the right side gives us

$$y - b = mx.$$

Finally, we solve for y by adding b to both sides.

$$y = mx + b$$

Thus, if a line's equation is written with y isolated on one side, the coefficient of x is the line's slope and the constant term is the y-intercept. This form of a line's equation is called the *slope-intercept form* of the line.

Slope-Intercept Form of the Equation of a Line

The **slope-intercept form of the equation** of a nonvertical line with slope m and y-intercept b is

$$y = mx + b.$$

The slope-intercept form of a line's equation, $y = mx + b$, can be expressed in function notation by replacing y with $f(x)$:

$$f(x) = mx + b.$$

We have seen that functions in this form are called **linear functions**. Thus, in the equation of a linear function, the coefficient of x is the line's slope and the constant term is the y-intercept. Here are two examples:

$$y = 2x - 4$$

The slope is 2. The y-intercept is -4.

$$f(x) = \frac{1}{2}x + 2.$$

The slope is $\frac{1}{2}$. The y-intercept is 2.

If a linear function's equation is in slope-intercept form, we can use the y-intercept and the slope to obtain its graph.

Graphing $y = mx + b$ Using the Slope and y-Intercept

1. Plot the point containing the y-intercept on the y-axis. This is the point $(0, b)$.
2. Obtain a second point using the slope, m. Write m as a fraction, and use rise over run, starting at the point containing the y-intercept, to plot this point.
3. Use a straightedge to draw a line through the two points. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

Study Tip

Writing the slope, m, as a fraction allows you to identify the rise (the fraction's numerator) and the run (the fraction's denominator).

EXAMPLE 4 Graphing Using the Slope and y-Intercept

Graph the linear function: $f(x) = -\frac{3}{2}x + 2.$

Solution The equation of the line is in the form $f(x) = mx + b$. We can find the slope, m, by identifying the coefficient of x. We can find the y-intercept, b, by identifying the constant term.

$$f(x) = -\frac{3}{2}x + 2$$

The slope is $-\frac{3}{2}$. The y-intercept is 2.

Now that we have identified the slope, $-\frac{3}{2}$, and the y-intercept, 2, we use the three-step procedure to graph the equation.

Step 1 Plot the point containing the y-intercept on the y-axis. The y-intercept is 2. We plot $(0, 2)$, shown in **Figure 1.41**.

Step 2 Obtain a second point using the slope, m. Write m as a fraction, and use rise over run, starting at the point containing the y-intercept, to plot this point. The slope, $-\frac{3}{2}$, is already written as a fraction.

$$m = -\frac{3}{2} = \frac{-3}{2} = \frac{\text{Rise}}{\text{Run}}$$

We plot the second point on the line by starting at $(0, 2)$, the first point. Based on the slope, we move 3 units *down* (the rise) and 2 units to the *right* (the run). This puts us at a second point on the line, $(2, -1)$, shown in **Figure 1.41**.

Step 3 Use a straightedge to draw a line through the two points. The graph of the linear function $f(x) = -\frac{3}{2}x + 2$ is shown as a blue line in **Figure 1.41**.

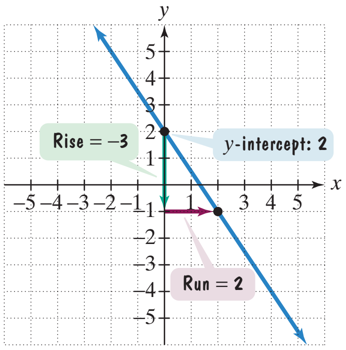


Figure 1.41 The graph of $f(x) = -\frac{3}{2}x + 2$

Check Point 4 Graph the linear function: $f(x) = \frac{3}{5}x + 1.$

4 Graph horizontal or vertical lines.

Equations of Horizontal and Vertical Lines

If a line is horizontal, its slope is zero: $m = 0$. Thus, the equation $y = mx + b$ becomes $y = b$, where b is the y-intercept. All horizontal lines have equations of the form $y = b$.

EXAMPLE 5 Graphing a Horizontal Line

Graph $y = -4$ in the rectangular coordinate system.

Solution All ordered pairs that are solutions of $y = -4$ have a value of y that is always -4. Any value can be used for x. In the table on the right, we have selected three of the possible values for x: -2, 0, and 3. The table shows that three ordered pairs that are solutions of $y = -4$ are $(-2, -4)$, $(0, -4)$, and $(3, -4)$. Drawing a line that passes through the three points gives the horizontal line shown in **Figure 1.42**.

x	$y = -4$	(x, y)
-2	-4	$(-2, -4)$
0	-4	$(0, -4)$
3	-4	$(3, -4)$

For all choices of x,

y is a constant -4.

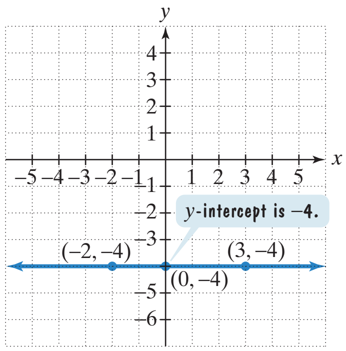


Figure 1.42 The graph of $y = -4$ or $f(x) = -4$

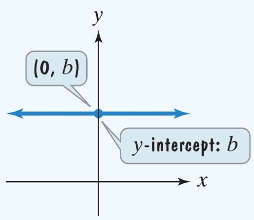
Check Point 5 Graph $y = 3$ in the rectangular coordinate system.

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b,$$

where b is the y-intercept of the line. The slope of a horizontal line is zero.



Because any vertical line can intersect the graph of a horizontal line $y = b$ only once, a horizontal line is the graph of a function. Thus, we can express the equation $y = b$ as $f(x) = b$. This linear function is often called a **constant function**.

Next, let's see what we can discover about the graph of an equation of the form $x = a$ by looking at an example.

EXAMPLE 6 Graphing a Vertical Line

Graph the linear equation: $x = 2$.

Solution All ordered pairs that are solutions of $x = 2$ have a value of x that is always 2. Any value can be used for y. In the table on the right, we have selected three of the possible values for y: -2, 0, and 3. The table shows that three ordered pairs that are solutions of $x = 2$ are $(2, -2)$, $(2, 0)$, and $(2, 3)$. Drawing a line that passes through the three points gives the vertical line shown in **Figure 1.43**.

For all choices of y,

$x = 2$	y	(x, y)
2	-2	$(2, -2)$
2	0	$(2, 0)$
2	3	$(2, 3)$

x is always 2.

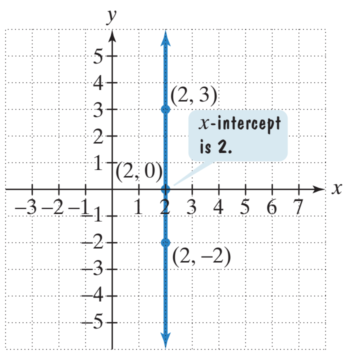


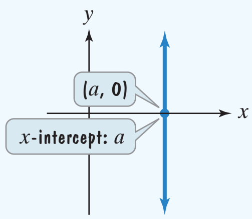
Figure 1.43 The graph of $x = 2$

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$x = a,$$

where a is the x-intercept of the line. The slope of a vertical line is undefined.



Does a vertical line represent the graph of a linear function? No. Look at the graph of $x = 2$ in **Figure 1.43**. A vertical line drawn through (2, 0) intersects the graph infinitely many times. This shows that infinitely many outputs are associated with the input 2. **No vertical line represents a linear function.**

Check Point 6 Graph the linear equation: $x = -3$.

5 Recognize and use the general form of a line's equation.

The General Form of the Equation of a Line

The vertical line whose equation is $x = 5$ cannot be written in slope-intercept form, $y = mx + b$, because its slope is undefined. However, every line has an equation that can be expressed in the form $Ax + By + C = 0$. For example, $x = 5$ can be expressed as $1x + 0y - 5 = 0$, or $x - 5 = 0$. The equation $Ax + By + C = 0$ is called the *general form* of the equation of a line.

General Form of the Equation of a Line

Every line has an equation that can be written in the **general form**

$$Ax + By + C = 0,$$

where A, B, and C are real numbers, and A and B are not both zero.

If the equation of a line is given in general form, it is possible to find the slope, m, and the y-intercept, b, for the line. We solve the equation for y, transforming it into the slope-intercept form $y = mx + b$. In this form, the coefficient of x is the slope of the line and the constant term is its y-intercept.

EXAMPLE 7 **Finding the Slope and the y-Intercept**

Find the slope and the y-intercept of the line whose equation is $3x + 2y - 4 = 0$.

Solution The equation is given in general form. We begin by rewriting it in the form $y = mx + b$. We need to solve for y.

Our goal is to isolate y.

$$3x + 2y - 4 = 0$$ This is the given equation.

$$2y = -3x + 4$$ Isolate the term containing y by adding $-3x + 4$ to both sides.

$$\frac{2y}{2} = \frac{-3x + 4}{2}$$ Divide both sides by 2.

$$y = -\frac{3}{2}x + 2$$ On the right, divide each term in the numerator by 2 to obtain slope-intercept form.

slope ($-\frac{3}{2}$) y-intercept (2)

The coefficient of x, $-\frac{3}{2}$, is the slope and the constant term, 2, is the y-intercept. This is the form of the equation that we graphed in **Figure 1.41** on page 183.

Check Point 7 Find the slope and the y-intercept of the line whose equation is $3x + 6y - 12 = 0$. Then use the y-intercept and the slope to graph the equation.

6 Use intercepts to graph the general form of a line's equation.

Using Intercepts to Graph $Ax + By + C = 0$

Example 7 and Check Point 7 illustrate that one way to graph the general form of a line's equation is to convert to slope-intercept form, $y = mx + b$. Then use the slope and the y-intercept to obtain the graph.

A second method for graphing $Ax + By + C = 0$ uses intercepts. This method does not require rewriting the general form in a different form.

Using Intercepts to Graph $Ax + By + C = 0$

1. Find the x-intercept. Let $y = 0$ and solve for x. Plot the point containing the x-intercept on the x-axis.
2. Find the y-intercept. Let $x = 0$ and solve for y. Plot the point containing the y-intercept on the y-axis.
3. Use a straightedge to draw a line through the two points containing the intercepts. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

EXAMPLE 8 Using Intercepts to Graph a Linear Equation

Graph using intercepts: $4x - 3y - 6 = 0$.

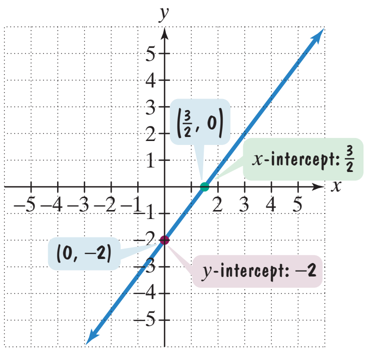


Figure 1.44 The graph of $4x - 3y - 6 = 0$

Solution

Step 1 Find the x-intercept. Let $y = 0$ and solve for x.

$$4x - 3 \cdot 0 - 6 = 0 \quad \text{Replace } y \text{ with 0 in } 4x - 3y - 6 = 0.$$
$$4x - 6 = 0 \quad \text{Simplify.}$$
$$4x = 6 \quad \text{Add 6 to both sides.}$$
$$x = \frac{6}{4} = \frac{3}{2} \quad \text{Divide both sides by 4.}$$

The x-intercept is $\frac{3}{2}$, so the line passes through $\left(\frac{3}{2}, 0\right)$ or $(1.5, 0)$, as shown in **Figure 1.44**.

Step 2 Find the y-intercept. Let $x = 0$ and solve for y.

$$4 \cdot 0 - 3y - 6 = 0 \quad \text{Replace } x \text{ with 0 in } 4x - 3y - 6 = 0.$$
$$-3y - 6 = 0 \quad \text{Simplify.}$$
$$-3y = 6 \quad \text{Add 6 to both sides.}$$
$$y = -2 \quad \text{Divide both sides by } -3.$$

The y-intercept is -2, so the line passes through $(0, -2)$, as shown in **Figure 1.44**.

Step 3 Graph the equation by drawing a line through the two points containing the intercepts. The graph of $4x - 3y - 6 = 0$ is shown in **Figure 1.44**.

Check Point 8 Graph using intercepts: $3x - 2y - 6 = 0$.

We've covered a lot of territory. Let's take a moment to summarize the various forms for equations of lines.

Equations of Lines

1. Point-slope form	$y - y_1 = m(x - x_1)$
2. Slope-intercept form	$y = mx + b$ or $f(x) = mx + b$
3. Horizontal line	$y = b$
4. Vertical line	$x = a$
5. General form	$Ax + By + C = 0$

7 Model data with linear functions and make predictions.

Applications

Linear functions are useful for modeling data that fall on or near a line.

EXAMPLE 9 **Modeling Global Warming**

The amount of carbon dioxide in the atmosphere, measured in parts per million, has been increasing as a result of the burning of oil and coal. The buildup of gases and particles traps heat and raises the planet's temperature. The bar graph in **Figure 1.45(a)** gives the average atmospheric concentration of carbon dioxide and the average global temperature for six selected years. The data are displayed as a set of six points in a rectangular coordinate system in **Figure 1.45(b)**.

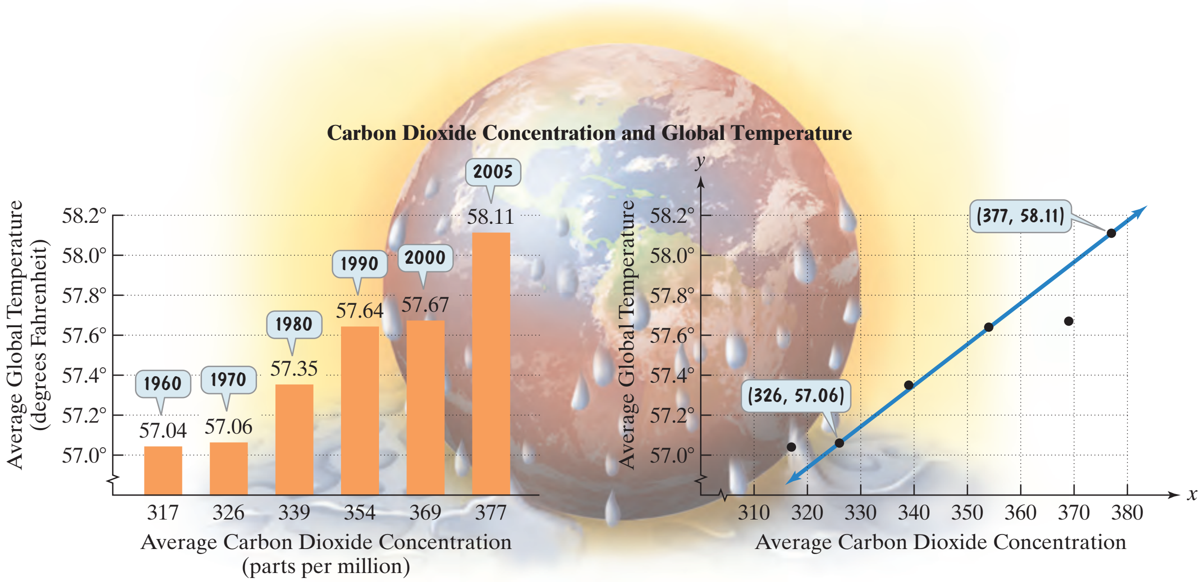


Figure 1.45(a)

Source: National Oceanic and Atmospheric Administration

Figure 1.45(b)

a. Shown on the scatter plot in **Figure 1.45(b)** is a line that passes through or near the six points. Write the slope-intercept form of this equation using function notation.

b. The preindustrial concentration of atmospheric carbon dioxide was 280 parts per million. The United Nation's Intergovernmental Panel on Climate Change predicts global temperatures will rise between 2°F and 5°F if carbon dioxide concentration doubles from the preindustrial level. Compared to the average global temperature of 58.11°F for 2005, how well does the function from part (a) model this prediction?

Solution

a. The line in **Figure 1.45(b)** passes through (326, 57.06) and (377, 58.11). We start by finding its slope.

$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{58.11 - 57.06}{377 - 326} = \frac{1.05}{51} \approx 0.02$$

The slope, approximately 0.02, indicates that for each increase of one part per million in carbon dioxide concentration, the average global temperature is increasing by approximately 0.02°F.

Now we write the line's equation in slope-intercept form.

$$y - y_1 = m(x - x_1)$$ Begin with the point-slope form.

$$y - 57.06 = 0.02(x - 326)$$ Either ordered pair, (326, 57.06) or (377, 58.11), can be (x_1, y_1). Let $(x_1, y_1) = (326, 57.06)$. From above, $m \approx 0.02$.

$$y - 57.06 = 0.02x - 6.52$$ Apply the distributive property: $0.02(326) = 6.52$.

$$y = 0.02x + 50.54$$ Add 57.06 to both sides and solve for y.

A linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of x parts per million is

$$f(x) = 0.02x + 50.54.$$

b. If carbon dioxide concentration doubles from its preindustrial level of 280 parts per million, which many experts deem very likely, the concentration will reach 280×2, or 560 parts per million. We use the linear function to predict average global temperature at this concentration.

$$f(x) = 0.02x + 50.54$$ Use the function from part (a).

$$f(560) = 0.02(560) + 50.54$$ Substitute 560 for x.

$$= 11.2 + 50.54 = 61.74$$

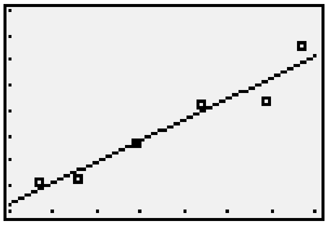
Our model projects an average global temperature of 61.74°F for a carbon dioxide concentration of 560 parts per million. Compared to the average global temperature of 58.11° for 2005 shown in **Figure 1.45(a)** on the previous page, this is an increase of

$$61.74°F - 58.11°F = 3.63°F.$$

This is consistent with a rise between 2°F and 5°F as predicted by the Intergovernmental Panel on Climate Change.

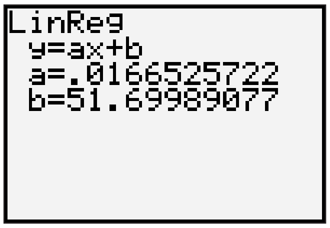
Technology

You can use a graphing utility to obtain a model for a scatter plot in which the data points fall on or near a straight line. After entering the data in **Figure 1.45(a)** on the previous page, a graphing utility displays a scatter plot of the data and the regression line, that is, the line that best fits the data.



[310, 380, 10] by [56.8, 58.4, 0.2]

Also displayed is the regression line's equation.



Check Point 9 Use the data points (317, 57.04) and (354, 57.64), shown, but not labeled, in **Figure 1.45(b)** on the previous page to obtain a linear function that models average global temperature, $f(x)$, for an atmospheric carbon dioxide concentration of x parts per million. Round m to three decimal places and b to one decimal place. Then use the function to project average global temperature at a concentration of 600 parts per million.

Exercise Set 1.4

Practice Exercises

In Exercises 1–10, find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

1. (4, 7) and (8, 10)
2. (2, 1) and (3, 4)
3. (−2, 1) and (2, 2)
4. (−1, 3) and (2, 4)
5. (4, −2) and (3, −2)
6. (4, −1) and (3, −1)
7. (−2, 4) and (−1, −1)
8. (6, −4) and (4, −2)
9. (5, 3) and (5, −2)
10. (3, −4) and (3, 5)

In Exercises 11–38, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

11. Slope = 2, passing through (3, 5)
12. Slope = 4, passing through (1, 3)
13. Slope = 6, passing through (−2, 5)
14. Slope = 8, passing through (4, −1)
15. Slope = −3, passing through (−2, −3)
16. Slope = −5, passing through (−4, −2)
17. Slope = −4, passing through (−4, 0)
18. Slope = −2, passing through (0, −3)

19. Slope $= -1$, passing through $\left(-\frac{1}{2}, -2\right)$

20. Slope $= -1$, passing through $\left(-4, -\frac{1}{4}\right)$

21. Slope $= \frac{1}{2}$, passing through the origin

22. Slope $= \frac{1}{3}$, passing through the origin

23. Slope $= -\frac{2}{3}$, passing through $(6, -2)$

24. Slope $= -\frac{3}{5}$, passing through $(10, -4)$

25. Passing through $(1, 2)$ and $(5, 10)$

26. Passing through $(3, 5)$ and $(8, 15)$

27. Passing through $(-3, 0)$ and $(0, 3)$

28. Passing through $(-2, 0)$ and $(0, 2)$

29. Passing through $(-3, -1)$ and $(2, 4)$

30. Passing through $(-2, -4)$ and $(1, -1)$

31. Passing through $(-3, -2)$ and $(3, 6)$

32. Passing through $(-3, 6)$ and $(3, -2)$

33. Passing through $(-3, -1)$ and $(4, -1)$

34. Passing through $(-2, -5)$ and $(6, -5)$

35. Passing through $(2, 4)$ with x-intercept $= -2$

36. Passing through $(1, -3)$ with x-intercept $= -1$

37. x-intercept $= -\frac{1}{2}$ and y-intercept $= 4$

38. x-intercept $= 4$ and y-intercept $= -2$

In Exercises 39–48, give the slope and y-intercept of each line whose equation is given. Then graph the linear function.

39. $y = 2x + 1$

40. $y = 3x + 2$

41. $f(x) = -2x + 1$

42. $f(x) = -3x + 2$

43. $f(x) = \frac{3}{4}x - 2$

44. $f(x) = \frac{3}{4}x - 3$

45. $y = -\frac{3}{5}x + 7$

46. $y = -\frac{2}{5}x + 6$

47. $g(x) = -\frac{1}{2}x$

48. $g(x) = -\frac{1}{3}x$

In Exercises 49–58, graph each equation in a rectangular coordinate system.

49. $y = -2$

50. $y = 4$

51. $x = -3$

52. $x = 5$

53. $y = 0$

54. $x = 0$

55. $f(x) = 1$

56. $f(x) = 3$

57. $3x - 18 = 0$

58. $3x + 12 = 0$

In Exercises 59–66,

a. *Rewrite the given equation in slope-intercept form.*

b. *Give the slope and y-intercept.*

c. *Use the slope and y-intercept to graph the linear function.*

59. $3x + y - 5 = 0$

60. $4x + y - 6 = 0$

61. $2x + 3y - 18 = 0$

62. $4x + 6y + 12 = 0$

63. $8x - 4y - 12 = 0$

64. $6x - 5y - 20 = 0$

65. $3y - 9 = 0$

66. $4y + 28 = 0$

In Exercises 67–72, use intercepts to graph each equation.

67. $6x - 2y - 12 = 0$

68. $6x - 9y - 18 = 0$

69. $2x + 3y + 6 = 0$

70. $3x + 5y + 15 = 0$

71. $8x - 2y + 12 = 0$

72. $6x - 3y + 15 = 0$

Practice Plus

In Exercises 73–76, find the slope of the line passing through each pair of points or state that the slope is undefined. Assume that all variables represent positive real numbers. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

73. $(0, a)$ and $(b, 0)$

74. $(-a, 0)$ and $(0, -b)$

75. (a, b) and $(a, b + c)$

76. $(a - b, c)$ and $(a, a + c)$

In Exercises 77–78, give the slope and y-intercept of each line whose equation is given. Assume that $B \neq 0$.

77. $Ax + By = C$

78. $Ax = By - C$

In Exercises 79–80, find the value of y if the line through the two given points is to have the indicated slope.

79. $(3, y)$ and $(1, 4)$, $m = -3$

80. $(-2, y)$ and $(4, -4)$, $m = \frac{1}{3}$

In Exercises 81–82, graph each linear function.

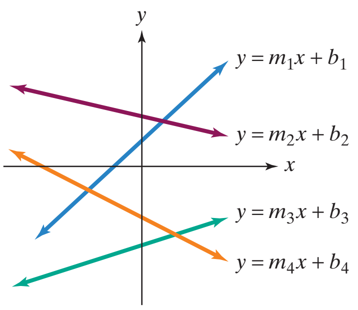
81. $3x - 4f(x) - 6 = 0$

82. $6x - 5f(x) - 20 = 0$

83. If one point on a line is $(3, -1)$ and the line's slope is -2, find the y-intercept.

84. If one point on a line is $(2, -6)$ and the line's slope is $-\frac{3}{2}$, find the y-intercept.

Use the figure to make the lists in Exercises 85–86.



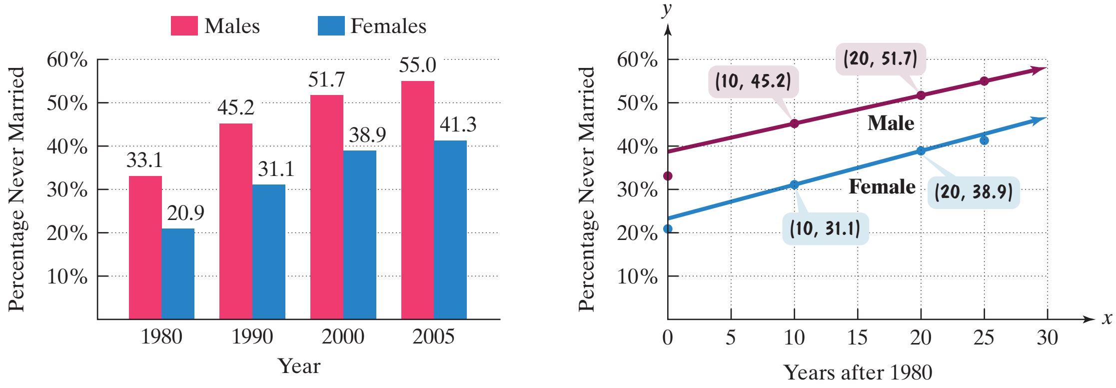
85. List the slopes m_1, m_2, m_3, and m_4 in order of decreasing size.

86. List the y-intercepts b_1, b_2, b_3, and b_4 in order of decreasing size.

Application Exercises

Americans are getting married later in life, or not getting married at all. In 2006, nearly half of Americans ages 25 through 29 were unmarried. The bar graph at the top of the next page shows the percentage of never-married men and women in this age group. The data are displayed on the next page as two sets of four points each, one scatter plot for the percentage of never-married American men and one for the percentage of never-married American women. Also shown for each scatter plot is a line that passes through or near the four points. Use these lines to solve Exercises 87–88.

Percentage of United States Population Never Married, Ages 25–29



Source: U.S. Census Bureau

87. In this exercise, you will use the blue line for the women shown on the scatter plot to develop a model for the percentage of never-married American females ages 25–29.

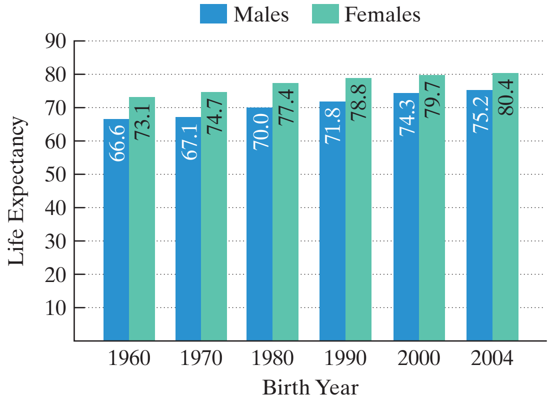
- **a.** Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the percentage of never-married American females ages 25–29, y, x years after 1980.
- **b.** Write the equation from part (a) in slope-intercept form. Use function notation.
- **c.** Use the linear function to predict the percentage of never-married American females, ages 25–29, in 2020.

88. In this exercise, you will use the red line for the men shown on the scatter plot to develop a model for the percentage of never-married American males ages 25–29.

- **a.** Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the percentage of never-married American males ages 25–29, y, x years after 1980.
- **b.** Write the equation from part (a) in slope-intercept form. Use function notation.
- **c.** Use the linear function to predict the percentage of never-married American males, ages 25–29, in 2015.

The bar graph gives the life expectancy for American men and women born in six selected years. In Exercises 89–90, you will use the data to obtain models for life expectancy and make predictions about how long American men and women will live in the future.

Life Expectancy in the United States, by Year of Birth



Source: National Center for Health Statistics

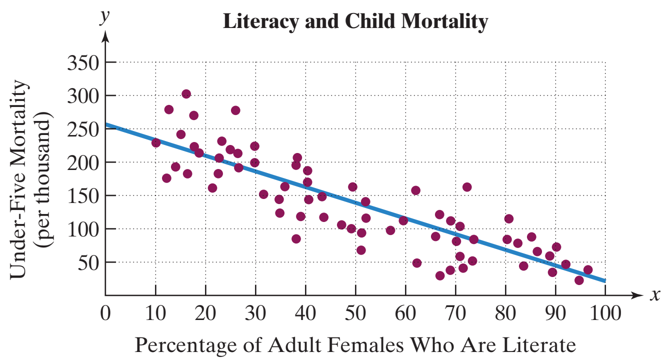
89. Use the data for males shown in the bar graph at the bottom of the previous column to solve this exercise.

- **a.** Let x represent the number of birth years after 1960 and let y represent male life expectancy. Create a scatter plot that displays the data as a set of six points in a rectangular coordinate system.
- **b.** Draw a line through the two points that show male life expectancies for 1980 and 2000. Use the coordinates of these points to write a linear function that models life expectancy, $E(x)$, for American men born x years after 1960.
- **c.** Use the function from part (b) to project the life expectancy of American men born in 2020.

90. Use the data for females shown in the bar graph at the bottom of the previous column to solve this exercise.

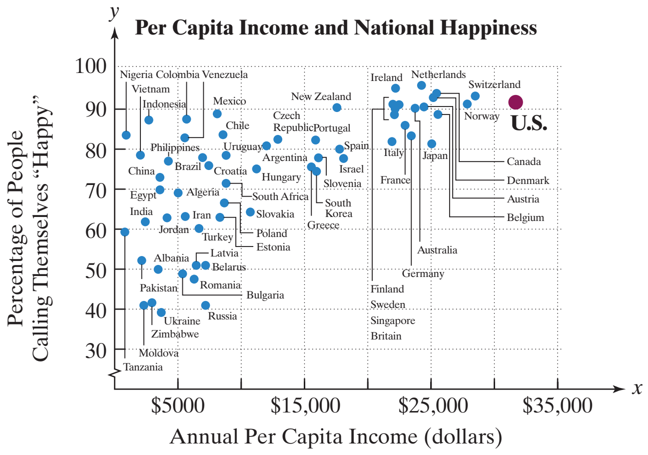
- **a.** Let x represent the number of birth years after 1960 and let y represent female life expectancy. Create a scatter plot that displays the data as a set of six points in a rectangular coordinate system.
- **b.** Draw a line through the two points that show female life expectancies for 1970 and 2000. Use the coordinates of these points to write a linear function that models life expectancy, $E(x)$, for American women born x years after 1960. Round the slope to two decimal places.
- **c.** Use the function from part (b) to project the life expectancy of American women born in 2020.

91. Shown, again, is the scatter plot that indicates a relationship between the percentage of adult females in a country who are literate and the mortality of children under five. Also shown is a line that passes through or near the points. Find a linear function that models the data by finding the slope-intercept form of the line's equation. Use the function to make a prediction about child mortality based on the percentage of adult females in a country who are literate.



Source: United Nations

92. Just as money doesn't buy happiness for individuals, the two don't necessarily go together for countries either. However, the scatter plot does show a relationship between a country's annual per capita income and the percentage of people in that country who call themselves "happy."



Source: Richard Layard, *Happiness: Lessons from a New Science,* Penguin, 2005

Draw a line that fits the data so that the spread of the data points around the line is as small as possible. Use the coordinates of two points along your line to write the slope-intercept form of its equation. Express the equation in function notation and use the linear function to make a prediction about national happiness based on per capita income.

Writing in Mathematics

93. What is the slope of a line and how is it found?

94. Describe how to write the equation of a line if the coordinates of two points along the line are known.

95. Explain how to derive the slope-intercept form of a line's equation, $y = mx + b$, from the point-slope form

$$y - y_1 = m(x - x_1).$$

96. Explain how to graph the equation $x = 2$. Can this equation be expressed in slope-intercept form? Explain.

97. Explain how to use the general form of a line's equation to find the line's slope and y-intercept.

98. Explain how to use intercepts to graph the general form of a line's equation.

99. Take another look at the scatter plot in Exercise 91. Although there is a relationship between literacy and child mortality, we cannot conclude that increased literacy causes child mortality to decrease. Offer two or more possible explanations for the data in the scatter plot.

Technology Exercises

Use a graphing utility to graph each equation in Exercises 100–103. Then use the TRACE *feature to trace along the line and find the coordinates of two points. Use these points to compute the line's slope. Check your result by using the coefficient of x in the line's equation.*

100. $y = 2x + 4$

101. $y = -3x + 6$

102. $y = -\frac{1}{2}x - 5$

103. $y = \frac{3}{4}x - 2$

104. Is there a relationship between wine consumption and deaths from heart disease? The table gives data from 19 developed countries.

France

Country	A	B	C	D	E	F	G
Liters of alcohol from drinking wine, per person per year (x)	2.5	3.9	2.9	2.4	2.9	0.8	9.1
Deaths from heart disease, per 100,000 people per year (y)	211	167	131	191	220	297	71

U.S.

Country	H	I	J	K	L	M	N	O	P	Q	R	S
(x)	0.8	0.7	7.9	1.8	1.9	0.8	6.5	1.6	5.8	1.3	1.2	2.7
(y)	211	300	107	167	266	227	86	207	115	285	199	172

Source: New York Times

a. Use the statistical menu of your graphing utility to enter the 19 ordered pairs of data items shown in the table.

b. Use the scatter plot capability to draw a scatter plot of the data.

c. Select the linear regression option. Use your utility to obtain values for a and b for the equation of the regression line, $y = ax + b$. You may also be given a **correlation coefficient**, r. Values of r close to 1 indicate that the points can be described by a linear relationship and the regression line has a positive slope. Values of r close to -1 indicate that the points can be described by a linear relationship and the regression line has a negative slope. Values of r close to 0 indicate no linear relationship between the variables. In this case, a linear model does not accurately describe the data.

d. Use the appropriate sequence (consult your manual) to graph the regression equation on top of the points in the scatter plot.

Critical Thinking Exercises

Make Sense? *In Exercises 105–108, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

105. The graph of my linear function at first increased, reached a maximum point, and then decreased.

106. A linear function that models tuition and fees at public four-year colleges from 2000 through 2006 has negative slope.

107. Because the variable m does not appear in $Ax + By + C = 0$, equations in this form make it impossible to determine the line's slope.

108. The federal minimum wage was $5.15 per hour from 1997 through 2006, so $f(x) = 5.15$ models the minimum wage, $f(x)$, in dollars, for the domain $\{1997, 1998, 1999, \ldots, 2006\}$.

In Exercises 109–112, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

109. The equation $y = mx + b$ shows that no line can have a y-intercept that is numerically equal to its slope.

110. Every line in the rectangular coordinate system has an equation that can be expressed in slope-intercept form.

111. The graph of the linear function $5x + 6y - 30 = 0$ is a line passing through the point $(6, 0)$ with slope $-\frac{5}{6}$.

112. The graph of $x = 7$ in the rectangular coordinate system is the single point $(7, 0)$.

In Exercises 113–114, find the coefficients that must be placed in each shaded area so that the function's graph will be a line satisfying the specified conditions.

113. $\square x + \square y - 12 = 0$; x-intercept $= -2$; y-intercept $= 4$

114. $\square x + \square y - 12 = 0$; y-intercept $= -6$; slope $= \frac{1}{2}$

115. Prove that the equation of a line passing through $(a, 0)$ and $(0, b)(a \neq 0, b \neq 0)$ can be written in the form $\frac{x}{a} + \frac{y}{b} = 1$. Why is this called the *intercept form* of a line?

116. Excited about the success of celebrity stamps, post office officials were rumored to have put forth a plan to institute two new types of thermometers. On these new scales, $°E$ represents degrees Elvis and $°M$ represents degrees Madonna. If it is known that $40°E = 25°M$, $280°E = 125°M$, and degrees Elvis is linearly related to degrees Madonna, write an equation expressing E in terms of M.

Group Exercise

117. In Exercises 87–88, we used the data in a bar graph to develop linear functions that modeled the percentage of never-married American females and males, ages 25–29. For this group exercise, you might find it helpful to pattern your work after Exercises 87 and 88. Group members should begin by consulting an almanac, newspaper, magazine, or the Internet to find data that appear to lie approximately on or near a line. Working by hand or using a graphing utility, group members should construct scatter plots for the data that were assembled. If working by hand, draw a line that approximately fits the data in each scatter plot and then write its equation as a function in slope-intercept form. If using a graphing utility, obtain the equation of each regression line. Then use each linear function's equation to make predictions about what might occur in the future. Are there circumstances that might affect the accuracy of the prediction? List some of these circumstances.

Preview Exercises

Exercises 118–120 will help you prepare for the material covered in the next section.

118. Write the slope-intercept form of the equation of the line passing through $(-3, 1)$ whose slope is the same as the line whose equation is $y = 2x + 1$.

119. Write an equation in general form of the line passing through $(3, -5)$ whose slope is the negative reciprocal (the reciprocal with the opposite sign) of $-\frac{1}{4}$.

120. If $f(x) = x^2$, find

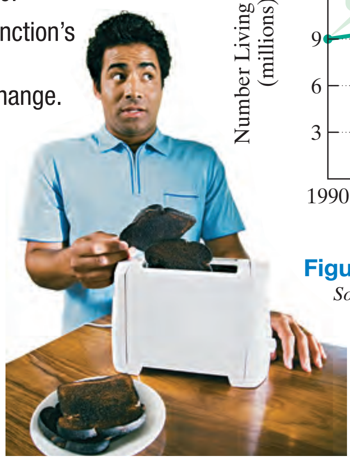
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1},$$

where $x_1 = 1$ and $x_2 = 4$.

Section 1.5 More on Slope

Objectives

1. Find slopes and equations of parallel and perpendicular lines.
2. Interpret slope as rate of change.
3. Find a function's average rate of change.



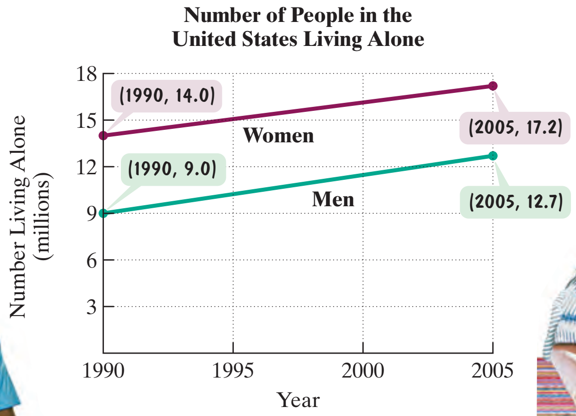


Figure 1.46
Source: U.S. Census Bureau

A best guess at the future of our nation indicates that the numbers of men and women living alone will increase each year. **Figure 1.46** shows that in 2005, 12.7 million men and 17.2 million women lived alone, an increase over the numbers displayed in the graph for 1990.