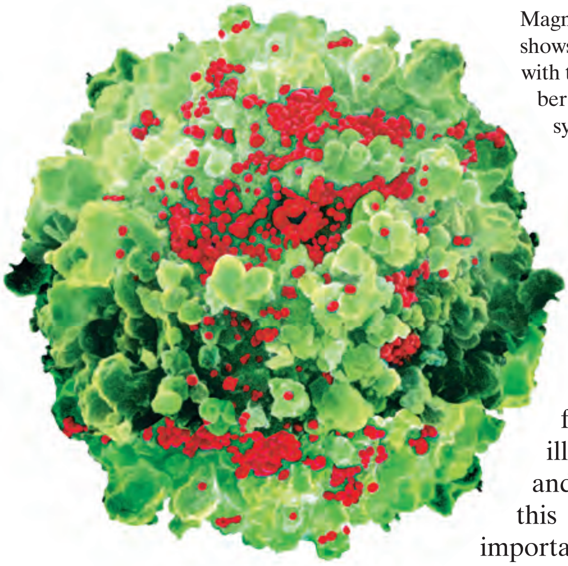
Section 1.2

Basics of Functions and Their Graphs

Objectives

1. Find the domain and range of a relation.
2. Determine whether a relation is a function.
3. Determine whether an equation represents a function.
4. Evaluate a function.
5. Graph functions by plotting points.
6. Use the vertical line test to identify functions.
7. Obtain information about a function from its graph.
8. Identify the domain and range of a function from its graph.
9. Identify intercepts from a function's graph.



Magnified 6000 times, this color-scanned image shows a T-lymphocyte blood cell (green) infected with the HIV virus (red). Depletion of the number of T cells causes destruction of the immune system.

The average number of T cells in a person with HIV is a *function* of time after infection. In this section, you will be introduced to the basics of functions and their graphs. We will analyze the graph of a function using an example that illustrates the progression of HIV and T cell count. Much of our work in this course will be devoted to the important topic of functions and how they model your world.

Relations

1 Find the domain and range of a relation.

The top five U.S. last names shown in **Table 1.1** account for nearly 4% of the entire population. The table indicates a correspondence between a last name and the percentage of Americans who share that name. We can write this correspondence using a set of ordered pairs:

$$\{(\text{Smith}, 1.006\%), (\text{Johnson}, 0.810\%), (\text{Williams}, 0.699\%), (\text{Brown}, 0.621\%), (\text{Jones}, 0.621\%)\}.$$

These braces indicate we are representing a set.

Table 1.1 Top U.S. Last Names

Name	% of All Names
Smith	1.006%
Johnson	0.810%
Williams	0.699%
Brown	0.621%
Jones	0.621%

Source: Russell Ash, *The Top 10 of Everything*

The mathematical term for a set of ordered pairs is a *relation*.

Definition of a Relation

A **relation** is any set of ordered pairs. The set of all first components of the ordered pairs is called the **domain** of the relation and the set of all second components is called the **range** of the relation.

EXAMPLE 1 Finding the Domain and Range of a Relation

Find the domain and range of the relation:

$$\{(\text{Smith}, 1.006\%), (\text{Johnson}, 0.810\%), (\text{Williams}, 0.699\%),$$
$$(\text{Brown}, 0.621\%), (\text{Jones}, 0.621\%)\}.$$

Solution The domain is the set of all first components. Thus, the domain is

$$\{\text{Smith}, \text{Johnson}, \text{Williams}, \text{Brown}, \text{Jones}\}.$$

Table 1.1 (repeated) Top U.S. Last Names

Name	% of All Names
Smith	1.006%
Johnson	0.810%
Williams	0.699%
Brown	0.621%
Jones	0.621%

The range is the set of all second components. Thus, the range is

$$\{1.006\%, 0.810\%, 0.699\%, 0.621\%\}.$$

Although Brown and Jones are both shared by 0.621% of the U.S. population, it is not necessary to list 0.621% twice.

Check Point 1 Find the domain and range of the relation:

$$\{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (36, 17.4)\}.$$

As you worked Check Point 1, did you wonder if there was a rule that assigned the "inputs" in the domain to the "outputs" in the range? For example, for the ordered pair (30, 13.2), how does the output 13.2 depend on the input 30? The ordered pair is based on the data in **Figure 1.13(a)**, which shows the percentage of first-year U.S. college students claiming no religious affiliation.

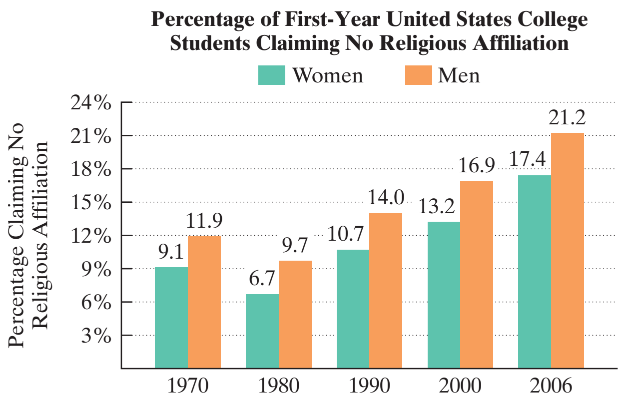


Figure 1.13(a) Data for women and men
Source: John Macionis, *Sociology*, Twelfth Edition, Prentice Hall, 2008

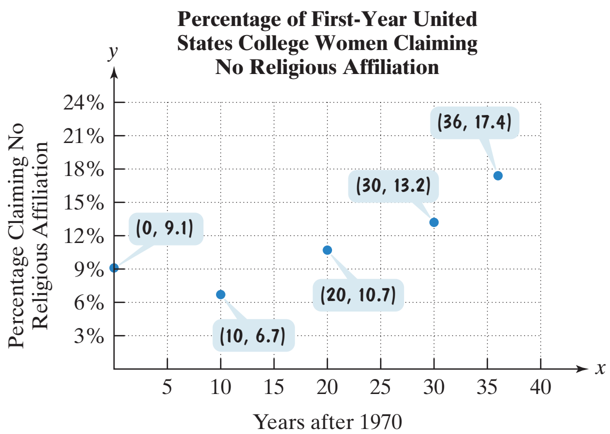


Figure 1.13(b) Visually representing the relation for the women's data

In **Figure 1.13(b)**, we used the data for college women to create the following ordered pairs:

$$\left(\text{years after 1970}, \begin{array}{l}\text{percentage of first-year college}\\ \text{women claiming no religious}\\ \text{affiliation}\end{array}\right).$$

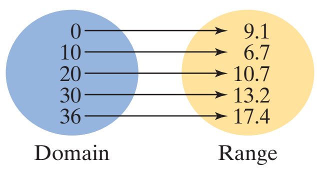
Consider, for example, the ordered pair (30, 13.2).

$$(30, 13.2)$$

30 years after 1970, or in 2000,

13.2% of first-year college women claimed no religious affiliation.

The five points in **Figure 1.13(b)** visually represent the relation formed from the women's data. Another way to visually represent the relation is as follows:



2 Determine whether a relation is a function.

Table 1.1 Top U.S. Last Names

Name	% of All Names
Smith	1.006%
Johnson	0.810%
Williams	0.699%
Brown	0.621%
Jones	0.621%

Functions

Table 1.1, repeated in the margin, shows the top five U.S. last names and the percentage of Americans who share those names. We've used this information to define two relations. **Figure 1.14(a)** shows a correspondence between last names and percents sharing those names. **Figure 1.14(b)** shows a correspondence between percents sharing last names and those last names.

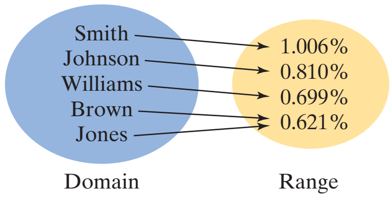


Figure 1.14(a) Names correspond to percents.

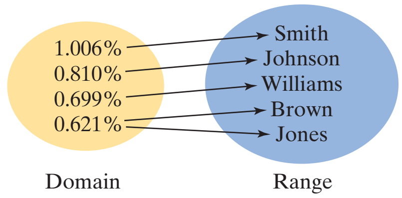


Figure 1.14(b) Percents correspond to names.

A relation in which each member of the domain corresponds to exactly one member of the range is a **function**. Can you see that the relation in **Figure 1.14(a)** is a function? Each last name in the domain corresponds to exactly one percent in the range. If we know the last name, we can be sure of the percentage of Americans sharing that name. Notice that more than one element in the domain can correspond to the same element in the range: Brown and Jones are both shared by 0.621% of Americans.

Is the relation in **Figure 1.14(b)** a function? Does each member of the domain correspond to precisely one member of the range? This relation is not a function because there is a member of the domain that corresponds to two different members of the range:

(0.621%, Brown) (0.621%, Jones).

The member of the domain 0.621% corresponds to both Brown and Jones in the range. If we know that the percentage of Americans sharing a last name is 0.621%, we cannot be sure of that last name. Because **a function is a relation in which no two ordered pairs have the same first component and different second components**, the ordered pairs (0.621%, Brown) and (0.621%, Jones) are not ordered pairs of a function.

Same first component

(0.621%, Brown) (0.621%, Jones)

Different second components

Definition of a Function

A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, such that each element in the domain corresponds to *exactly one* element in the range.

In Check Point 1, we considered a relation that gave a correspondence between years after 1970 and the percentage of first-year college women claiming no religious affiliation. Can you see that this relation is a function?

Each element in the domain

$\{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (36, 17.4)\}$

corresponds to exactly one element in the range.

However, Example 2 illustrates that not every correspondence between sets is a function.

EXAMPLE 2 Determining Whether a Relation Is a Function

Determine whether each relation is a function:

a. $\{(1, 6), (2, 6), (3, 8), (4, 9)\}$ **b.** $\{(6, 1), (6, 2), (8, 3), (9, 4)\}$.

Solution We begin by making a figure for each relation that shows the domain and the range (**Figure 1.15**).

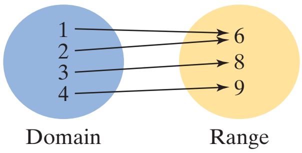


Figure 1.15(a)

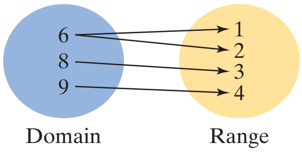


Figure 1.15(b)

a. **Figure 1.15(a)** shows that every element in the domain corresponds to exactly one element in the range. The element 1 in the domain corresponds to the element 6 in the range. Furthermore, 2 corresponds to 6, 3 corresponds to 8, and 4 corresponds to 9. No two ordered pairs in the given relation have the same first component and different second components. Thus, the relation is a function.

b. **Figure 1.15(b)** shows that 6 corresponds to both 1 and 2. If any element in the domain corresponds to more than one element in the range, the relation is not a function. This relation is not a function; two ordered pairs have the same first component and different second components.

Same first component

$(6, 1)$ $(6, 2)$

Different second components

Look at **Figure 1.15(a)** again. The fact that 1 and 2 in the domain correspond to the same number, 6, in the range does not violate the definition of a function. **A function can have two different first components with the same second component.** By contrast, a relation is not a function when two different ordered pairs have the same first component and different second components. Thus, the relation in **Figure 1.15(b)** is not a function.

Study Tip

If a relation is a function, reversing the components in each of its ordered pairs may result in a relation that is not a function.

Check Point 2 Determine whether each relation is a function:

a. $\{(1, 2), (3, 4), (5, 6), (5, 8)\}$ **b.** $\{(1, 2), (3, 4), (6, 5), (8, 5)\}$.

3 Determine whether an equation represents a function.

Functions as Equations

Functions are usually given in terms of equations rather than as sets of ordered pairs. For example, here is an equation that models the percentage of first-year college women claiming no religious affiliation as a function of time:

$$y = 0.013x^2 - 0.21x + 8.7.$$

The variable x represents the number of years after 1970. The variable y represents the percentage of first-year college women claiming no religious affiliation. The variable y is a function of the variable x. For each value of x, there is one and only one value of y. The variable x is called the **independent variable** because it can be assigned any value from the domain. Thus, x can be assigned any nonnegative integer representing the number of years after 1970. The variable y is called the **dependent variable** because its value depends on x. The percentage claiming no religious affiliation depends on the number of years after 1970. The value of the dependent variable, y, is calculated after selecting a value for the independent variable, x.

We have seen that not every set of ordered pairs defines a function. Similarly, not all equations with the variables x and y define functions. If an equation is solved for y and more than one value of y can be obtained for a given x, then the equation does not define y as a function of x.

EXAMPLE 3 Determining Whether an Equation Represents a Function

Determine whether each equation defines y as a function of x:

a. $x^2 + y = 4$ **b.** $x^2 + y^2 = 4$.

Solution Solve each equation for y in terms of x. If two or more values of y can be obtained for a given x, the equation is not a function.

a.

$$x^2 + y = 4 \quad \text{This is the given equation.}$$

$$x^2 + y - x^2 = 4 - x^2 \quad \text{Solve for } y \text{ by subtracting } x^2 \text{ from both sides.}$$

$$y = 4 - x^2 \quad \text{Simplify.}$$

From this last equation we can see that for each value of x, there is one and only one value of y. For example, if $x = 1$, then $y = 4 - 1^2 = 3$. The equation defines y as a function of x.

b.

$$x^2 + y^2 = 4 \quad \text{This is the given equation.}$$

$$x^2 + y^2 - x^2 = 4 - x^2 \quad \text{Isolate } y^2 \text{ by subtracting } x^2 \text{ from both sides.}$$

$$y^2 = 4 - x^2 \quad \text{Simplify.}$$

$$y = \pm\sqrt{4 - x^2} \quad \text{Apply the square root property: If } u^2 = d, \text{ then } u = \pm\sqrt{d}.$$

The $\pm$ in this last equation shows that for certain values of x (all values between -2 and 2), there are two values of y. For example, if $x = 1$, then $y = \pm\sqrt{4 - 1^2} = \pm\sqrt{3}$. For this reason, the equation does not define y as a function of x.

Check Point 3 Solve each equation for y and then determine whether the equation defines y as a function of x:

a. $2x + y = 6$ **b.** $x^2 + y^2 = 1$.

4 Evaluate a function.

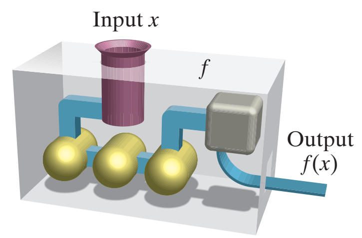


Figure 1.16 A "function machine" with inputs and outputs

Function Notation

If an equation in x and y gives one and only one value of y for each value of x, then the variable y is a function of the variable x. When an equation represents a function, the function is often named by a letter such as f, g, h, F, G, or H. Any letter can be used to name a function. Suppose that f names a function. Think of the domain as the set of the function's inputs and the range as the set of the function's outputs. As shown in **Figure 1.16**, input is represented by x and the output by $f(x)$. The special notation $\boldsymbol{f(x)}$, read "f of x" or "f at x," represents the **value of the function at the number $\boldsymbol{x}$**.

Let's make this clearer by considering a specific example. We know that the equation

$$y = 0.013x^2 - 0.21x + 8.7$$

defines y as a function of x. We'll name the function f. Now, we can apply our new function notation.

We read this equation as "f of x equals $0.013x^2 - 0.21x + 8.7$."

Input	Output	Equation
x	$f(x)$	$f(x) = 0.013x^2 - 0.21x + 8.7$

Study Tip

The notation $f(x)$ does *not* mean "f times x." The notation describes the value of the function at x.

Suppose we are interested in finding $f(30)$, the function's output when the input is 30. To find the value of the function at 30, we substitute 30 for x. We are **evaluating the function** at 30.

$$
\begin{aligned}
f(x) &= 0.013x^2 - 0.21x + 8.7 && \text{This is the given function.}\\
f(30) &= 0.013(30)^2 - 0.21(30) + 8.7 && \text{Replace each occurrence of x with 30.}\\
&= 0.013(900) - 0.21(30) + 8.7 && \text{Evaluate the exponential expression: } 30^2 = 30 \cdot 30 = 900.\\
&= 11.7 - 6.3 + 8.7 && \text{Perform the multiplications.}\\
f(30) &= 14.1 && \text{Subtract and add from left to right.}
\end{aligned}
$$

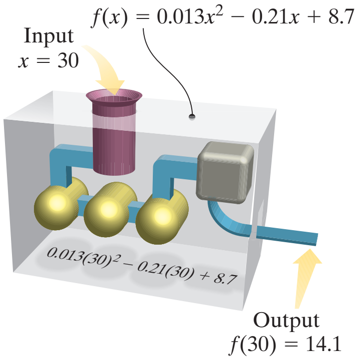


Figure 1.17 A function machine at work

The statement $f(30) = 14.1$, read "f of 30 equals 14.1," tells us that the value of the function at 30 is 14.1. When the function's input is 30, its output is 14.1. **Figure 1.17** illustrates the input and output in terms of a function machine.

$$f(30) = 14.1$$

30 years after 1970, or in 2000,

14.1% of first-year college women claimed no religious affiliation.

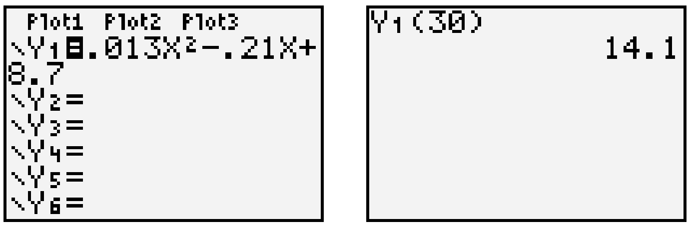
We have seen that in 2000, 13.2% actually claimed nonaffiliation, so our function that models the data slightly overestimates the percent for 2000.

Technology

Graphing utilities can be used to evaluate functions. The screens below show the evaluation of

$$f(x) = 0.013x^2 - 0.21x + 8.7$$

at 30 on a TI-84 Plus graphing calculator. The function f is named Y_1.



We used $f(x) = 0.013x^2 - 0.21x + 8.7$ to find $f(30)$. To find other function values, such as $f(40)$ or $f(55)$, substitute the specified input value, 40 or 55, for x in the function's equation.

If a function is named f and x represents the independent variable, the notation $f(x)$ corresponds to the y-value for a given x. Thus,

$$f(x) = 0.013x^2 - 0.21x + 8.7 \quad \text{and} \quad y = 0.013x^2 - 0.21x + 8.7$$

define the same function. This function may be written as

$$y = f(x) = 0.013x^2 - 0.21x + 8.7.$$

EXAMPLE 4 Evaluating a Function

If $f(x) = x^2 + 3x + 5$, evaluate each of the following:

a. $f(2)$ **b.** $f(x + 3)$ **c.** $f(-x)$.

Solution We substitute 2, $x + 3$, and $-x$ for x in the equation for f. When replacing x with a variable or an algebraic expression, you might find it helpful to think of the function's equation as

$$f(\boxed{x}) = \boxed{x}^2 + 3\boxed{x} + 5.$$

a. We find $f(2)$ by substituting 2 for x in the equation.

$$f(2) = 2^2 + 3 \cdot 2 + 5 = 4 + 6 + 5 = 15$$

Thus, $f(2) = 15$.

b. We find $f(x + 3)$ by substituting $x + 3$ for x in the equation.

$$f(x + 3) = (x + 3)^2 + 3(x + 3) + 5$$

Equivalently,

$$f(x + 3) = (x + 3)^2 + 3(x + 3) + 5$$

$$= x^2 + 6x + 9 + 3x + 9 + 5$$

Square $x + 3$ using $(A + B)^2 = A^2 + 2AB + B^2$.
Distribute 3 throughout the parentheses.

$$= x^2 + 9x + 23.$$

Combine like terms.

c. We find $f(-x)$ by substituting $-x$ for x in the equation.

$$f(-x) = (-x)^2 + 3(-x) + 5$$

Equivalently,

$$f(-x) = (-x)^2 + 3(-x) + 5$$
$$= x^2 - 3x + 5.$$

Discovery

Using $f(x) = x^2 + 3x + 5$ and the answers in parts (b) and (c):

1. Is $f(x + 3)$ equal to $f(x) + f(3)$?
2. Is $f(-x)$ equal to $-f(x)$?

Check Point 4 If $f(x) = x^2 - 2x + 7$, evaluate each of the following:

a. $f(-5)$ **b.** $f(x + 4)$ **c.** $f(-x)$.

5 Graph functions by plotting points.

Graphs of Functions

The **graph of a function** is the graph of its ordered pairs. For example, the graph of $f(x) = 2x$ is the set of points (x, y) in the rectangular coordinate system satisfying $y = 2x$. Similarly, the graph of $g(x) = 2x + 4$ is the set of points (x, y) in the rectangular coordinate system satisfying the equation $y = 2x + 4$. In the next example, we graph both of these functions in the same rectangular coordinate system.

EXAMPLE 5 Graphing Functions

Graph the functions $f(x) = 2x$ and $g(x) = 2x + 4$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2.

Solution We begin by setting up a partial table of coordinates for each function. Then, we plot the five points in each table and connect them, as shown in **Figure 1.18**. The graph of each function is a straight line. Do you see a relationship between the two graphs? The graph of g is the graph of f shifted vertically up by 4 units.

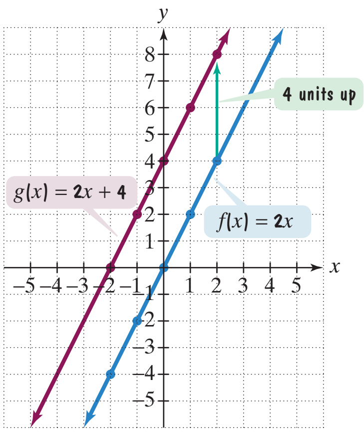


Figure 1.18

x	$f(x) = 2x$	(x, y) or $(x, f(x))$
-2	$f(-2) = 2(-2) = -4$	$(-2, -4)$
-1	$f(-1) = 2(-1) = -2$	$(-1, -2)$
0	$f(0) = 2 \cdot 0 = 0$	$(0, 0)$
1	$f(1) = 2 \cdot 1 = 2$	$(1, 2)$
2	$f(2) = 2 \cdot 2 = 4$	$(2, 4)$

Choose x. Compute $f(x)$ by evaluating f at x. Form the ordered pair.

x	$g(x) = 2x + 4$	(x, y) or $(x, g(x))$
-2	$g(-2) = 2(-2) + 4 = 0$	$(-2, 0)$
-1	$g(-1) = 2(-1) + 4 = 2$	$(-1, 2)$
0	$g(0) = 2 \cdot 0 + 4 = 4$	$(0, 4)$
1	$g(1) = 2 \cdot 1 + 4 = 6$	$(1, 6)$
2	$g(2) = 2 \cdot 2 + 4 = 8$	$(2, 8)$

Choose x. Compute $g(x)$ by evaluating g at x. Form the ordered pair.

The graphs in Example 5 are straight lines. All functions with equations of the form $f(x) = mx + b$ graph as straight lines. Such functions, called **linear functions**, will be discussed in detail in Section 1.4.

Technology

We can use a graphing utility to check the tables and the graphs in Example 5 for the functions

$$f(x) = 2x \quad \text{and} \quad g(x) = 2x + 4.$$

Enter $y_1 = 2x$ in the $\boxed{y=}$ screen.

Enter $y_2 = 2x + 4$ in the $\boxed{y=}$ screen.

Checking Tables

We entered −2 for the starting x-value and 1 as an increment between x-values to check our tables in Example 5.

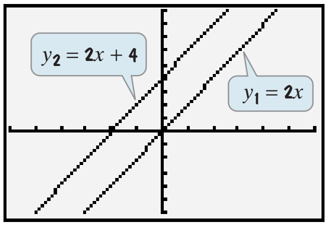
X	Y1	Y2
-2	-4	0
-1	-2	2
0	0	4
1	2	6
2	4	8
3	6	10
4	8	12

X=-2

Use the first five ordered pairs (x, y_1) to check the first table.

Use the first five ordered pairs (x, y_2) to check the second table.

Checking Graphs



$[-6, 6, 1]$ by $[-6, 9, 1]$

We selected this viewing rectangle, or window, to match **Figure 1.18.**

Check Point 5 Graph the functions $f(x) = 2x$ and $g(x) = 2x - 3$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2. How is the graph of g related to the graph of f?

6 Use the vertical line test to identify functions.

The Vertical Line Test

Not every graph in the rectangular coordinate system is the graph of a function. The definition of a function specifies that no value of x can be paired with two or more different values of y. Consequently, if a graph contains two or more different points with the same first coordinate, the graph cannot represent a function. This is illustrated in **Figure 1.19**. Observe that points sharing a common first coordinate are vertically above or below each other.

This observation is the basis of a useful test for determining whether a graph defines y as a function of x. The test is called the **vertical line test**.

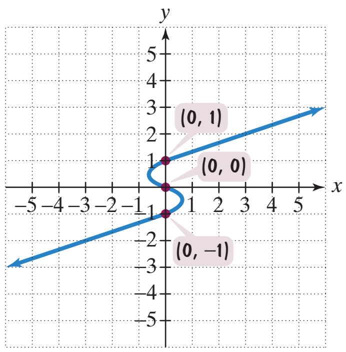


Figure 1.19 y is not a function of x because 0 is paired with three values of y, namely, 1, 0, and -1.

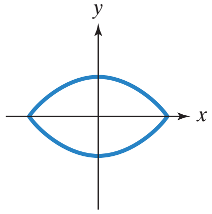
The Vertical Line Test for Functions

If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.

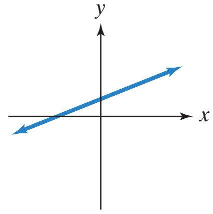
EXAMPLE 6 Using the Vertical Line Test

Use the vertical line test to identify graphs in which y is a function of x.

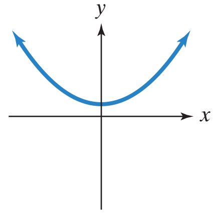
a.



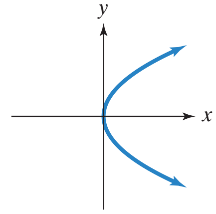
b.



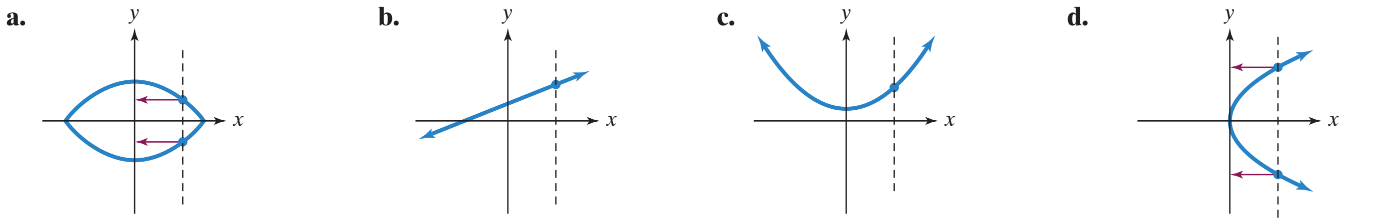
c.



d.



Solution y is a function of x for the graphs in (b) and (c).



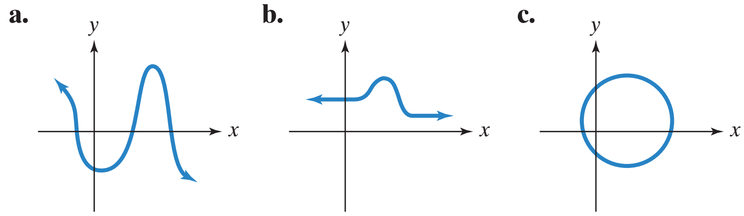
a. y is **not a function of** x. Two values of y correspond to an x-value.

b. y **is a function of** x.

c. y **is a function of** x.

d. y is **not a function of** x. Two values of y correspond to an x-value.

Check Point 6 Use the vertical line test to identify graphs in which y is a function of x.



7 Obtain information about a function from its graph.

Obtaining Information from Graphs

You can obtain information about a function from its graph. At the right or left of a graph, you will find closed dots, open dots, or arrows.

- A closed dot indicates that the graph does not extend beyond this point and the point belongs to the graph.
- An open dot indicates that the graph does not extend beyond this point and the point does not belong to the graph.
- An arrow indicates that the graph extends indefinitely in the direction in which the arrow points.

EXAMPLE 7 Analyzing the Graph of a Function

The human immunodeficiency virus, or HIV, infects and kills helper T cells. Because T cells stimulate the immune system to produce antibodies, their destruction disables the body's defenses against other pathogens. By counting the number of T cells that remain active in the body, the progression of HIV can be monitored. The fewer helper T cells, the more advanced the disease. **Figure 1.20** shows a graph that is used to monitor the average progression of the disease. The average number of T cells, $f(x)$, is a function of time after infection, x.

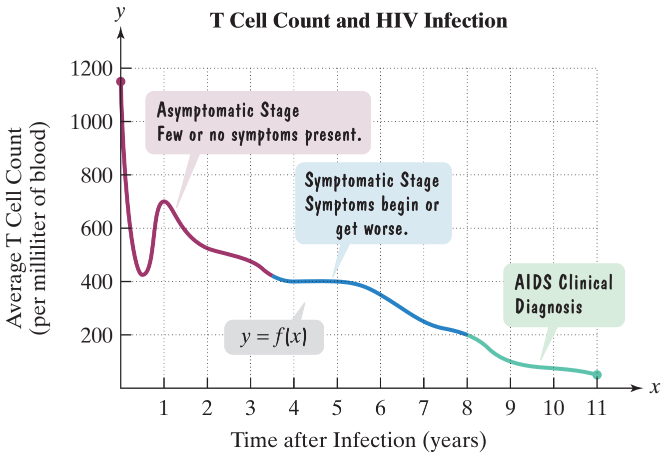


Figure 1.20

Source: B. E. Pruitt et al., *Human Sexuality*, Prentice Hall, 2007

a. Explain why f represents the graph of a function.

b. Use the graph to find $f(8)$.

c. For what value of x is $f(x) = 350$?

d. Describe the general trend shown by the graph.

Solution

a. No vertical line can be drawn that intersects the graph of f more than once. By the vertical line test, f represents the graph of a function.

b. To find $f(8)$, or f of 8, we locate 8 on the x-axis. **Figure 1.21** shows the point on the graph of f for which 8 is the first coordinate. From this point, we look to the y-axis to find the corresponding y-coordinate. We see that the y-coordinate is 200. Thus,

$$f(8) = 200.$$

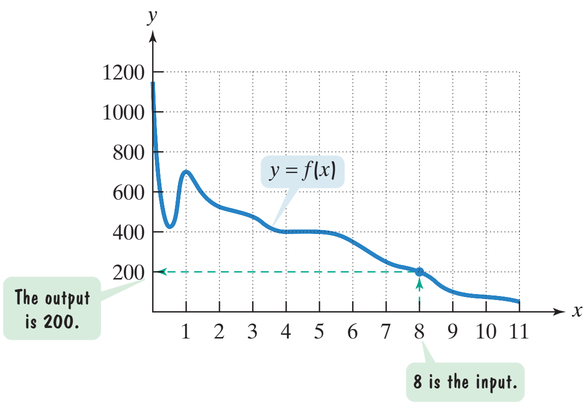


Figure 1.21 Finding $f(8)$

When the time after infection is 8 years, the average T cell count is 200 cells per milliliter of blood. (AIDS clinical diagnosis is given at a T cell count of 200 or below.)

c. To find the value of x for which $f(x) = 350$, we find the approximate location of 350 on the y-axis. **Figure 1.22** shows that there is one point on the graph of f for which 350 is the second coordinate. From this point, we look to the x-axis to find the corresponding x-coordinate. We see that the x-coordinate is 6. Thus,

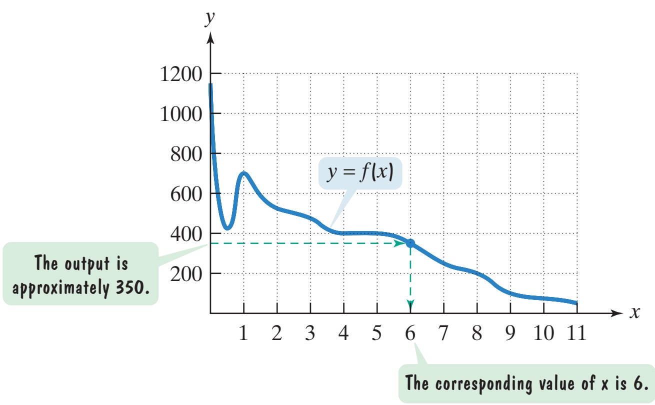


Figure 1.22 Finding x for which $f(x) = 350$

$$f(x) = 350 \text{ for } x = 6.$$

An average T cell count of 350 occurs 6 years after infection.

d. **Figure 1.23** uses voice balloons to describe the general trend shown by the graph.

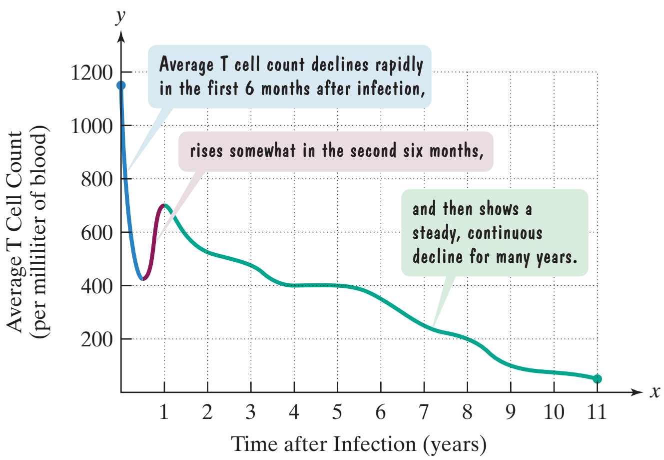


Figure 1.23 Describing changing T cell count over time in a person infected with HIV

Check Point 7

a. Use the graph of f in **Figure 1.20** on page 155 to find $f(5)$.

b. For what value of x is $f(x) = 100$?

c. Estimate the minimum average T cell count during the asymptomatic stage.

8 Identify the domain and range of a function from its graph.

Identifying Domain and Range from a Function's Graph

Study Tip

Throughout this discussion, we will be using interval notation. Recall that square brackets indicate endpoints that are included in an interval. Parentheses indicate endpoints that are not included in an interval. Parentheses are always used with ∞ or $-\infty$. For more detail on interval notation, see Section P.9, pages 115–116.

Figure 1.24 illustrates how the graph of a function is used to determine the function's domain and its range.

Domain: set of inputs

Found on the x-axis

Range: set of outputs

Found on the y-axis

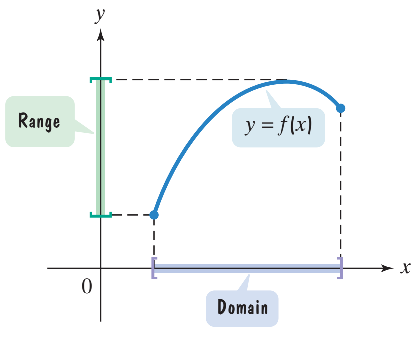


Figure 1.24 Domain and range of f

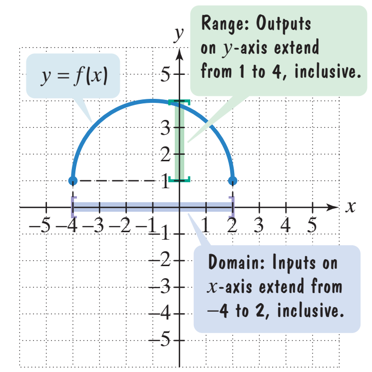


Figure 1.25 Domain and range of f

Let's apply these ideas to the graph of the function shown in **Figure 1.25**. To find the domain, look for all the inputs on the x-axis that correspond to points on the graph. Can you see that they extend from -4 to 2, inclusive? The function's domain can be represented as follows:

Using Set-Builder Notation	Using Interval Notation
$\{x \mid -4 \le x \le 2\}$	$[-4, 2]$.
The set of all x / such that / x is greater than or equal to -4 and less than or equal to 2.	The square brackets indicate -4 and 2 are included. Note the square brackets on the x-axis in **Figure 1.25**.

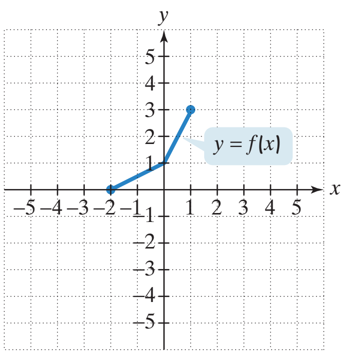
To find the range, look for all the outputs on the y-axis that correspond to points on the graph. They extend from 1 to 4, inclusive. The function's range can be represented as follows:

Using Set-Builder Notation	Using Interval Notation
$\{y \mid 1 \le y \le 4\}$	$[1, 4]$.
The set of all y / such that / y is greater than or equal to 1 and less than or equal to 4.	The square brackets indicate 1 and 4 are included. Note the square brackets on the y-axis in **Figure 1.25**.

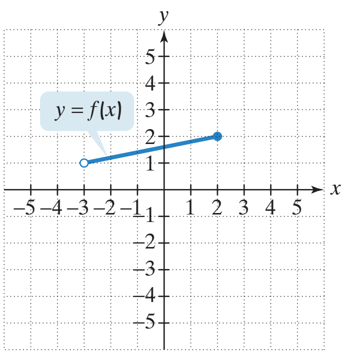
EXAMPLE 8 Identifying the Domain and Range of a Function from Its Graph

Use the graph of each function shown on the next page to identify its domain and its range.

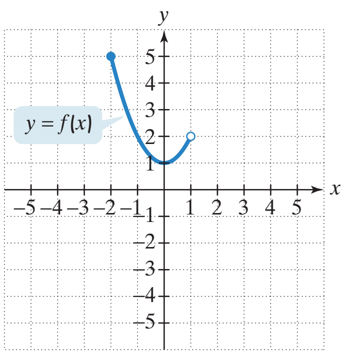
a.



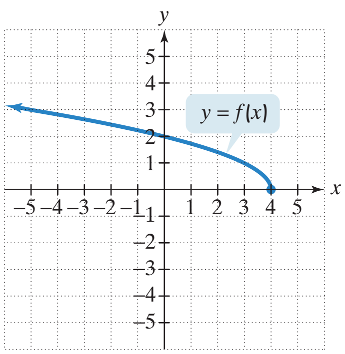
b.



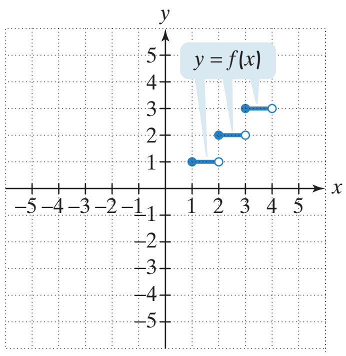
c.



d.

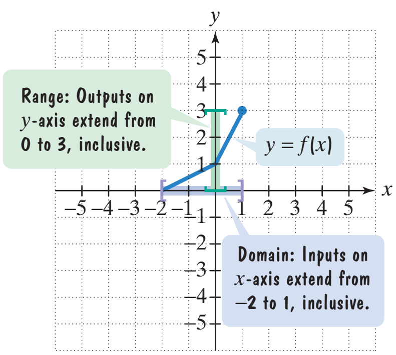


e.



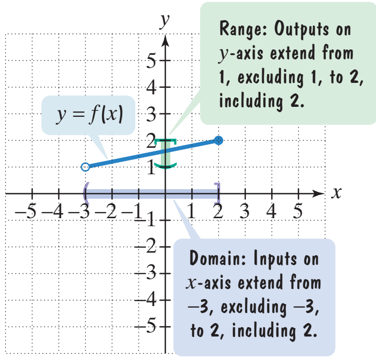
Solution For the graph of each function, the domain is highlighted in purple on the x-axis and the range is highlighted in green on the y-axis.

a.



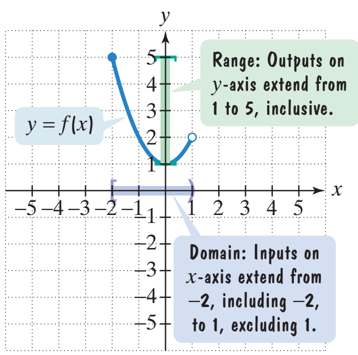
Domain $= \{x | -2 \le x \le 1\}$ or $[-2, 1]$
Range $= \{y | 0 \le y \le 3\}$ or $[0, 3]$

b.



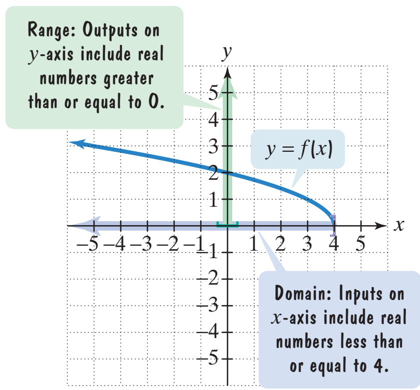
Domain $= \{x | -3 < x \le 2\}$ or $(-3, 2]$
Range $= \{y | 1 < y \le 2\}$ or $(1, 2]$

c.



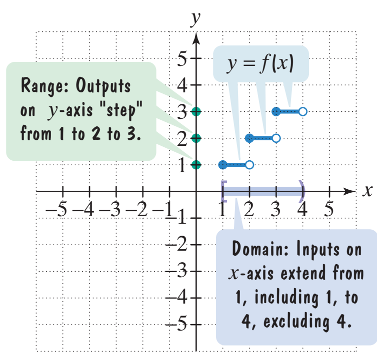
Domain $= \{x | -2 \le x < 1\}$ or $[-2, 1)$
Range $= \{y | 1 \le y \le 5\}$ or $[1, 5]$

d.



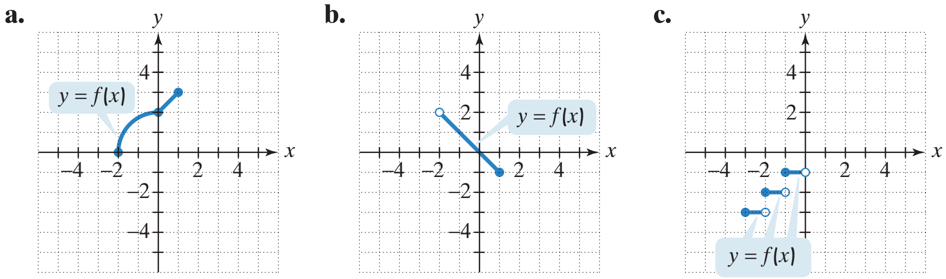
Domain $= \{x | x \le 4\}$ or $(-\infty, 4]$
Range $= \{y | y \ge 0\}$ or $[0, \infty)$

e.



Domain $= \{x | 1 \le x < 4\}$ or $[1, 4)$
Range $= \{y | y = 1, 2, 3\}$

Check Point 8 Use the graph of each function to identify its domain and its range.



9 Identify intercepts from a function's graph.

Identifying Intercepts from a Function's Graph

Figure 1.26 illustrates how we can identify intercepts from a function's graph. To find the x-intercepts, look for the points at which the graph crosses the x-axis. There are three such points: $(-2, 0)$, $(3, 0)$, and $(5, 0)$. Thus, the x-intercepts are -2, 3, and 5. We express this in function notation by writing $f(-2) = 0$, $f(3) = 0$, and $f(5) = 0$. We say that -2, 3, and 5 are the *zeros of the function*. The **zeros of a function** f are the x-values for which $f(x) = 0$. Thus, the real zeros are the x-intercepts.

To find the y-intercept, look for the point at which the graph crosses the y-axis. This occurs at $(0, 3)$. Thus, the y-intercept is 3. We express this in function notation by writing $f(0) = 3$.

By the definition of a function, for each value of x we can have at most one value for y. What does this mean in terms of intercepts? **A function can have more than one x-intercept but at most one y-intercept.**

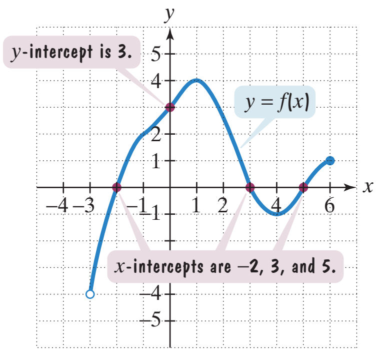


Figure 1.26 Identifying intercepts

Exercise Set 1.2

Practice Exercises

In Exercises 1–10, determine whether each relation is a function. Give the domain and range for each relation.

1. $\{(1, 2), (3, 4), (5, 5)\}$ **2.** $\{(4, 5), (6, 7), (8, 8)\}$

3. $\{(3, 4), (3, 5), (4, 4), (4, 5)\}$

4. $\{(5, 6), (5, 7), (6, 6), (6, 7)\}$

5. $\{(3, -2), (5, -2), (7, 1), (4, 9)\}$

6. $\{(10, 4), (-2, 4), (-1, 1), (5, 6)\}$

7. $\{(-3, -3), (-2, -2), (-1, -1), (0, 0)\}$

8. $\{(-7, -7), (-5, -5), (-3, -3), (0, 0)\}$

9. $\{(1, 4), (1, 5), (1, 6)\}$

10. $\{(4, 1), (5, 1), (6, 1)\}$

In Exercises 11–26, determine whether each equation defines y as a function of x.

11. $x + y = 16$ **12.** $x + y = 25$

13. $x^2 + y = 16$ **14.** $x^2 + y = 25$

15. $x^2 + y^2 = 16$ **16.** $x^2 + y^2 = 25$

17. $x = y^2$ **18.** $4x = y^2$

19. $y = \sqrt{x + 4}$ **20.** $y = -\sqrt{x + 4}$

21. $x + y^3 = 8$ **22.** $x + y^3 = 27$

23. $xy + 2y = 1$ **24.** $xy - 5y = 1$

25. $|x| - y = 2$ **26.** $|x| - y = 5$

In Exercises 27–38, evaluate each function at the given values of the independent variable and simplify.

27. $f(x) = 4x + 5$
a. $f(6)$ **b.** $f(x + 1)$ **c.** $f(-x)$

28. $f(x) = 3x + 7$
a. $f(4)$ **b.** $f(x + 1)$ **c.** $f(-x)$

29. $g(x) = x^2 + 2x + 3$
a. $g(-1)$ **b.** $g(x + 5)$ **c.** $g(-x)$

30. $g(x) = x^2 - 10x - 3$
a. $g(-1)$ **b.** $g(x + 2)$ **c.** $g(-x)$

31. $h(x) = x^4 - x^2 + 1$
a. $h(2)$ **b.** $h(-1)$
c. $h(-x)$ **d.** $h(3a)$

32. $h(x) = x^3 - x + 1$
a. $h(3)$ **b.** $h(-2)$
c. $h(-x)$ **d.** $h(3a)$

33. $f(r) = \sqrt{r + 6} + 3$
a. $f(-6)$ **b.** $f(10)$ **c.** $f(x - 6)$

34. $f(r) = \sqrt{25 - r} - 6$

a. $f(16)$ **b.** $f(-24)$ **c.** $f(25 - 2x)$

35. $f(x) = \dfrac{4x^2 - 1}{x^2}$

a. $f(2)$ **b.** $f(-2)$ **c.** $f(-x)$

36. $f(x) = \dfrac{4x^3 + 1}{x^3}$

a. $f(2)$ **b.** $f(-2)$ **c.** $f(-x)$

37. $f(x) = \dfrac{x}{|x|}$

a. $f(6)$ **b.** $f(-6)$ **c.** $f(r^2)$

38. $f(x) = \dfrac{|x + 3|}{x + 3}$

a. $f(5)$ **b.** $f(-5)$ **c.** $f(-9 - x)$

In Exercises 39–50, graph the given functions, f and g, in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2. Once you have obtained your graphs, describe how the graph of g is related to the graph of f.

39. $f(x) = x, g(x) = x + 3$

40. $f(x) = x, g(x) = x - 4$

41. $f(x) = -2x, g(x) = -2x - 1$

42. $f(x) = -2x, g(x) = -2x + 3$

43. $f(x) = x^2, g(x) = x^2 + 1$

44. $f(x) = x^2, g(x) = x^2 - 2$

45. $f(x) = |x|, g(x) = |x| - 2$

46. $f(x) = |x|, g(x) = |x| + 1$

47. $f(x) = x^3, g(x) = x^3 + 2$

48. $f(x) = x^3, g(x) = x^3 - 1$

49. $f(x) = 3, g(x) = 5$

50. $f(x) = -1, g(x) = 4$

In Exercises 51–54, graph the given square root functions, f and g, in the same rectangular coordinate system. Use the integer values of x given to the right of each function to obtain ordered pairs. Because only nonnegative numbers have square roots that are real numbers, be sure that each graph appears only for values of x that cause the expression under the radical sign to be greater than or equal to zero. Once you have obtained your graphs, describe how the graph of g is related to the graph of f.

51. $f(x) = \sqrt{x} \quad (x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x} - 1 \quad (x = 0, 1, 4, 9)$

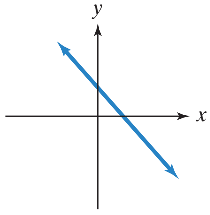
52. $f(x) = \sqrt{x} \quad (x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x} + 2 \quad (x = 0, 1, 4, 9)$

53. $f(x) = \sqrt{x} \quad (x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x - 1} \quad (x = 1, 2, 5, 10)$

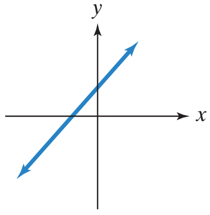
54. $f(x) = \sqrt{x} \quad (x = 0, 1, 4, 9)$ and
$g(x) = \sqrt{x + 2} \quad (x = -2, -1, 2, 7)$

In Exercises 55–64, use the vertical line test to identify graphs in which y is a function of x.

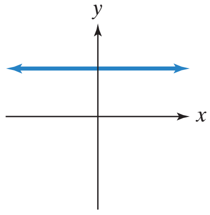
55.



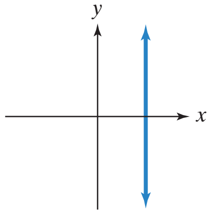
56.



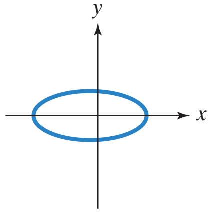
57.



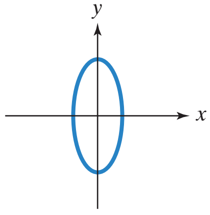
58.



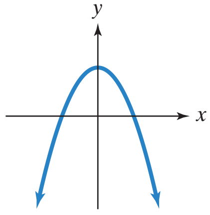
59.



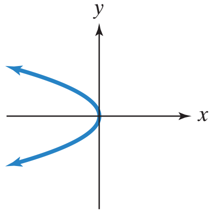
60.



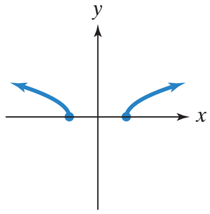
61.



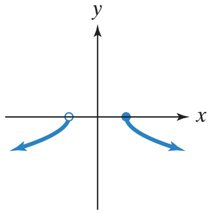
62.



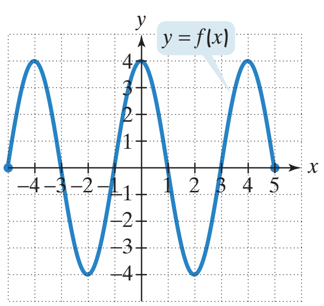
63.



64.



In Exercises 65–70, use the graph of f to find each indicated function value.



65. $f(-2)$

66. $f(2)$

67. $f(4)$

68. $f(-4)$

69. $f(-3)$

70. $f(-1)$

Use the graph of g to solve Exercises 71–76.

71. Find $g(-4)$.

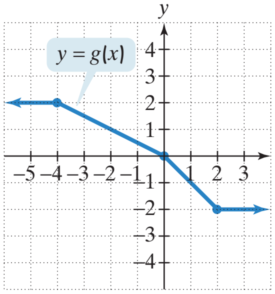
72. Find $g(2)$.

73. Find $g(-10)$.

74. Find $g(10)$.

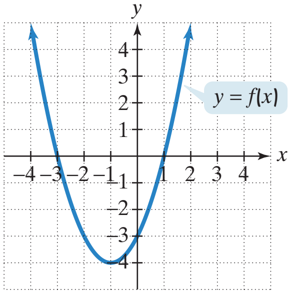
75. For what value of x is $g(x) = 1$?

76. For what value of x is $g(x) = -1$?



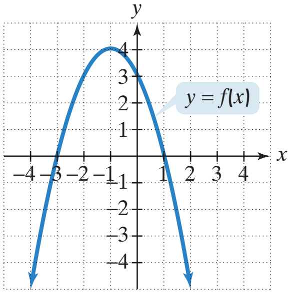
In Exercises 77–92, use the graph to determine **a.** *the function's domain;* **b.** *the function's range;* **c.** *the x-intercepts, if any;* **d.** *the y-intercept, if any; and* **e.** *the missing function values, indicated by question marks, below each graph.*

77.



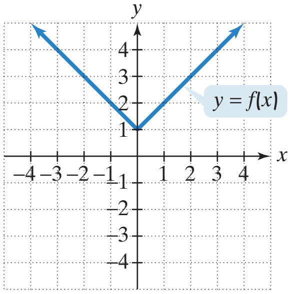
$f(-2) = ?\quad f(2) = ?$

78.



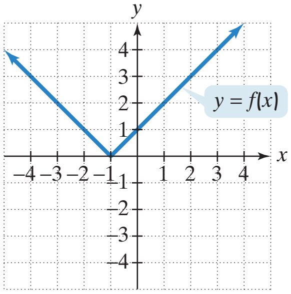
$f(-2) = ?\quad f(2) = ?$

79.



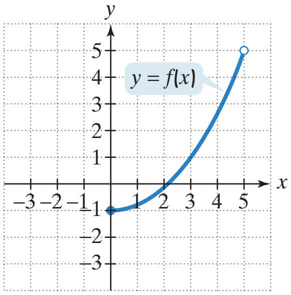
$f(-1) = ?\quad f(3) = ?$

80.



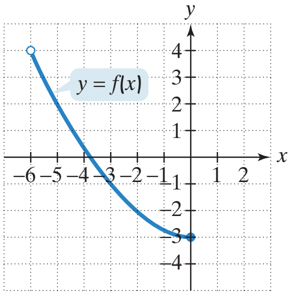
$f(-4) = ?\quad f(3) = ?$

81.



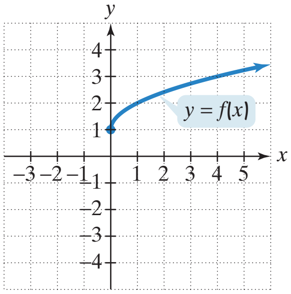
$f(3) = ?$

82.



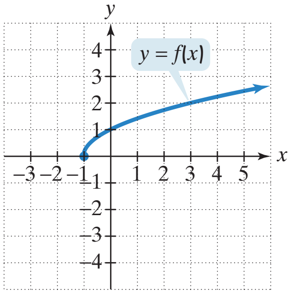
$f(-5) = ?$

83.



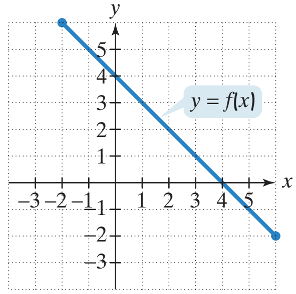
$f(4) = ?$

84.



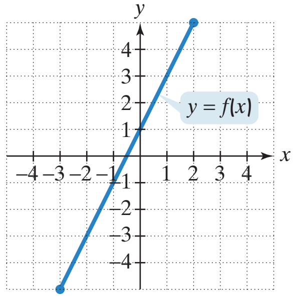
$f(3) = ?$

85.



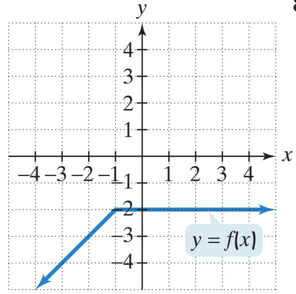
$f(-1) = ?$

86.



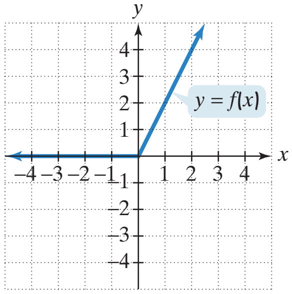
$f(-2) = ?$

87.



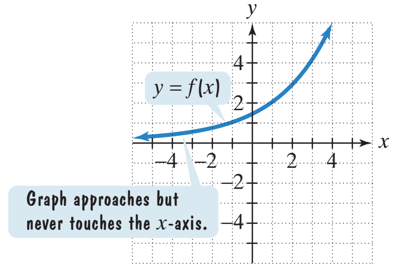
$f(-4) = ?\quad f(4) = ?$

88.



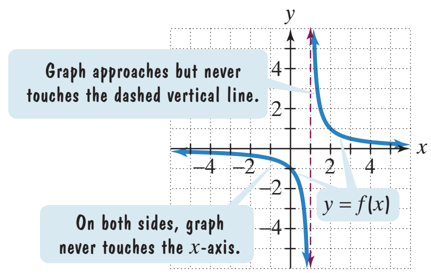
$f(-2) = ?\quad f(2) = ?$

89.



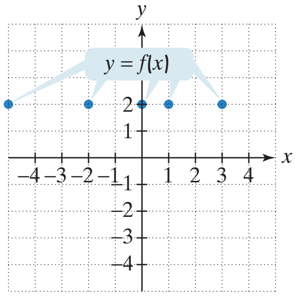
$f(4) = ?$

90.



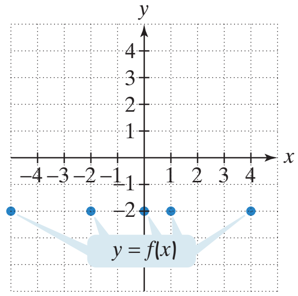
$f(2) = ?$

91.



$f(-5) + f(3) = ?$

92.



$f(-5) + f(4) = ?$

Practice Plus

In Exercises 93–94, let $f(x) = x^2 - x + 4$ *and* $g(x) = 3x - 5$.

93. Find $g(1)$ and $f(g(1))$. **94.** Find $g(-1)$ and $f(g(-1))$.

In Exercises 95–96, let f and g be defined by the following table:

x	$f(x)$	$g(x)$
-2	6	0
-1	3	4
0	-1	1
1	-4	-3
2	0	-6

95. Find $\sqrt{f(-1) - f(0)} - [g(2)]^2 + f(-2) \div g(2) \cdot g(-1)$.

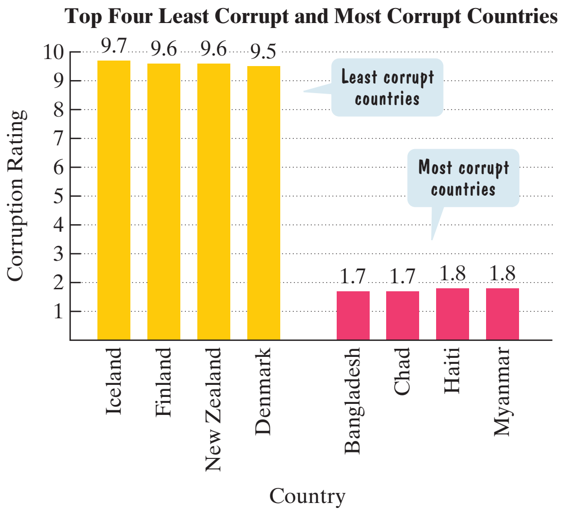
96. Find $|f(1) - f(0)| - [g(1)]^2 + g(1) \div f(-1) \cdot g(2)$.

In Exercises 97–98, find $f(-x) - f(x)$ *for the given function f. Then simplify the expression.*

97. $f(x) = x^3 + x - 5$ **98.** $f(x) = x^2 - 3x + 7$

Application Exercises

The Corruption Perceptions Index uses perceptions of the general public, business people, and risk analysts to rate countries by how likely they are to accept bribes. The ratings are on a scale from 0 to 10, where higher scores represent less corruption. The graph shows the corruption ratings for the world's least corrupt and most corrupt countries. (The rating for the United States is 7.6.) Use the graph to solve Exercises 99–100.



Source: Transparency International, *Corruption Perceptions Index*

99. Use the four least corrupt countries to solve this exercise.

a. Write a set of four ordered pairs in which countries correspond to corruption ratings. Each ordered pair should be in the form

(country, corruption rating).

b. Is the relation in part (a) a function? Explain your answer.

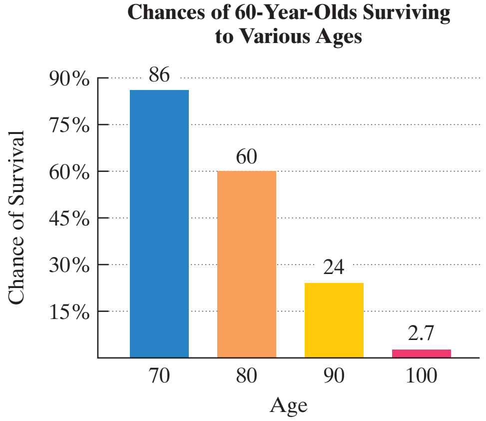
c. Write a set of four ordered pairs in which corruption ratings correspond to countries. Each ordered pair should be in the form

(corruption rating, country).

d. Is the relation in part (c) a function? Explain your answer.

100. Repeat parts (a) through (d) in Exercise 99 using the four most corrupt countries.

The bar graph shows your chances of surviving to various ages once you reach 60.



Source: National Center for Health Statistics

The functions

$$f(x) = -2.9x + 286$$
$$\text{and} \quad g(x) = 0.01x^2 - 4.9x + 370$$

model the chance, as a percent, that a 60-year-old will survive to age x. Use this information to solve Exercises 101–102.

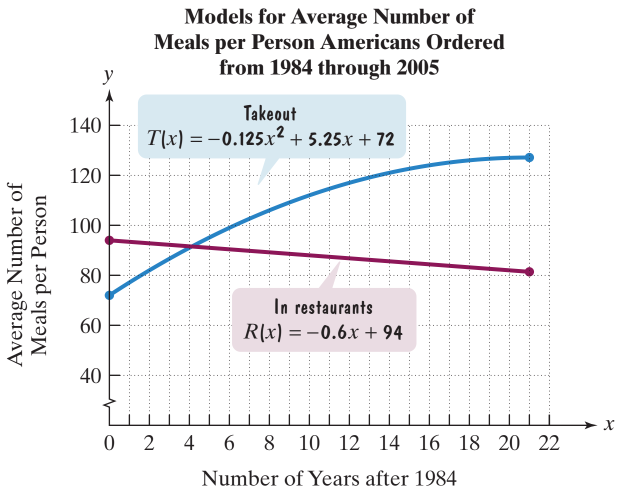
101. a. Find and interpret $f(70)$. **b.** Find and interpret $g(70)$.

c. Which function serves as a better model for the chance of surviving to age 70?

102. a. Find and interpret $f(90)$. **b.** Find and interpret $g(90)$.

c. Which function serves as a better model for the chance of surviving to age 90?

To go, please. The graphs show that more Americans are ordering their food to go, instead of dining inside restaurants. A quadratic function models the average number of meals per person that Americans ordered for takeout and a linear function models the average number of meals ordered for eating in restaurants. In each model, x represents the number of years after 1984. Use the graphs and the displayed equations to solve Exercises 103–104.



Source: NPD Group

(In Exercises 103–104, refer to the graphs and their equations at the bottom of the previous page.)

103. **a.** Use the equation for function T to find and interpret $T(20)$. How is this shown on the graph of T?

b. Use the equation for function R to find and interpret $R(0)$. How is this shown on the graph of R?

c. According to the graphs, in which year did the average number of takeout orders approximately equal the average number of in-restaurant orders? Use the equations for T and R to find the average number of meals per person for each kind of order in that year.

104. **a.** Use the equation for function T to find and interpret $T(18)$. How is this shown on the graph of T?

b. Use the equation for function R to find and interpret $R(20)$. How is this shown on the graph of R?

In Exercises 105–108, you will be developing functions that model given conditions.

105. A company that manufactures bicycles has a fixed cost of \$100,000. It costs \$100 to produce each bicycle. The total cost for the company is the sum of its fixed cost and variable costs. Write the total cost, C, as a function of the number of bicycles produced, x. Then find and interpret $C(90)$.

106. A car was purchased for \$22,500. The value of the car decreased by \$3200 per year for the first six years. Write a function that describes the value of the car, V, after x years, where $0 \le x \le 6$. Then find and interpret $V(3)$.

107. You commute to work a distance of 40 miles and return on the same route at the end of the day. Your average rate on the return trip is 30 miles per hour faster than your average rate on the outgoing trip. Write the total time, T, in hours, devoted to your outgoing and return trips as a function of your rate on the outgoing trip, x. Then find and interpret $T(30)$. Hint:

$$\text{Time traveled} = \frac{\text{Distance traveled}}{\text{Rate of travel}}.$$

108. A chemist working on a flu vaccine needs to mix a 10% sodium-iodine solution with a 60% sodium-iodine solution to obtain a 50-milliliter mixture. Write the amount of sodium iodine in the mixture, S, in milliliters, as a function of the number of milliliters of the 10% solution used, x. Then find and interpret $S(30)$.

Writing in Mathematics

109. What is a relation? Describe what is meant by its domain and its range.

110. Explain how to determine whether a relation is a function. What is a function?

111. How do you determine if an equation in x and y defines y as a function of x?

112. Does $f(x)$ mean f times x when referring to a function f? If not, what does $f(x)$ mean? Provide an example with your explanation.

113. What is the graph of a function?

114. Explain how the vertical line test is used to determine whether a graph represents a function.

115. Explain how to identify the domain and range of a function from its graph.

116. For people filing a single return, federal income tax is a function of adjusted gross income because for each value of adjusted gross income there is a specific tax to be paid. By contrast, the price of a house is not a function of the lot size on which the house sits because houses on same-sized lots can sell for many different prices.

a. Describe an everyday situation between variables that is a function.

b. Describe an everyday situation between variables that is not a function.

Technology Exercise

117. Use a graphing utility to verify any five pairs of graphs that you drew by hand in Exercises 39–54.

Critical Thinking Exercises

Make Sense? *In Exercises 118–121, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

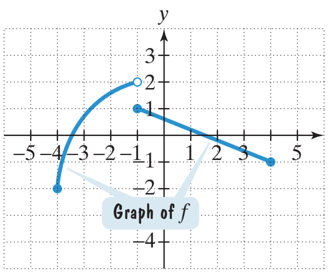
118. My body temperature is a function of the time of day.

119. Using $f(x) = 3x + 2$, I found $f(50)$ by applying the distributive property to $(3x + 2)50$.

120. I graphed a function showing how paid vacation days depend on the number of years a person works for a company. The domain was the number of paid vacation days.

121. I graphed a function showing how the average number of annual physician visits depends on a person's age. The domain was the average number of annual physician visits.

Use the graph of f to determine whether each statement in Exercises 122–125 is true or false.



122. The domain of f is $[-4, -1) \cup (-1, 4]$.

123. The range of f is $[-2, 2]$.

124. $f(-1) - f(4) = 2$

125. $f(0) = 2.1$

126. If $f(x) = 3x + 7$, find $\dfrac{f(a + h) - f(a)}{h}$.

127. Give an example of a relation with the following characteristics: The relation is a function containing two ordered pairs. Reversing the components in each ordered pair results in a relation that is not a function.

128. If $f(x + y) = f(x) + f(y)$ and $f(1) = 3$, find $f(2)$, $f(3)$, and $f(4)$. Is $f(x + y) = f(x) + f(y)$ for all functions?

Preview Exercises

Exercises 129–131 will help you prepare for the material covered in the next section.

129. The function $C(t) = 20 + 0.40\,(t - 60)$ describes the monthly cost, $C(t)$, in dollars, for a cellular phone plan for t calling minutes, where $t > 60$. Find and interpret $C(100)$.

130. Use point plotting to graph $f(x) = x + 2$ if $x \le 1$.

131. Simplify: $2(x + h)^2 + 3(x + h) + 5 - (2x^2 + 3x + 5)$.