Functions and Graphs

1

A vast expanse of open water at the top of our world was once covered with ice. The melting of the Arctic ice caps has forced polar bears to swim as far as 40 miles, causing them to drown in significant numbers. Such deaths were rare in the past. There is strong scientific consensus that human activities are changing the Earth's climate. Scientists now believe that there is a striking correlation between atmospheric carbon dioxide concentration and global temperature. As both of these variables increase at significant rates, there are warnings of a planetary emergency that threatens to condemn coming generations to a catastrophically diminished future.*

In this chapter, you'll learn to approach our climate crisis mathematically by creating formulas, called functions, that model data for average global temperature and carbon dioxide concentration over time. Understanding the concept of a function will give you a new perspective on many situations, ranging from global warming to using mathematics in a way that is similar to making a movie.

A mathematical model involving global warming is developed in Example 9 in Section 2.3. Using mathematics in a way that is similar to making a movie is discussed in the essay on page 199.

**Sources:* Al Gore, *An Inconvenient Truth*, Rodale, 2006; *Time*, April 3, 2006

Section 1.1

Graphs and Graphing Utilities

Objectives

1. Plot points in the rectangular coordinate system.
2. Graph equations in the rectangular coordinate system.
3. Interpret information about a graphing utility's viewing rectangle or table.
4. Use a graph to determine intercepts.
5. Interpret information given by graphs.

The beginning of the seventeenth century was a time of innovative ideas and enormous intellectual progress in Europe. English theatergoers enjoyed a succession of exciting new plays by Shakespeare. William Harvey proposed the radical notion that the heart was a pump for blood rather than the center of emotion. Galileo, with his newfangled invention called the telescope, supported the theory of Polish astronomer Copernicus that the sun, not the Earth, was the center of the solar system. Monteverdi was writing the world's first grand operas. French mathematicians Pascal and Fermat invented a new field of mathematics called probability theory.

Into this arena of intellectual electricity stepped French aristocrat René Descartes (1596–1650). Descartes (pronounced "day cart"), propelled by the creativity surrounding him, developed a new branch of mathematics that brought together algebra and geometry in a unified way—a way that visualized numbers as points on a graph, equations as geometric figures, and geometric figures as equations. This new branch of mathematics, called *analytic geometry,* established Descartes as one of the founders of modern thought and among the most original mathematicians and philosophers of any age. We begin this section by looking at Descartes's deceptively simple idea, called the **rectangular coordinate system** or (in his honor) the **Cartesian coordinate system**.

Points and Ordered Pairs

1 Plot points in the rectangular coordinate system.

Descartes used two number lines that intersect at right angles at their zero points, as shown in **Figure 1.1**. The horizontal number line is the ***x*-axis.** The vertical number line is the ***y*-axis**. The point of intersection of these axes is their zero points, called the **origin**. Positive numbers are shown to the right and above the origin. Negative numbers are shown to the left and below the origin. The axes divide the plane into four quarters, called **quadrants**. The points located on the axes are not in any quadrant.

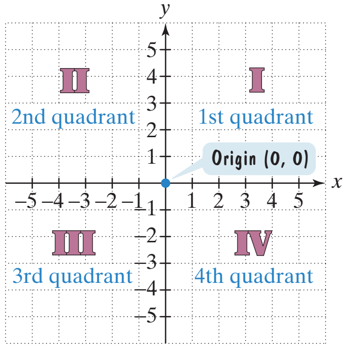


Figure 1.1 The rectangular coordinate system

Each point in the rectangular coordinate system corresponds to an **ordered pair** of real numbers, (x, y). Examples of such pairs are $(-5, 3)$ and $(3, -5)$. The first number in each pair, called the ***x*-coordinate**, denotes the distance and direction from the origin along the x-axis. The second number in each pair, called the ***y*-coordinate**, denotes vertical distance and direction along a line parallel to the y-axis or along the y-axis itself.

Figure 1.2 shows how we **plot**, or locate, the points corresponding to the ordered pairs $(-5, 3)$ and $(3, -5)$. We plot $(-5, 3)$ by going 5 units from 0 to the left along the x-axis. Then we go 3 units up parallel to the y-axis. We plot $(3, -5)$ by going 3 units from 0 to the right along the x-axis and 5 units down parallel to the y-axis. The phrase "the points corresponding to the ordered pairs $(-5, 3)$ and $(3, -5)$" is often abbreviated as "the points $(-5, 3)$ and $(3, -5)$."

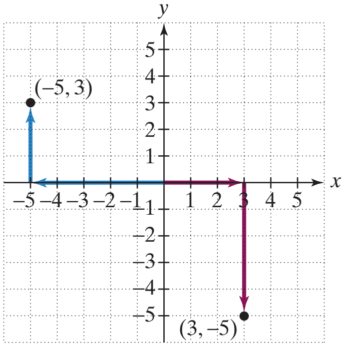


Figure 1.2 Plotting $(-5, 3)$ and $(3, -5)$

Study Tip

The phrase *ordered pair* is used because order is important. The order in which coordinates appear makes a difference in a point's location. This is illustrated in **Figure 1.2**.

EXAMPLE 1 Plotting Points in the Rectangular Coordinate System

Plot the points: $A(-3, 5)$, $B(2, -4)$, $C(5, 0)$, $D(-5, -3)$, $E(0, 4)$, and $F(0, 0)$.

Solution See **Figure 1.3**. We move from the origin and plot the points in the following way:

$A(-3, 5)$: 3 units left, 5 units up
$B(2, -4)$: 2 units right, 4 units down
$C(5, 0)$: 5 units right, 0 units up or down
$D(-5, -3)$: 5 units left, 3 units down
$E(0, 4)$: 0 units right or left, 4 units up
$F(0, 0)$: 0 units right or left, 0 units up or down

Notice that the origin is represented by (0, 0).

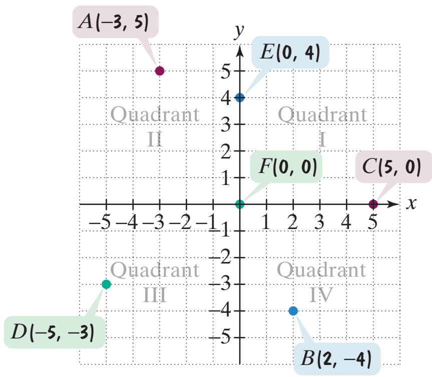


Figure 1.3 Plotting points

Reminder: Answers to all Check Point exercises are given in the answer section. Check your answer before continuing your reading to verify that you understand the concept.

Check Point 1 Plot the points: $A(-2, 4)$, $B(4, -2)$, $C(-3, 0)$, and $D(0, -3)$.

2 Graph equations in the rectangular coordinate system.

Graphs of Equations

A relationship between two quantities can be expressed as an **equation in two variables**, such as

$$y = 4 - x^2.$$

A **solution of an equation in two variables**, x and y, is an ordered pair of real numbers with the following property: When the x-coordinate is substituted for x and the y-coordinate is substituted for y in the equation, we obtain a true statement. For example, consider the equation $y = 4 - x^2$ and the ordered pair $(3, -5)$. When 3 is substituted for x and -5 is substituted for y, we obtain the statement $-5 = 4 - 3^2$, or $-5 = 4 - 9$, or $-5 = -5$. Because this statement is true, the ordered pair $(3, -5)$ is a solution of the equation $y = 4 - x^2$. We also say that $(3, -5)$ **satisfies** the equation.

We can generate as many ordered-pair solutions as desired to $y = 4 - x^2$ by substituting numbers for x and then finding the corresponding values for y. For example, suppose we let $x = 3$:

Start with x. Compute y. Form the ordered pair (x, y).

x	$y = 4 - x^2$	Ordered Pair (x, y)
3	$y = 4 - 3^2 = 4 - 9 = -5$	$(3, -5)$

Let $x = 3$. $(3, -5)$ is a solution of $y = 4 - x^2$.

The **graph of an equation in two variables** is the set of all points whose coordinates satisfy the equation. One method for graphing such equations is the **point-plotting method**. First, we find several ordered pairs that are solutions of the equation. Next, we plot these ordered pairs as points in the rectangular coordinate system. Finally, we connect the points with a smooth curve or line. This often gives us a picture of all ordered pairs that satisfy the equation.

EXAMPLE 2 Graphing an Equation Using the Point-Plotting Method

Graph $y = 4 - x^2$. Select integers for x, starting with -3 and ending with 3.

Solution For each value of x, we find the corresponding value for y.

Start with x.

Compute y.

Form the ordered pair (x, y).

We selected integers from -3 to 3, inclusive, to include three negative numbers, 0, and three positive numbers. We also wanted to keep the resulting computations for y relatively simple.

x	$y = 4 - x^2$	Ordered Pair (x, y)
-3	$y = 4-(-3)^2 = 4 - 9 = -5$	$(-3, -5)$
-2	$y = 4-(-2)^2 = 4 - 4 = 0$	$(-2, 0)$
-1	$y = 4-(-1)^2 = 4 - 1 = 3$	$(-1, 3)$
0	$y = 4 - 0^2 = 4 - 0 = 4$	$(0, 4)$
1	$y = 4 - 1^2 = 4 - 1 = 3$	$(1, 3)$
2	$y = 4 - 2^2 = 4 - 4 = 0$	$(2, 0)$
3	$y = 4 - 3^2 = 4 - 9 = -5$	$(3, -5)$

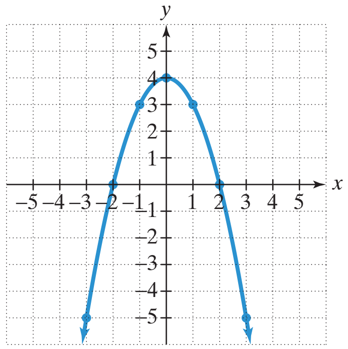


Figure 1.4 The graph of $y = 4 - x^2$

Now we plot the seven points and join them with a smooth curve, as shown in **Figure 1.4**. The graph of $y = 4 - x^2$ is a curve where the part of the graph to the right of the y-axis is a reflection of the part to the left of it and vice versa. The arrows on the left and the right of the curve indicate that it extends indefinitely in both directions.

Check Point 2 Graph $y = 4 - x$. Select integers for x, starting with -3 and ending with 3.

EXAMPLE 3 Graphing an Equation Using the Point-Plotting Method

Graph $y = |x|$. Select integers for x, starting with -3 and ending with 3.

Solution For each value of x, we find the corresponding value for y.

x	$y = \|x\|$	Ordered Pair (x, y)
-3	$y = \|-3\| = 3$	$(-3, 3)$
-2	$y = \|-2\| = 2$	$(-2, 2)$
-1	$y = \|-1\| = 1$	$(-1, 1)$
0	$y = \|0\| = 0$	$(0, 0)$
1	$y = \|1\| = 1$	$(1, 1)$
2	$y = \|2\| = 2$	$(2, 2)$
3	$y = \|3\| = 3$	$(3, 3)$

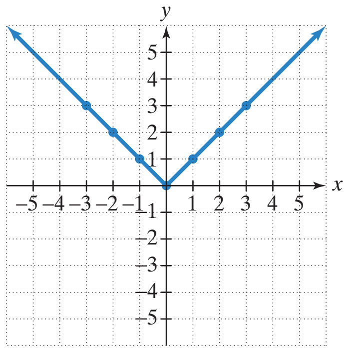


Figure 1.5 The graph of $y = |x|$

We plot the points and connect them, resulting in the graph shown in **Figure 1.5**. The graph is V-shaped and centered at the origin. For every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. This shows that the absolute value of a positive number is the same as the absolute value of its opposite.

Check Point 3 Graph $y = |x + 1|$. Select integers for x, starting with -4 and ending with 2.

3 Interpret information about a graphing utility's viewing rectangle or table.

Graphing Equations and Creating Tables Using a Graphing Utility

Graphing calculators and graphing software packages for computers are referred to as **graphing utilities** or graphers. A graphing utility is a powerful tool that quickly generates the graph of an equation in two variables. **Figures 1.6(a)** and **1.6(b)** show two such graphs for the equations in Examples 2 and 3.

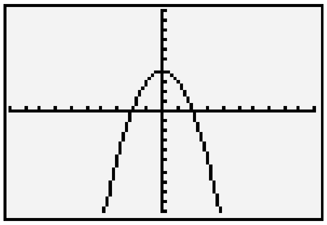


Figure 1.6(a) The graph of $y = 4 - x^2$

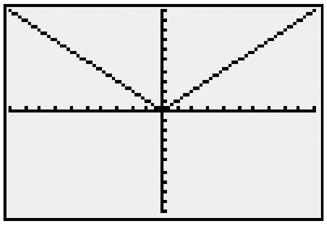


Figure 1.6(b) The graph of $y = |x|$

Study Tip

Even if you are not using a graphing utility in the course, read this part of the section. Knowing about viewing rectangles will enable you to understand the graphs that we display in the technology boxes throughout the book.

What differences do you notice between these graphs and the graphs that we drew by hand? They do seem a bit "jittery." Arrows do not appear on the left and right ends of the graphs. Furthermore, numbers are not given along the axes. For both graphs in **Figure 1.6**, the x-axis extends from -10 to 10 and the y-axis also extends from -10 to 10. The distance represented by each consecutive tick mark is one unit. We say that the **viewing rectangle**, or the **viewing window**, is $[-10, 10, 1]$ by $[-10, 10, 1]$.

$[-10,$ — The minimum x-value along the x-axis is -10.

$10,$ — The maximum x-value along the x-axis is 10.

$1]$ — Distance between consecutive tick marks on the x-axis is one unit.

by

$[-10,$ — The minimum y-value along the y-axis is -10.

$10,$ — The maximum y-value along the y-axis is 10.

$1]$ — Distance between consecutive tick marks on the y-axis is one unit.

To graph an equation in x and y using a graphing utility, enter the equation and specify the size of the viewing rectangle. The size of the viewing rectangle sets minimum and maximum values for both the x- and y-axes. Enter these values, as well as the values representing the distances between consecutive tick marks, on the respective axes. The $[-10, 10, 1]$ by $[-10, 10, 1]$ viewing rectangle used in **Figure 1.6** is called the **standard viewing rectangle**.

EXAMPLE 4 Understanding the Viewing Rectangle

What is the meaning of a $[-2, 3, 0.5]$ by $[-10, 20, 5]$ viewing rectangle?

Solution We begin with $[-2, 3, 0.5]$, which describes the x-axis. The minimum x-value is -2 and the maximum x-value is 3. The distance between consecutive tick marks is 0.5.

Next, consider $[-10, 20, 5]$, which describes the y-axis. The minimum y-value is -10 and the maximum y-value is 20. The distance between consecutive tick marks is 5.

Figure 1.7 illustrates a $[-2, 3, 0.5]$ by $[-10, 20, 5]$ viewing rectangle. To make things clearer, we've placed numbers by each tick mark. These numbers do not appear on the axes when you use a graphing utility to graph an equation.

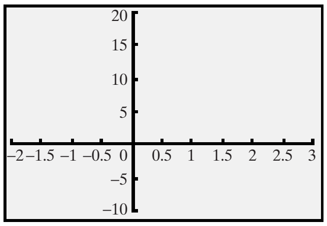


Figure 1.7 A $[-2, 3, 0.5]$ by $[-10, 20, 5]$ viewing rectangle

Check Point 4 What is the meaning of a $[-100, 100, 50]$ by $[-100, 100, 10]$ viewing rectangle? Create a figure like the one in **Figure 1.7** that illustrates this viewing rectangle.

On most graphing utilities, the display screen is two-thirds as high as it is wide. By using a square setting, you can equally space the x and y tick marks. (This does not occur in the standard viewing rectangle.) Graphing utilities can also *zoom in* and *zoom out*. When you zoom in, you see a smaller portion of the graph, but you do so in greater detail. When you zoom out, you see a larger portion of the graph. Thus, zooming out may help you to develop a better understanding of the overall character of the graph. With practice, you will become more comfortable with graphing equations in two variables using your graphing utility. You will also develop a better sense of the size of the viewing rectangle that will reveal needed information about a particular graph.

Graphing utilities can also be used to create tables showing solutions of equations in two variables. Use the Table Setup function to choose the starting value of x and to input the increment, or change, between the consecutive x-values. The corresponding y-values are calculated based on the equation(s) in two variables in the [Y=] screen. In **Figure 1.8**, we used a TI-84 Plus to create a table for $y = 4 - x^2$ and $y = |x|$, the equations in Examples 2 and 3.

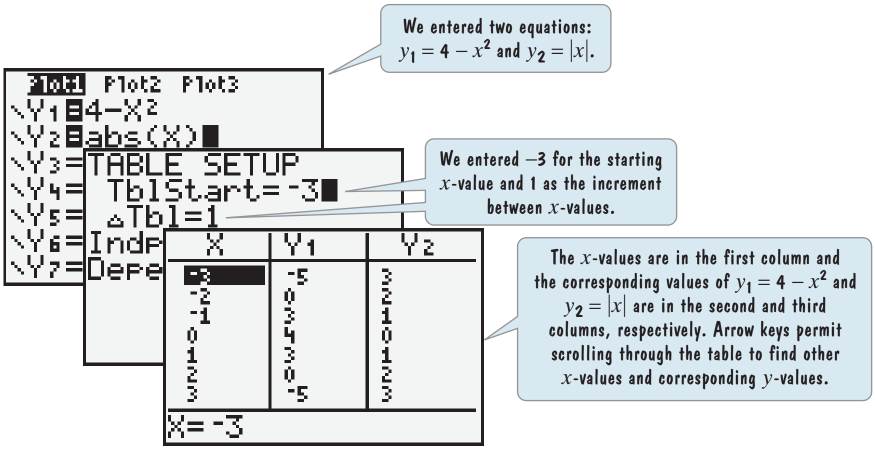


Figure 1.8 Creating a table for $y_1 = 4 - x^2$ and $y_2 = |x|$

4 Use a graph to determine intercepts.

Intercepts

An ***x*-intercept** of a graph is the x-coordinate of a point where the graph intersects the x-axis. For example, look at the graph of $y = 4 - x^2$ in **Figure 1.9**. The graph crosses the x-axis at $(-2, 0)$ and $(2, 0)$. Thus, the x-intercepts are -2 and 2. **The *y*-coordinate corresponding to an *x*-intercept is always zero.**

A ***y*-intercept** of a graph is the y-coordinate of a point where the graph intersects the y-axis. The graph of $y = 4 - x^2$ in **Figure 1.9** shows that the graph crosses the y-axis at $(0, 4)$. Thus, the y-intercept is 4. **The *x*-coordinate corresponding to a *y*-intercept is always zero.**

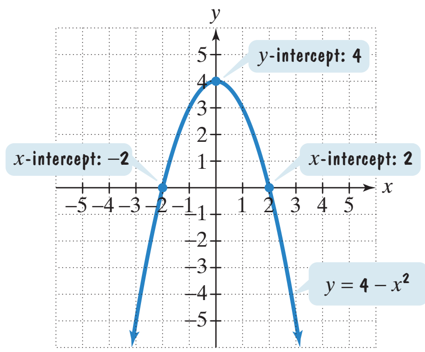


Figure 1.9 Intercepts of $y = 4 - x^2$

Study Tip

Mathematicians tend to use two ways to describe intercepts. Did you notice that we are using single numbers? If a is an x-intercept of a graph, then the graph passes through the point $(a, 0)$. If b is a y-intercept of a graph, then the graph passes through the point $(0, b)$.

Some books state that the x-intercept is the *point* $(a, 0)$ and the x-intercept is *at a* on the x-axis. Similarly, the y-intercept is the *point* $(0, b)$ and the y-intercept is *at b* on the y-axis. In these descriptions, the intercepts are the actual points where the graph intersects the axes.

Although we'll describe intercepts as single numbers, we'll immediately state the point on the x- or y-axis that the graph passes through. Here's the important thing to keep in mind:

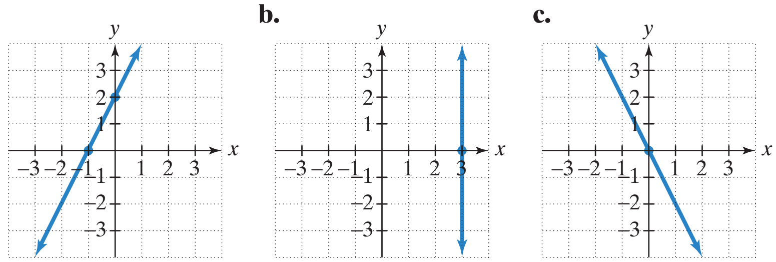
x-intercept: The corresponding value of y is 0.

y-intercept: The corresponding value of x is 0.

EXAMPLE 5 Identifying Intercepts

Identify the x- and y-intercepts.

a. **b.** **c.**



Solution

a. The graph crosses the x-axis at $(-1, 0)$. Thus, the x-intercept is -1. The graph crosses the y-axis at $(0, 2)$. Thus, the y-intercept is 2.

b. The graph crosses the x-axis at $(3, 0)$, so the x-intercept is 3. This vertical line does not cross the y-axis. Thus, there is no y-intercept.

c. This graph crosses the x- and y-axes at the same point, the origin. Because the graph crosses both axes at $(0, 0)$, the x-intercept is 0 and the y-intercept is 0.

Check Point 5 Identify the x- and y-intercepts.

a. **b.** **c.**

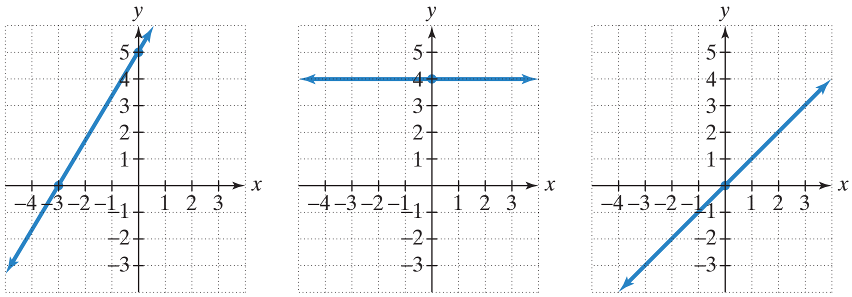


Figure 1.10 illustrates that a graph may have no intercepts or several intercepts.

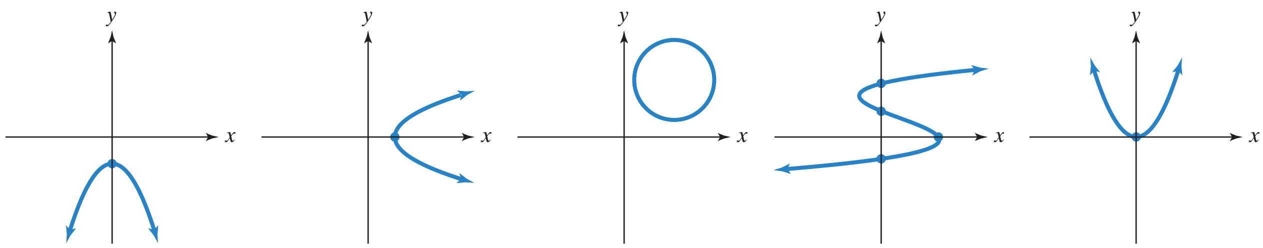


Figure 1.10 No x-intercept, One y-intercept; One x-intercept, No y-intercept; No intercepts; One x-intercept, Three y-intercepts; The same x-intercept and y-intercept

5 Interpret information given by graphs.

Interpreting Information Given by Graphs

Line graphs are often used to illustrate trends over time. Some measure of time, such as months or years, frequently appears on the horizontal axis. Amounts are generally listed on the vertical axis. Points are drawn to represent the given information. The graph is formed by connecting the points with line segments.

A line graph displays information in the first quadrant of a rectangular coordinate system. By identifying points on line graphs and their coordinates, you can interpret specific information given by the graph.

EXAMPLE 6 Age at Marriage and the Probability of Divorce

Divorce rates are considerably higher for couples who marry in their teens. The line graphs in **Figure 1.11** show the percentages of marriages ending in divorce based on the wife's age at marriage.

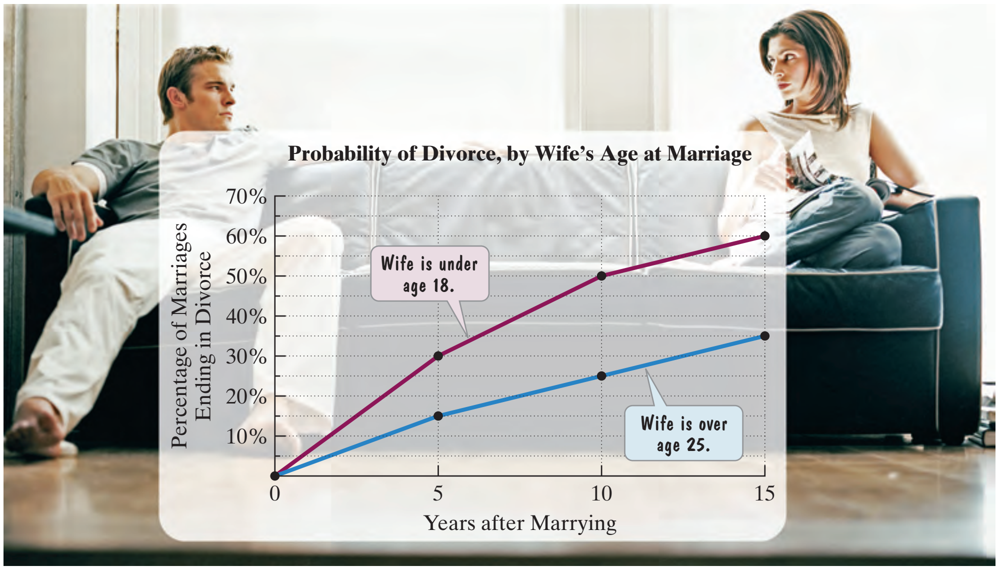


Figure 1.11
Source: B. E. Pruitt et al., *Human Sexuality*, Prentice Hall, 2007

Here are two mathematical models that approximate the data displayed by the line graphs:

Wife is under 18 at time of marriage. | Wife is over 25 at time of marriage.

$$d = 4n + 5 \qquad d = 2.3n + 1.5.$$

In each model, the variable n is the number of years after marriage and the variable d is the percentage of marriages ending in divorce.

a. Use the appropriate formula to determine the percentage of marriages ending in divorce after 10 years when the wife is over 25 at the time of marriage.

b. Use the appropriate line graph in **Figure 1.11** to determine the percentage of marriages ending in divorce after 10 years when the wife is over 25 at the time of marriage.

c. Does the value given by the mathematical model underestimate or overestimate the actual percentage of marriages ending in divorce after 10 years as shown by the graph? By how much?

Solution

a. Because the wife is over 25 at the time of marriage, we use the formula on the right, $d = 2.3n + 1.5$. To find the percentage of marriages ending in divorce after 10 years, we substitute 10 for n and evaluate the formula.

$d = 2.3n + 1.5$ — This is one of the two given mathematical models.

$d = 2.3(10) + 1.5$ — Replace n with 10.

$d = 23 + 1.5$ — Multiply: $2.3(10) = 23$.

$d = 24.5$ — Add.

The model indicates that 24.5% of marriages end in divorce after 10 years when the wife is over 25 at the time of marriage.

b. Now let's use the line graph that shows the percentage of marriages ending in divorce when the wife is over 25 at the time of marriage. The graph is shown again in **Figure 1.12**. To find the percentage of marriages ending in divorce after 10 years:

- Locate 10 on the horizontal axis and locate the point above 10.
- Read across to the corresponding percent on the vertical axis.

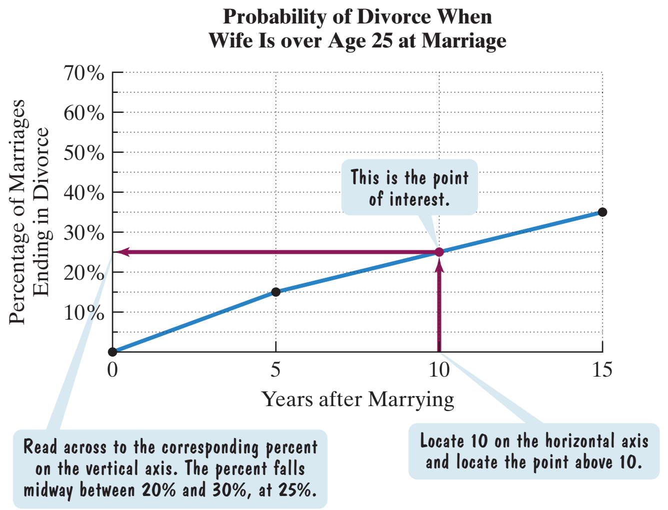


Figure 1.12

The actual data displayed by the graph indicate that 25% of these marriages end in divorce after 10 years.

c. The value obtained by evaluating the mathematical model, 24.5%, is close to, but slightly less than, the actual percentage of divorces, 25.0%. The difference between these percents is 25.0% − 24.5%, or 0.5%. The value given by the mathematical model, 24.5%, underestimates the actual percent, 25%, by only 0.5, providing a fairly accurate description of the data.

Check Point 6

a. Use the appropriate formula from Example 6 to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

b. Use the appropriate line graph in **Figure 1.11** to determine the percentage of marriages ending in divorce after 15 years when the wife is under 18 at the time of marriage.

c. Does the value given by the mathematical model underestimate or overestimate the actual percentage of marriages ending in divorce after 15 years as shown by the graph? By how much?

Exercise Set 1.1

Practice Exercises

In Exercises 1–12, plot the given point in a rectangular coordinate system.

1. $(1, 4)$ **2.** $(2, 5)$ **3.** $(-2, 3)$

4. $(-1, 4)$ **5.** $(-3, -5)$ **6.** $(-4, -2)$

7. $(4, -1)$ **8.** $(3, -2)$ **9.** $(-4, 0)$

10. $(0, -3)$ **11.** $\left(\frac{7}{2}, -\frac{3}{2}\right)$ **12.** $\left(-\frac{5}{2}, \frac{3}{2}\right)$

Graph each equation in Exercises 13–28. Let $x = -3, -2, -1, 0, 1, 2,$ and 3.

13. $y = x^2 - 2$ **14.** $y = x^2 + 2$ **15.** $y = x - 2$

16. $y = x + 2$ **17.** $y = 2x + 1$ **18.** $y = 2x - 4$

19. $y = -\frac{1}{2}x$ **20.** $y = -\frac{1}{2}x + 2$ **21.** $y = 2|x|$

22. $y = -2|x|$ **23.** $y = |x| + 1$ **24.** $y = |x| - 1$

25. $y = 9 - x^2$ **26.** $y = -x^2$ **27.** $y = x^3$

28. $y = x^3 - 1$

In Exercises 29–32, match the viewing rectangle with the correct figure. Then label the tick marks in the figure to illustrate this viewing rectangle.

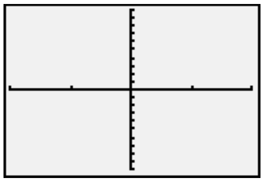
29. $[-5, 5, 1]$ by $[-5, 5, 1]$

30. $[-10, 10, 2]$ by $[-4, 4, 2]$

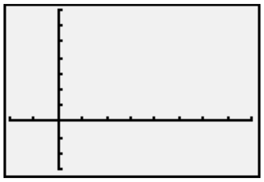
31. $[-20, 80, 10]$ by $[-30, 70, 10]$

32. $[-40, 40, 20]$ by $[-1000, 1000, 100]$

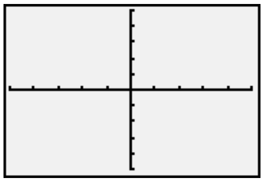
a.



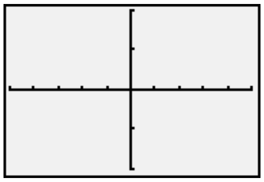
b.



c.



d.



The table of values was generated by a graphing utility with a TABLE feature. Use the table to solve Exercises 33–40.

X	Y1	Y2
-3	9	5
-2	4	4
-1	1	3
0	0	2
1	1	1
2	4	0
3	9	-1

X= -3

33. Which equation corresponds to Y_2 in the table?

a. $y_2 = x + 8$ **b.** $y_2 = x - 2$

c. $y_2 = 2 - x$ **d.** $y_2 = 1 - 2x$

34. Which equation corresponds to Y_1 in the table?

a. $y_1 = -3x$ **b.** $y_1 = x^2$

c. $y_1 = -x^2$ **d.** $y_1 = 2 - x$

35. Does the graph of Y_2 pass through the origin?

36. Does the graph of Y_1 pass through the origin?

37. At which point does the graph of Y_2 cross the x-axis?

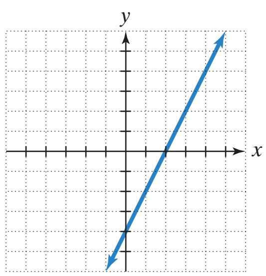
38. At which point does the graph of Y_2 cross the y-axis?

39. At which points do the graphs of Y_1 and Y_2 intersect?

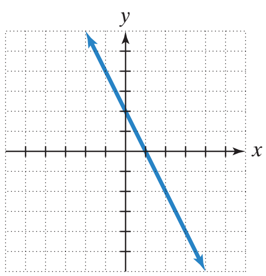
40. For which values of x is $Y_1 = Y_2$?

In Exercises 41–46, use the graph to **a.** *determine the x-intercepts, if any;* **b.** *determine the y-intercepts, if any. For each graph, tick marks along the axes represent one unit each.*

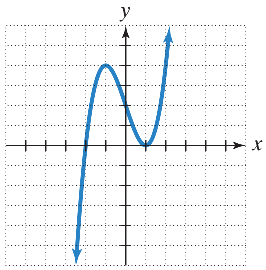
41.



42.



43.



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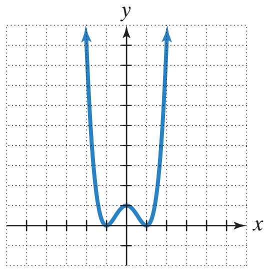

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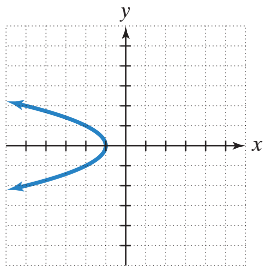

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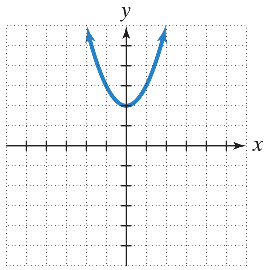


Practice Plus

In Exercises 47–50, write each English sentence as an equation in two variables. Then graph the equation.

47. The y-value is four more than twice the x-value.

48. The y-value is the difference between four and twice the x-value.

49. The y-value is three decreased by the square of the x-value.

50. The y-value is two more than the square of the x-value.

In Exercises 51–54, graph each equation.

51. $y = 5$ (Let $x = -3, -2, -1, 0, 1, 2,$ and 3.)

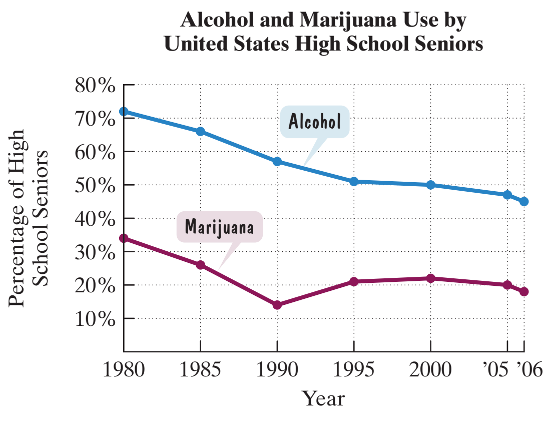
52. $y = -1$ (Let $x = -3, -2, -1, 0, 1, 2,$ and 3.)

53. $y = \frac{1}{x}$ (Let $x = -2, -1, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, 1,$ and 2.)

54. $y = -\frac{1}{x}$ (Let $x = -2, -1, -\frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \frac{1}{2}, 1,$ and 2.)

Application Exercises

The graphs show the percentage of high school seniors who used alcohol or marijuana during the 30 days prior to being surveyed for the University of Michigan's Monitoring the Future *study.*



Source: U.S. Department of Health and Human Services

The data can be described by the following mathematical models:

Percentage of seniors using alcohol: $A = -n + 70$

Number of years after 1980

Percentage of seniors using marijuana: $M = -0.4n + 28.$

Use this information to solve Exercises 55–56.

55. a. Use the appropriate line graph to determine the percentage of seniors who used marijuana in 2005.

b. Use the appropriate formula to determine the percentage of seniors who used marijuana in 2005. Does the formula underestimate or overestimate the actual percentage displayed by the graph? By how much?

c. Use the appropriate line graph to estimate the percentage of seniors who used alcohol in 2006.

d. Use the appropriate formula to determine the percentage of seniors who used alcohol in 2006. How does this compare with your estimate in part (c)?

e. For the period from 1980 through 2006, in which year was marijuana use by seniors at a minimum? Estimate the percentage of seniors who used marijuana in that year.

56. a. Use the appropriate line graph to determine the percentage of seniors who used alcohol in 2000.

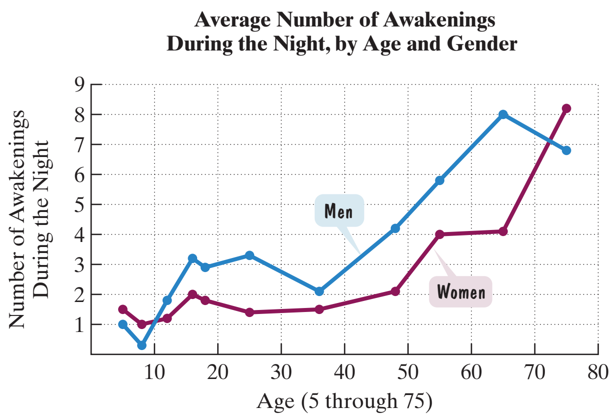
b. Use the appropriate formula to determine the percentage of seniors who used alcohol in 2000. What do you observe?

c. Use the appropriate line graph to estimate the percentage of seniors who used marijuana in 2000.

d. Use the appropriate formula to determine the percentage of seniors who used marijuana in 2000. How does this compare with your estimate in part (c)?

e. For the period from 1980 through 2006, in which year was alcohol use by seniors at a maximum? Estimate the percentage of seniors who used alcohol in that year.

Contrary to popular belief, older people do not need less sleep than younger adults. However, the line graphs show that they awaken more often during the night. The numerous awakenings are one reason why some elderly individuals report that sleep is less restful than it had been in the past. Use the line graphs to solve Exercises 57–60.



Source: Stephen Davis and Joseph Palladino, *Psychology*, 5th Edition, Prentice Hall, 2007

57. At which age, estimated to the nearest year, do women have the least number of awakenings during the night? What is the average number of awakenings at that age?

58. At which age do men have the greatest number of awakenings during the night? What is the average number of awakenings at that age?

59. Estimate, to the nearest tenth, the difference between the average number of awakenings during the night for 25-year-old men and 25-year-old women.

60. Estimate, to the nearest tenth, the difference between the average number of awakenings during the night for 18-year-old men and 18-year-old women.

Writing in Mathematics

61. What is the rectangular coordinate system?

62. Explain how to plot a point in the rectangular coordinate system. Give an example with your explanation.

63. Explain why $(5, -2)$ and $(-2, 5)$ do not represent the same point.

64. Explain how to graph an equation in the rectangular coordinate system.

65. What does a $[-20, 2, 1]$ by $[-4, 5, 0.5]$ viewing rectangle mean?

Technology Exercise

66. Use a graphing utility to verify each of your hand-drawn graphs in Exercises 13–28. Experiment with the size of the viewing rectangle to make the graph displayed by the graphing utility resemble your hand-drawn graph as much as possible.

Critical Thinking Exercises

Make Sense? *In Exercises 67–70, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

67. The rectangular coordinate system provides a geometric picture of what an equation in two variables looks like.

68. There is something wrong with my graphing utility because it is not displaying numbers along the x- and y-axes.

69. I used the ordered pairs $(-2, 2)$, $(0, 0)$, and $(2, 2)$ to graph a straight line.

70. I used the ordered pairs

(time of day, calories that I burned)

to obtain a graph that is a horizontal line.

In Exercises 71–74, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

71. If the product of a point's coordinates is positive, the point must be in quadrant I.

72. If a point is on the x-axis, it is neither up nor down, so $x = 0$.

73. If a point is on the y-axis, its x-coordinate must be 0.

74. The ordered pair $(2, 5)$ satisfies $3y - 2x = -4$.

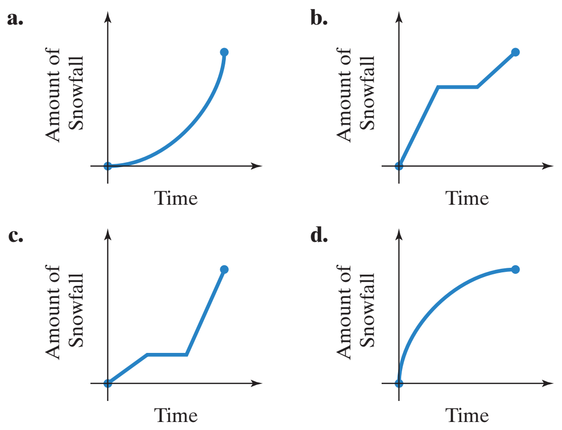
In Exercises 75–78, match the story with the correct figure. The figures are labeled (a), (b), (c), and (d).

75. As the blizzard got worse, the snow fell harder and harder.

76. The snow fell more and more softly.

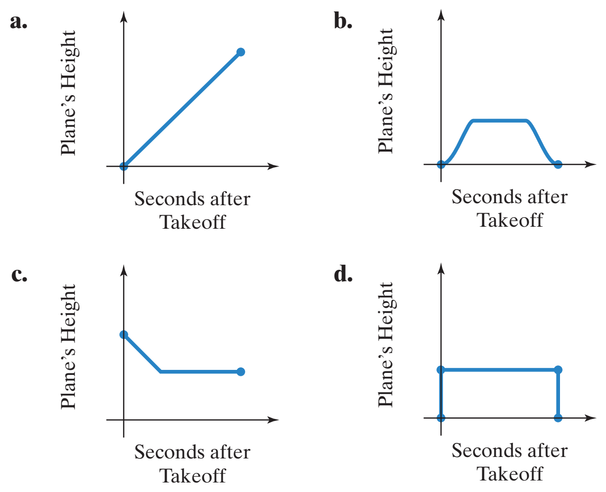
77. It snowed hard, but then it stopped. After a short time, the snow started falling softly.

78. It snowed softly, and then it stopped. After a short time, the snow started falling hard.

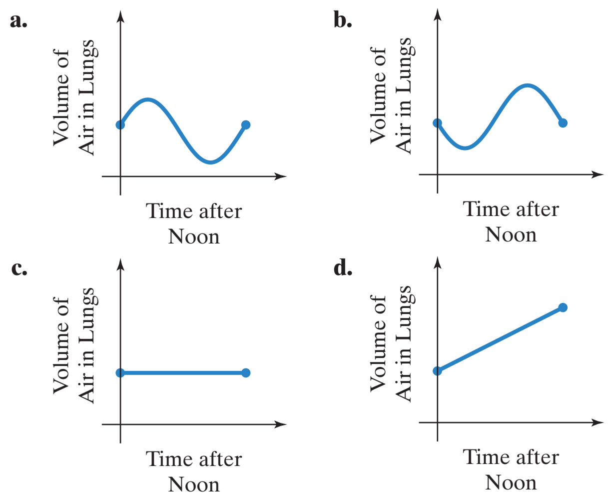


In Exercises 79–82, select the graph that best illustrates each story.

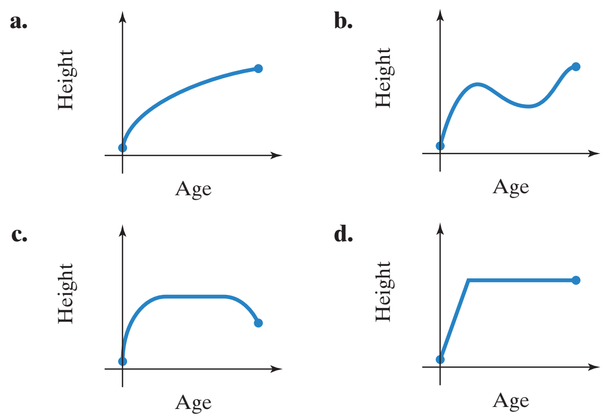
79. An airplane flew from Miami to San Francisco.



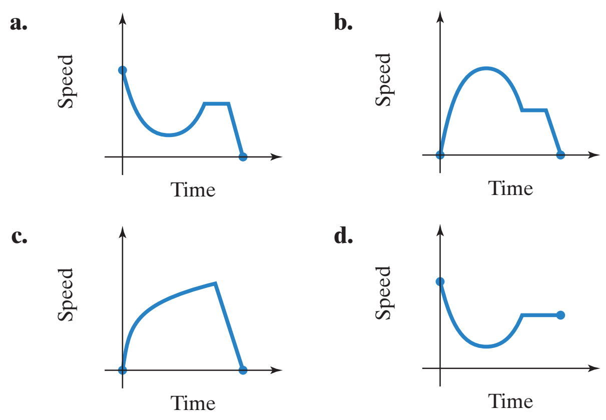
80. At noon, you begin to breathe in.



81. Measurements are taken of a person's height from birth to age 100.



82. You begin your bike ride by riding down a hill. Then you ride up another hill. Finally, you ride along a level surface before coming to a stop.



Preview Exercises

Exercises 83–85 will help you prepare for the material covered in the next section.

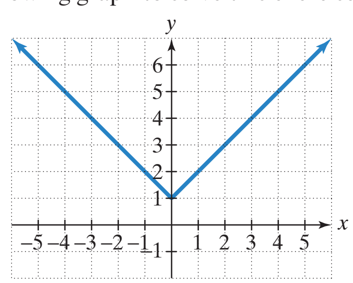
83. Here are two sets of ordered pairs:

$$\text{set 1: } \{(1, 5), (2, 5)\}$$
$$\text{set 2: } \{(5, 1), (5, 2)\}.$$

In which set is each x-coordinate paired with only one y-coordinate?

84. Graph $y = 2x$ and $y = 2x + 4$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2.

85. Use the following graph to solve this exercise.



a. What is the y-coordinate when the x-coordinate is 2?

b. What are the x-coordinates when the y-coordinate is 4?

c. Describe the x-coordinates of all points on the graph.

d. Describe the y-coordinates of all points on the graph.