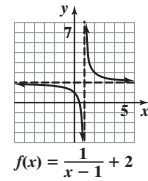
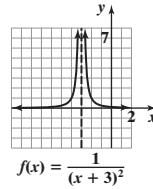
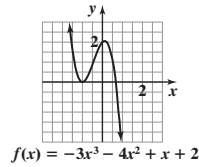


18.  $f(x) = 2x^4 - 2$

19.  $-1$  and  $\frac{2}{3}$

20.  $(-\infty, -3) \cup (-3, \infty)$

21.  $(-\infty, 1) \cup (1, \infty)$

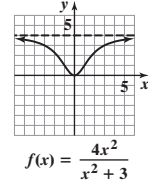
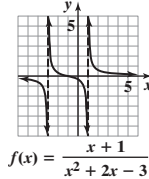
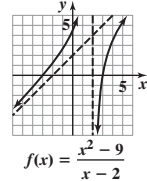
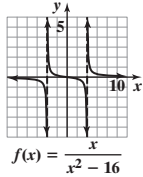


22. domain:  $\{x|x \neq 4, x \neq -4\}$

23. domain:  $\{x|x \neq 2\}$

24. domain:  $\{x|x \neq -3, x \neq 1\}$

25. domain: all real numbers



26. a.  $\bar{C}(x) = \frac{300,000 + 10x}{x}$

b.  $y = 10$ ; As the number of satellite radio players increases, the average cost approaches \$10.

27.  $(-3, 4)$



28.  $(-\infty, 3) \cup [10, \infty)$



29. 45 foot-candles

### Cumulative Review Exercises (Chapters P-2)

1. domain:  $(-2, 2)$ ; range:  $[0, \infty)$

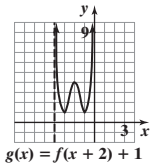
2.  $-1$  and  $1$ , both of multiplicity 2

3. 0

4. 3

5.  $x \rightarrow -2^+$ ;  $x \rightarrow 2^-$

6.



7.  $\{2, -1\}$

8.  $\left\{\frac{5 + \sqrt{13}}{6}, \frac{5 - \sqrt{13}}{6}\right\}$

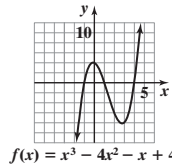
9.  $\left\{\frac{1}{3}, -\frac{2}{3}\right\}$

10.  $\{-3, -1, 2\}$

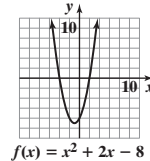
11.  $(-\infty, 1) \cup (4, \infty)$

12.  $(-\infty, -1) \cup \left(\frac{5}{3}, \infty\right)$

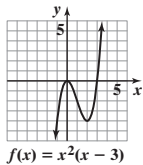
13.



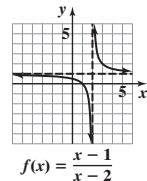
14.



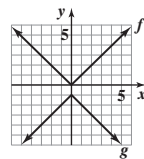
15.



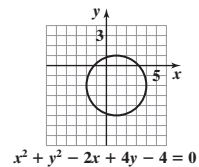
16.



17.



18.



19.  $(f \circ g)(x) = 32x^2 - 20x + 2$

20.  $4x + 2h - 1$

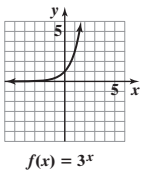
## CHAPTER 3

### Section 3.1

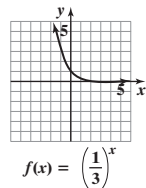
#### Check Point Exercises

1. approximately \$160; overestimates by \$11

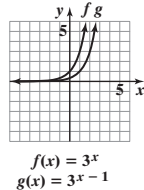
2.



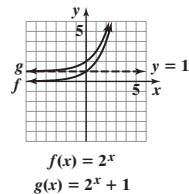
3.



4.



5.



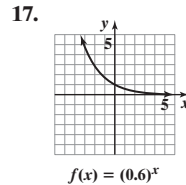
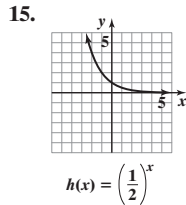
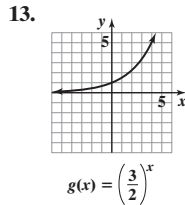
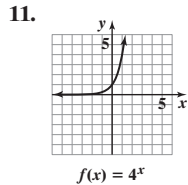
6. approximately 4446

7. a. \$14,859.47

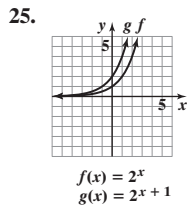
b. \$14,918.25

Exercise Set 3.1

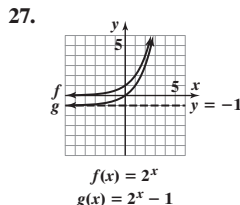
1. 10.556    3. 11.665    5. 0.125    7. 9.974    9. 0.387



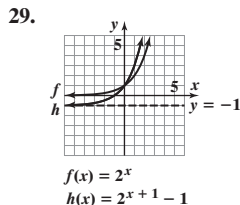
19.  $H(x) = -3^{-x}$   
 21.  $F(x) = -3^x$   
 23.  $h(x) = 3^x - 1$



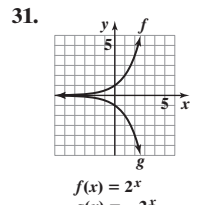
asymptote:  $y = 0$   
 domain:  $(-\infty, \infty)$   
 range:  $(0, \infty)$



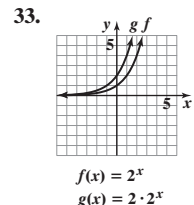
asymptote:  $y = -1$   
 domain:  $(-\infty, \infty)$   
 range:  $(-1, \infty)$



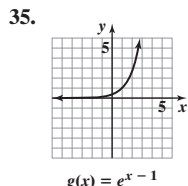
asymptote:  $y = -1$   
 domain:  $(-\infty, \infty)$   
 range:  $(-1, \infty)$



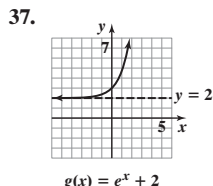
asymptote:  $y = 0$   
 domain:  $(-\infty, \infty)$   
 range:  $(-\infty, 0)$



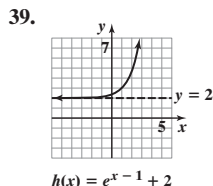
asymptote:  $y = 0$   
 domain:  $(-\infty, \infty)$   
 range:  $(0, \infty)$



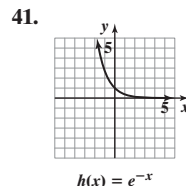
asymptote:  $y = 0$   
 domain:  $(-\infty, \infty)$   
 range:  $(0, \infty)$



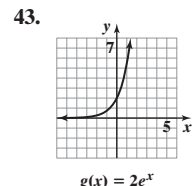
asymptote:  $y = 2$   
 domain:  $(-\infty, \infty)$   
 range:  $(2, \infty)$



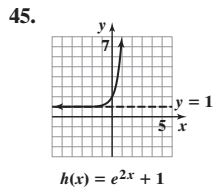
asymptote:  $y = 2$   
 domain:  $(-\infty, \infty)$   
 range:  $(2, \infty)$



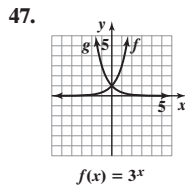
asymptote:  $y = 0$   
 domain:  $(-\infty, \infty)$   
 range:  $(0, \infty)$



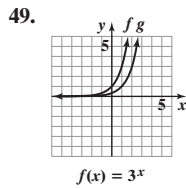
asymptote:  $y = 0$   
 domain:  $(-\infty, \infty)$   
 range:  $(0, \infty)$



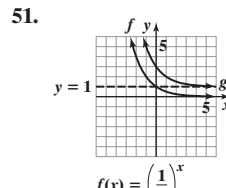
asymptote:  $y = 1$   
 domain:  $(-\infty, \infty)$   
 range:  $(1, \infty)$



asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 0$

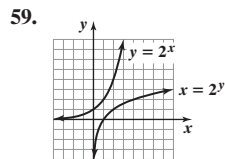
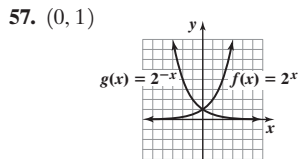


asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 0$



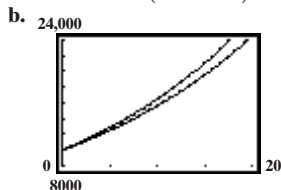
asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 1$

53. a. \$13,116.51  
 b. \$13,140.67  
 c. \$13,157.04  
 d. \$13,165.31  
 55. 7% compounded monthly



61.  $y = 4^x$     63.  $y = -e^x$     65. a. 574 million    b. 1148 million    c. 2295 million    d. 4590 million    e. It appears to double.  
 67. \$832,744    69. 3.249009585; 3.317278183; 3.321880096; 3.321995226; 3.321997068;  $2^{\sqrt{3}} \approx 3.321997085$ ; The closer the exponent is to  $\sqrt{3}$ , the closer the value is to  $2^{\sqrt{3}}$ .  
 71. a. approximately 8.9 million words    b. approximately 10.4 million words    c. linear model  
 73. a. 100%    b.  $\approx 68.5\%$     c.  $\approx 30.8\%$     d.  $\approx 20\%$     75. the function  $g$

81. a.  $A = 10,000 \left(1 + \frac{0.05}{4}\right)^{4t}$ ;  $A = 10,000 \left(1 + \frac{0.045}{12}\right)^{12t}$     83. does not make sense    85. does not make sense    87. false    89. false



5% interest compounded quarterly

91.  $y = 3^x$  is (d);  $y = 5^x$  is (c);  $y = \left(\frac{1}{3}\right)^x$  is (a);  $y = \left(\frac{1}{5}\right)^x$  is (b).

93. a.  $\cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \frac{e^{-x} + e^x}{2} = \frac{e^x + e^{-x}}{2} = \cosh x$

b.  $\sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^x}{2} = -\frac{e^x - e^{-x}}{2} = -\sinh x$

c.  $\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 \stackrel{?}{=} 1$

$\frac{e^{2x} + 2 + e^{-2x}}{4} - \frac{e^{2x} - 2 + e^{-2x}}{4} \stackrel{?}{=} 1$

$\frac{e^{2x} + 2 + e^{-2x} - e^{2x} + 2 - e^{-2x}}{4} \stackrel{?}{=} 1$

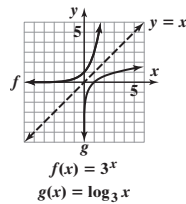
$\frac{4}{4} \stackrel{?}{=} 1$   
 $1 = 1$

94. We don't know how to solve  $x = 2^y$  for  $y$ .    95.  $\frac{1}{2}$     96.  $(-\infty, 3) \cup (3, \infty)$

**Section 3.2**

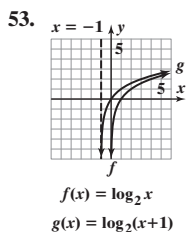
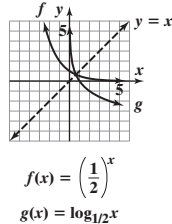
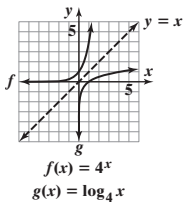
**Check Point Exercises**

1. a.  $7^3 = x$     b.  $b^2 = 25$     c.  $4^y = 26$     2. a.  $5 = \log_2 x$     b.  $3 = \log_b 27$     c.  $y = \log_e 33$     3. a. 2    b. -3    c.  $\frac{1}{2}$     d.  $\frac{1}{7}$   
 4. a. 1    b. 0    5. a. 8    b. 17    6. 7.  $(5, \infty)$     8. 80%    9. 4.0    10. a.  $(-\infty, 4)$     b.  $(-\infty, 0) \cup (0, \infty)$   
 11.  $34^\circ$ ; quite well

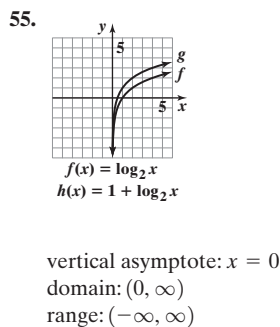


**Exercise Set 3.2**

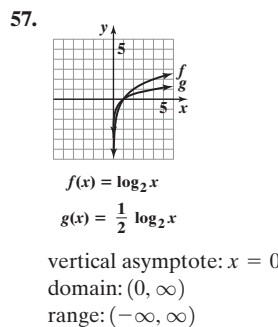
1.  $2^4 = 16$     3.  $3^2 = x$     5.  $b^5 = 32$     7.  $6^y = 216$     9.  $\log_2 8 = 3$     11.  $\log_2 \frac{1}{16} = -4$     13.  $\log_8 2 = \frac{1}{3}$     15.  $\log_{13} x = 2$   
 17.  $\log_b 1000 = 3$     19.  $\log_7 200 = y$     21. 2    23. 6    25. -1    27. -3    29.  $\frac{1}{2}$     31.  $-\frac{1}{2}$     33.  $\frac{1}{2}$     35. 1    37. 0    39. 7    41. 19  
 43. 45. 47.  $H(x) = 1 - \log_3 x$     49.  $h(x) = \log_3 x - 1$     51.  $g(x) = \log_3(x - 1)$



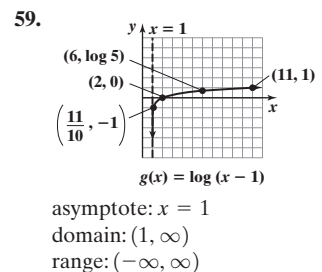
vertical asymptote:  $x = -1$   
 domain:  $(-1, \infty)$   
 range:  $(-\infty, \infty)$



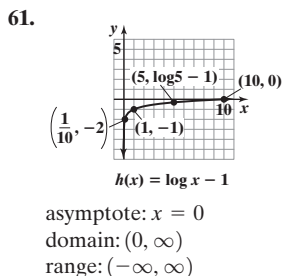
vertical asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$



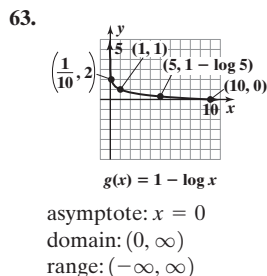
vertical asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$



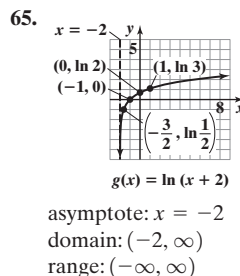
asymptote:  $x = 1$   
 domain:  $(1, \infty)$   
 range:  $(-\infty, \infty)$



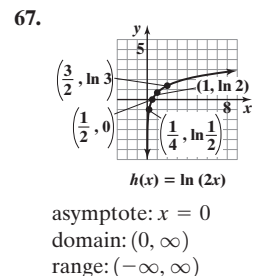
asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$



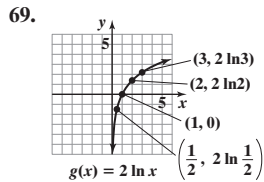
asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$



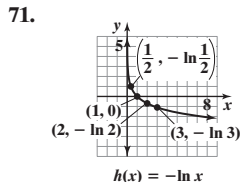
asymptote:  $x = -2$   
 domain:  $(-2, \infty)$   
 range:  $(-\infty, \infty)$



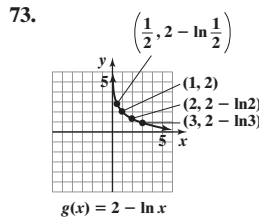
asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$



asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$



asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$



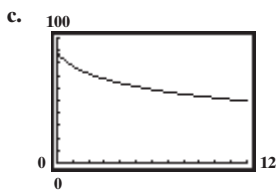
asymptote:  $x = 0$   
 domain:  $(0, \infty)$   
 range:  $(-\infty, \infty)$

75.  $(-4, \infty)$     77.  $(-\infty, 2)$     79.  $(-\infty, 2) \cup (2, \infty)$     81. 2    83. 7    85. 33    87. 0    89. 6    91. -6    93. 125    95.  $9x$

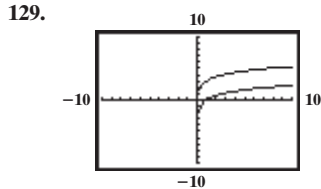
97.  $5x^2$     99.  $\sqrt{x}$     101.  $3^2 = x - 1; \{10\}$     103.  $4^{-3} = x; \left\{\frac{1}{64}\right\}$     105. 0    107. 2    109.  $(-\infty, -1) \cup (2, \infty)$     111.  $(-\infty, -1) \cup (5, \infty)$

113. 95.4%    115. a. 26.4%; underestimates by 1%    b. 25.2%

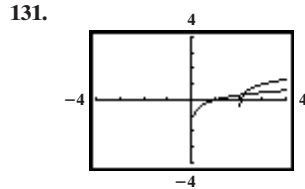
119. a. 88  
 b. 71.5; 63.9; 58.8; 55; 52; 49.5



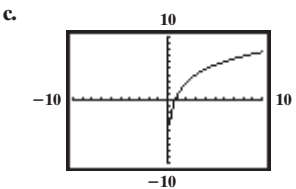
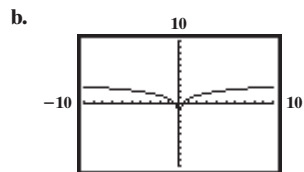
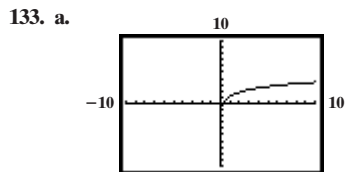
Material retention decreases as time passes.



$g(x)$  is  $f(x)$  shifted upward 3 units.



$g(x)$  is  $f(x)$  shifted right 2 units and upward 1 unit.



- d. They are the same.  
 $\log_b M + \log_b N$   
 e. the sum of the logarithms of its factors

135. makes sense    137. makes sense    139. false    141. false    143.  $\frac{4}{5}$     145.  $\log_3 40 > \log_4 60$     147. a. 5    b. 5  
 c.  $\log_2(8 \cdot 4) = \log_2 8 + \log_2 2$     148. a. 4    b. 4    c.  $\log_2\left(\frac{32}{2}\right) = \log_2 32 - \log_2 2$     149. a. 4    b. 4    c.  $\log_3 9^2 = 2\log_3 9$

### Section 3.3

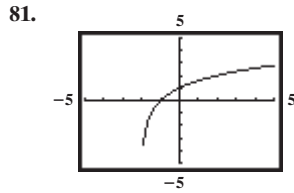
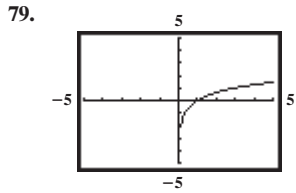
#### Check Point Exercises

1. a.  $\log_6 7 + \log_6 11$     b.  $2 + \log x$     2. a.  $\log_8 23 - \log_8 x$     b.  $5 - \ln 11$     3. a.  $9 \log_6 3$     b.  $\frac{1}{3} \ln x$     c.  $2 \log(x + 4)$   
 4. a.  $4 \log_b x + \frac{1}{3} \log_b y$     b.  $\frac{1}{2} \log_5 x - 2 - 3 \log_5 y$     5. a.  $\log 100 = 2$     b.  $\log \frac{7x + 6}{x}$     6. a.  $\ln x^2 \sqrt[3]{x + 5}$     b.  $\log \frac{(x - 3)^2}{x}$   
 c.  $\log_b \frac{\sqrt[4]{x}}{25y^{10}}$     7. 4.02    8. 4.02

#### Exercise Set 3.3

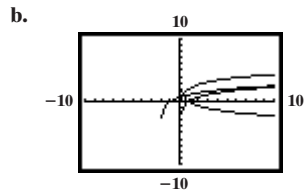
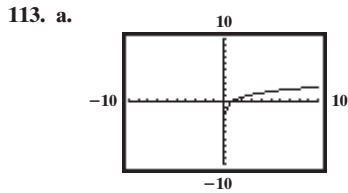
1.  $\log_5 7 + \log_5 3$     3.  $1 + \log_7 x$     5.  $3 + \log x$     7.  $1 - \log_7 x$     9.  $\log x - 2$     11.  $3 - \log_4 y$     13.  $2 - \ln 5$     15.  $3 \log_b x$   
 17.  $-6 \log N$     19.  $\frac{1}{5} \ln x$     21.  $2 \log_b x + \log_b y$     23.  $\frac{1}{2} \log_4 x - 3$     25.  $2 - \frac{1}{2} \log_6(x + 1)$     27.  $2 \log_b x + \log_b y - 2 \log_b z$   
 29.  $1 + \frac{1}{2} \log x$     31.  $\frac{1}{3} \log x - \frac{1}{3} \log y$     33.  $\frac{1}{2} \log_b x + 3 \log_b y - 3 \log_b z$     35.  $\frac{2}{3} \log_5 x + \frac{1}{3} \log_5 y - \frac{2}{3}$     37.  $3 \ln x + \frac{1}{2} \ln(x^2 + 1) - 4 \ln(x + 1)$   
 39.  $1 + 2 \log x + \frac{1}{3} \log(1 - x) - \log 7 - 2 \log(x + 1)$     41. 1    43.  $\ln(7x)$     45. 5    47.  $\log\left(\frac{2x + 5}{x}\right)$     49.  $\log(xy^3)$   
 51.  $\ln(x^{1/2}y)$  or  $\ln(y\sqrt{x})$     53.  $\log_b(x^2y^3)$     55.  $\ln\left(\frac{x^5}{y^2}\right)$     57.  $\ln\left(\frac{x^3}{y^{1/3}}\right)$  or  $\ln\left(\frac{x^3}{\sqrt[3]{y}}\right)$     59.  $\ln\left(\frac{x + 6}{x^3}\right)$     61.  $\ln\left(\frac{x^3y^5}{z^6}\right)$     63.  $\log\sqrt{xy}$

65.  $\log_5\left(\frac{\sqrt{xy}}{(x+1)^2}\right)$     67.  $\ln\sqrt[3]{\frac{(x+5)^2}{x(x^2-4)}}$     69.  $\log\frac{x(x^2-1)}{7(x+1)} = \log\frac{x(x-1)}{7}$     71. 1.5937    73. 1.6944    75. -1.2304    77. 3.6193

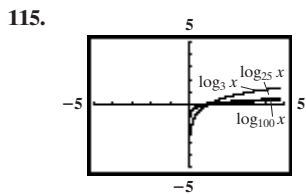


83.  $C - A$     85.  $3A$     87.  $\frac{1}{2}A - \frac{3}{2}C$     89. false;  $\ln e = 1$     91. false;  $\log_4(2x)^3 = 3 \log_4(2x)$     93. true    95. true

97. false;  $\log(x+3) - \log(2x) = \log\left(\frac{x+3}{2x}\right)$     99. true    101. true    103. a.  $D = 10\log\frac{I}{I_0}$     b. 20 decibels louder



$y = 2 + \log_3 x$  shifts the graph of  $y = \log_3 x$  two units upward;  
 $y = \log_3(x + 2)$  shifts the graph of  $y = \log_3 x$  two units left;  $y = -\log_3 x$  reflects the graph of  $y = \log_3 x$  about the  $x$ -axis.

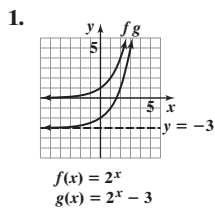


a. top graph:  $y = \log_{100} x$ ; bottom graph:  $y = \log_3 x$   
 b. top graph:  $y = \log_3 x$ ; bottom graph:  $y = \log_{100} x$   
 c. The graph of the equation with the largest  $b$  will be on the top in the interval  $(0, 1)$  and on the bottom in the interval  $(1, \infty)$ .

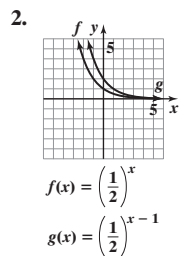
121. makes sense    123. makes sense

125. true    127. false    129.  $\log e = \log_{10} e = \frac{\ln e}{\ln 10} = \frac{1}{\ln 10}$     131.  $4x^3$     133.  $x = \frac{2a+3b}{a-2b}$     134.  $\left\{\frac{7 \pm \sqrt{61}}{2}\right\}$     135.  $\{-1, 3\}$

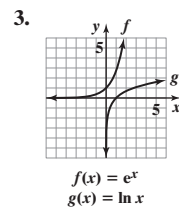
### Mid-Chapter 3 Check Point



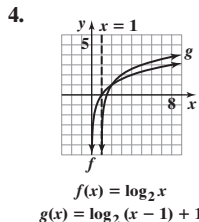
asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = -3$   
 domain of  $f =$  domain of  $g = (-\infty, \infty)$   
 range of  $f = (0, \infty)$ ; range of  $g = (-3, \infty)$



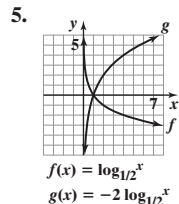
asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 0$   
 domain of  $f =$  domain of  $g = (-\infty, \infty)$   
 range of  $f =$  range of  $g = (0, \infty)$



asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $x = 0$   
 domain of  $f =$  range of  $g = (-\infty, \infty)$   
 range of  $f =$  domain of  $g = (0, \infty)$



asymptote of  $f$ :  $x = 0$   
 asymptote of  $g$ :  $x = 1$   
 domain of  $f = (0, \infty)$ ; domain of  $g = (1, \infty)$   
 range of  $f =$  range of  $g = (-\infty, \infty)$



asymptote of  $f$ :  $x = 0$   
 asymptote of  $g$ :  $x = 0$   
 domain of  $f =$  domain of  $g = (0, \infty)$   
 range of  $f =$  range of  $g = (-\infty, \infty)$

6.  $(-6, \infty)$     7.  $(0, \infty)$     8.  $(-\infty, -6) \cup (-6, \infty)$     9.  $(-\infty, \infty)$     10. 5    11. -2    12.  $\frac{1}{2}$     13.  $\frac{1}{3}$     14. 2

15. Evaluation not possible;  $\log_2 \frac{1}{8} = -3$  and  $\log_3(-3)$  is undefined.    16. 5    17.  $\sqrt{7}$     18. 13    19.  $-\frac{1}{2}$     20.  $\sqrt{\pi}$

21.  $\frac{1}{2} \log x + \frac{1}{2} \log y - 3$     22.  $19 + 20 \ln x$     23.  $\log_7\left(\frac{x^8}{\sqrt[3]{y}}\right)$     24.  $\log_5 x^9$     25.  $\ln\left[\frac{\sqrt{x}}{y^3(z-2)}\right]$     26. \$8

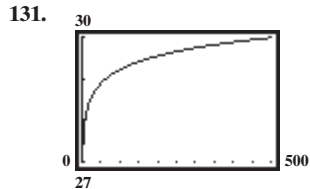
Section 3.4

Check Point Exercises

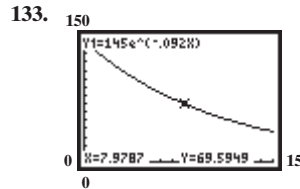
1. a. {3}    b. {-12}    2. a.  $\left\{\frac{\ln 134}{\ln 5}\right\}; \approx 3.04$     b.  $\{\log 8000\}; \approx 3.90$     3.  $\left\{\frac{\ln 9}{2}\right\}; \approx 1.10$     4.  $\left\{\frac{\ln 3 + \ln 7}{2 \ln 3 - \ln 7}\right\}; \approx 12.11$   
 5.  $\{0, \ln 7\}; \ln 7 \approx 1.95$     6. a. {12}    b.  $\left\{\frac{e^2}{3}\right\}$     7. {5}    8. {4, 5}    9. 0.01    10. 16.2 yr    11. \$11,000

Exercise Set 3.4

1. {6}    3. {3}    5. {3}    7. {2}    9.  $\left\{\frac{3}{5}\right\}$     11.  $\left\{\frac{3}{2}\right\}$     13. {4}    15. {5}    17.  $\left\{-\frac{1}{4}\right\}$     19. {13}    21. {-2}  
 23.  $\left\{\frac{\ln 3.91}{\ln 10}\right\}; \approx 0.59$     25.  $\{\ln 5.7\}; \approx 1.74$     27.  $\left\{\frac{\ln 17}{\ln 5}\right\}; \approx 1.76$     29.  $\left\{\ln \frac{23}{5}\right\}; \approx 1.53$     31.  $\left\{\frac{\ln 659}{5}\right\}; \approx 1.30$   
 33.  $\left\{\frac{\ln 793 - 1}{-5}\right\}; \approx -1.14$     35.  $\left\{\frac{\ln 10,478 + 3}{5}\right\}; \approx 2.45$     37.  $\left\{\frac{\ln 410}{\ln 7} - 2\right\}; \approx 1.09$     39.  $\left\{\frac{\ln 813}{0.3 \ln 7}\right\}; \approx 11.48$     41.  $\left\{\frac{3 \ln 5 + \ln 3}{\ln 3 - 2 \ln 5}\right\}; \approx -2.80$   
 43.  $\{0, \ln 2\}; \ln 2 \approx 0.69$     45.  $\left\{\frac{\ln 3}{2}\right\}; \approx 0.55$     47. {0}    49. {81}    51.  $\{e^2\}; \approx 7.39$     53. {59}    55.  $\left\{\frac{109}{27}\right\}$     57.  $\left\{\frac{62}{3}\right\}$   
 59.  $\left\{\frac{e^4}{2}\right\}; \approx 27.30$     61.  $\{e^{-1/2}\}; \approx 0.61$     63.  $\{e^2 - 3\}; \approx 4.39$     65.  $\left\{\frac{5}{4}\right\}$     67. {6}    69. {6}    71. {5}    73. {12}    75.  $\left\{\frac{4}{3}\right\}$     77.  $\emptyset$   
 79. {5}    81.  $\left\{\frac{2}{9}\right\}$     83. {28}    85. {2}    87.  $\emptyset$     89.  $\left\{\frac{11}{3}\right\}$     91.  $\left\{\frac{1}{2}\right\}$     93.  $\{e^3, e^{-3}\}$     95.  $\left\{\pm \sqrt{\frac{\ln 45}{\ln 3}}\right\}$     97.  $\left\{\frac{5 + \sqrt{37}}{2}\right\}$   
 99. {-2, 6}    101. a. 36.1 million    b. 2013    103. 118 ft; by the point (118, 1)    105. 8.2 yr    107. 16.8%    109. 8.7 yr    111. 15.7%  
 113. a. 69%; overestimates by 1%    b. 2010    115. 2.8 days; (2.8, 50)    117. a.  $10^{-5.6}$  mole per liter    b.  $10^{-2.4}$  mole per liter    c.  $10^{3.2}$  times greater  
 123. {2}    125. {4}    127. {2}    129.  $\{-1.391606, 1.6855579\}$

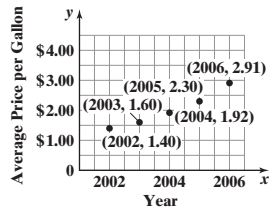


As distance from eye increases, barometric air pressure increases.



about 7.9 min

135. does not make sense    137. makes sense    139. false    141. true    143. after 36 yr    145.  $\{10^{-2}, 10^{3/2}\}, 10^{3/2} = 10\sqrt{10} \approx 31.62$   
 148. a. 10 million; 9.97 million; 9.94 million; 9.91 million    b. decreasing  
 149. ; logarithmic function    150. a. 3    b.  $e^{(\ln 3)x}$

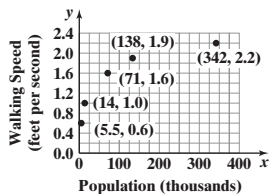


Section 3.5

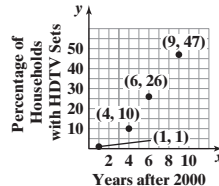
Check Point Exercises

1. a.  $A = 643 e^{0.021t}$     b. 2044    2. a.  $A = A_0 e^{-0.0248t}$     b. about 72 yr    3. a. 0.4 correct responses    b. 0.7 correct responses  
 c. 0.8 correct responses    4. a.  $T = 30 + 70e^{-0.0673t}$     b. 48°C    c. 39 min

5. ; logarithmic function



6. ; exponential    7.  $y = 4e^{(\ln 7.8)x}$ ;  $y = 4e^{2.054x}$

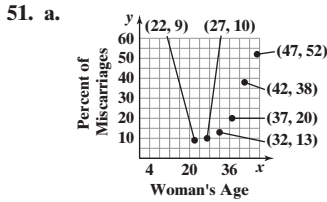


Exercise Set 3.5

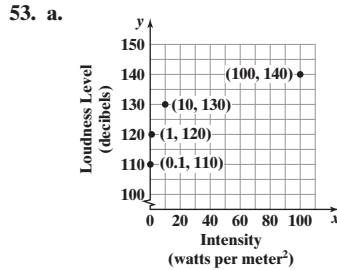
1. 127.5 million    3. Iraq; 2.7%    5. 2015    7. a.  $A = 6.04e^{0.01t}$     b. 2040    9. 118.7 million    11. 0.0121    13. -0.0053  
 15. approximately 8 grams    17. 8 grams after 10 seconds; 4 grams after 20 seconds; 2 grams after 30 seconds; 1 gram after 40 seconds; 0.5 gram after 50 seconds    19. approximately 15,679 years old    21. 12.6 yr    23. -0.000428; 0.0428% per year    25. -0.039608; 3.9608% per day

27. a.  $\frac{1}{2} = e^{1.31k}$  yields  $k = \frac{\ln\left(\frac{1}{2}\right)}{1.31} \approx -0.52912$ .    b. about 0.1069 billion or 106,900,000 years old    29. 7.1 yr    31. 5.5 hr

33.  $2A_0 = A_0 e^{kt}$ ;  $2 = e^{kt}$ ;  $\ln 2 = \ln e^{kt}$ ;  $\ln 2 = kt$ ;  $\frac{\ln 2}{k} = t$     35. a. 1%    b. about 69 yr    37. a. about 20 people    b. about 1080 people  
 c. 100,000 people    39. quite well    41. 2026    43. about 3.7%    45. about 48 years old    47. a.  $T = 45 + 25e^{-0.0916t}$     b. 51°F  
 c. 18 min    49. 26 min    50. 45 min



b. exponential function



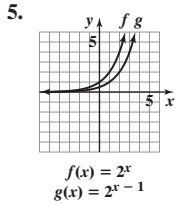
b. logarithmic function

55.  $y = 100e^{(\ln 4.6)x}$ ;  $y = 100e^{1.526x}$   
 57.  $y = 2.5e^{(\ln 0.7)x}$ ;  $y = 2.5e^{-0.357x}$   
 69.  $y = 193.1 + 23.574 \ln x$ ;  $r \approx 0.878$ ;  
 Fit is ok, but not great.  
 71.  $y = 195.871x^{0.097}$ ;  $r \approx 0.901$ ;  
 Fits data fairly well.  
 73. a.  $y = 3.46(1.02)^x$   
 b.  $y = 3.46e^{(\ln 1.02)x}$ ;  $y = 3.46e^{0.02x}$ ;  
 by approximately 2%

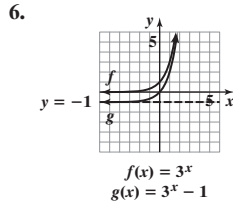
75. does not make sense    77. makes sense    79. true    81. true    83. about 8:02 a.m.    85.  $\left\{ \frac{5}{8} \right\}$     86.  $\frac{5\pi}{6}$     87.  $\frac{23\pi}{12}$

**Chapter 3 Review Exercises**

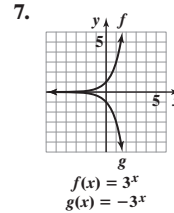
1.  $g(x) = 4^{-x}$     2.  $h(x) = -4^{-x}$     3.  $r(x) = -4^{-x} + 3$     4.  $f(x) = 4^x$



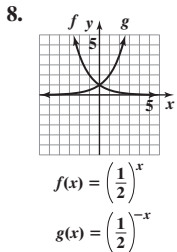
asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 0$   
 domain of  $f =$  domain of  $g = (-\infty, \infty)$   
 range of  $f =$  range of  $g = (0, \infty)$



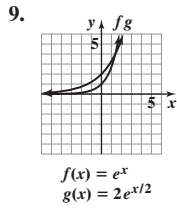
asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = -1$   
 domain of  $f =$  domain of  $g = (-\infty, \infty)$   
 range of  $f = (0, \infty)$ ; range of  $g = (-1, \infty)$



asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 0$   
 domain of  $f =$  domain of  $g = (-\infty, \infty)$   
 range of  $f = (0, \infty)$ ; range of  $g = (-\infty, 0)$

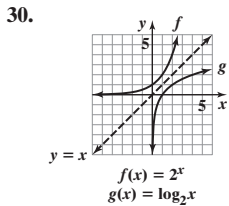


asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 0$   
 domain of  $f =$  domain of  $g = (-\infty, \infty)$   
 range of  $f =$  range of  $g = (0, \infty)$

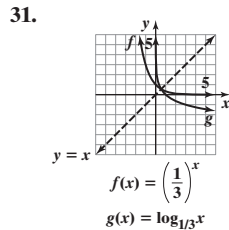


asymptote of  $f$ :  $y = 0$   
 asymptote of  $g$ :  $y = 0$   
 domain of  $f =$  domain of  $g = (-\infty, \infty)$   
 range of  $f =$  range of  $g = (0, \infty)$

10. 5.5% compounded semiannually    11. 7% compounded monthly    12. a. 200°    b. 120°; 119°    c. 70°; The temperature in the room is 70°.  
 13.  $49^{1/2} = 7$     14.  $4^3 = x$     15.  $3^y = 81$     16.  $\log_6 216 = 3$     17.  $\log_b 625 = 4$     18.  $\log_{13} 874 = y$     19. 3    20. -2    21. undefined;  
 $\log_b x$  is defined only for  $x > 0$ .    22.  $\frac{1}{2}$     23. 1    24. 8    25. 5    26.  $-\frac{1}{2}$     27. -2    28. -3    29. 0



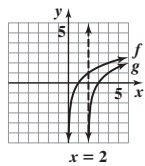
domain of  $f =$  range of  $g = (-\infty, \infty)$   
 range of  $f =$  domain of  $g = (0, \infty)$



domain of  $f =$  range of  $g = (-\infty, \infty)$   
 range of  $f =$  domain of  $g = (0, \infty)$

32.  $g(x) = \log(-x)$   
 33.  $r(x) = 1 + \log(2 - x)$   
 34.  $h(x) = \log(2 - x)$   
 35.  $f(x) = \log x$

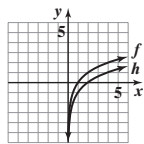
36.



$f(x) = \log_2 x$   
 $g(x) = \log_2(x - 2)$

x-intercept: (3, 0)  
 vertical asymptote:  $x = 2$   
 domain:  $(2, \infty)$ ; range:  $(-\infty, \infty)$

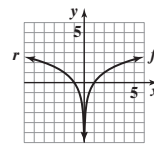
37.



$f(x) = \log_2 x$   
 $h(x) = -1 + \log_2 x$

x-intercept: (2, 0)  
 vertical asymptote:  $x = 0$   
 domain:  $(0, \infty)$ ; range:  $(-\infty, \infty)$

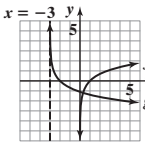
38.



$f(x) = \log_2 x$   
 $r(x) = \log_2(-x)$

x-intercept: (-1, 0)  
 vertical asymptote:  $x = 0$   
 domain:  $(-\infty, 0)$ ; range:  $(-\infty, \infty)$

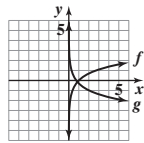
39.



$f(x) = \log x$   
 $g(x) = -\log(x + 3)$

asymptote of  $f$ :  $x = 0$   
 asymptote of  $g$ :  $x = -3$   
 domain of  $f = (0, \infty)$ ; domain of  $g = (-3, \infty)$   
 range of  $f = \text{range of } g = (-\infty, \infty)$

40.



$f(x) = \ln x$   
 $g(x) = -\ln(2x)$

asymptote of  $f$ :  $x = 0$   
 asymptote of  $g$ :  $x = 0$   
 domain of  $f = \text{domain of } g = (0, \infty)$   
 range of  $f = \text{range of } g = (-\infty, \infty)$

41.  $(-5, \infty)$

42.  $(-\infty, 3)$

43.  $(-\infty, 1) \cup (1, \infty)$

44.  $6x$

45.  $\sqrt{x}$

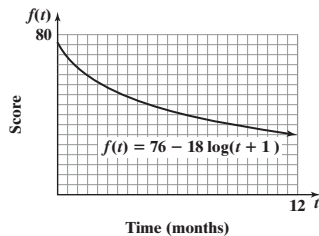
46.  $4x^2$

47. 3.0

48. a. 76    b.  $\approx 67, \approx 63, \approx 61, \approx 59, \approx 56$

49. about 9 weeks    50.  $2 + 3 \log_6 x$     51.  $\frac{1}{2} \log_4 x - 3$

c.



52.  $\log_2 x + 2 \log_2 y - 6$     53.  $\frac{1}{3} \ln x - \frac{1}{3}$     54.  $\log_b 21$     55.  $\log \frac{3}{x^3}$

56.  $\ln(x^3 y^4)$     57.  $\ln \frac{\sqrt{x}}{y}$     58. 6.2448    59. -0.1063

60. true    61. false;  $\log(x + 9) - \log(x + 1) = \log\left(\frac{x + 9}{x + 1}\right)$

62. false;  $4 \log_2 x = \log_2 x^4$     63. true    64.  $\{2\}$     65.  $\left\{\frac{2}{3}\right\}$     66.  $\{\log 7000\}$ ;  $\approx 3.85$

67.  $\left\{\frac{4}{5}\right\}$     68.  $\left\{\frac{\ln 12,143}{\ln 8}\right\}$ ;  $\approx 4.52$     69.  $\left\{\frac{1}{5} \ln 141\right\}$ ;  $\approx 0.99$     70.  $\left\{\frac{12 - \ln 130}{5}\right\}$ ;  $\approx 1.43$     71.  $\left\{\frac{\ln 37,500 - 2 \ln 5}{4 \ln 5}\right\}$ ;  $\approx 1.14$

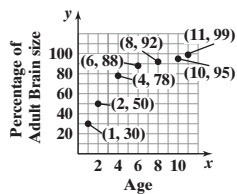
72.  $\left\{\frac{\ln 7 + 4 \ln 3}{2 \ln 7 - \ln 3}\right\}$ ;  $\approx 2.27$     73.  $\{\ln 3\}$ ;  $\approx 1.10$     74.  $\{23\}$     75.  $\left\{\frac{e^3}{2}\right\}$ ;  $\approx 10.04$     76.  $\{5\}$     77.  $\emptyset$     78.  $\{2\}$     79.  $\{4\}$

80. 5.5 mi    81. approximately 2086    82. approximately 8103 thousand or 8,103,000    83. 7.3 yr    84. 14.6 yr    85. about 22%

86. a. 0.045    b. 55.1 million    c. 2012    87. 325 days    88. a. 200 people    b. about 45,411 people    c. 500,000 people

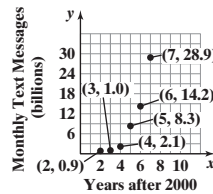
89. a.  $T = 65 + 120e^{-0.144t}$     b. 8 min

90. a.



b. logarithmic function

91. a.

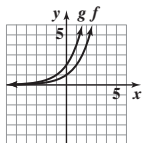


b. exponential function

92.  $y = 73e^{(\ln 2.6)x}$ ;  $y = 73e^{0.956x}$     93.  $y = 6.5e^{(\ln 0.43)x}$ ;  $y = 6.5e^{-0.844x}$

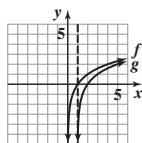
### Chapter 3 Test

1.



$f(x) = 2^x$   
 $g(x) = 2^{x+1}$

2.



$f(x) = \log_2 x$   
 $g(x) = \log_2(x - 1)$

3.  $5^3 = 125$     4.  $\log_{36} 6 = \frac{1}{2}$     5.  $(-\infty, 3)$     6.  $3 + 5 \log_4 x$

7.  $\frac{1}{3} \log_3 x - 4$     8.  $\log(x^6 y^2)$     9.  $\ln \frac{7}{x^3}$     10. 1.5741    11.  $\{-10\}$

12.  $\left\{\frac{\ln 1.4}{\ln 5}\right\}$     13.  $\left\{\frac{\ln 4}{0.005}\right\}$     14.  $\{0, \ln 5\}$     15.  $\{54.25\}$

16.  $\left\{\frac{e^4}{3}\right\}$     17.  $\{5\}$     18.  $\emptyset$     19. 120 db    20.  $5x$     21. 1    22. 0

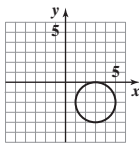
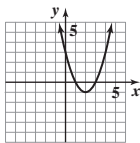
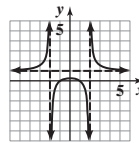
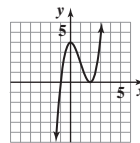
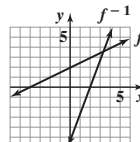
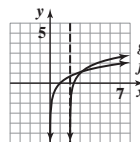
23. 6.5% compounded semiannually; \$221 more    24. 13.9 years    25. about 6.9%

26. a. 82.3 million    b. decreasing; The growth rate, -0.002, is negative.    c. 2008    27.  $A = 509e^{0.036t}$     28. 12.5 days    29. a. 14 elk

b. about 51 elk    c. 140 elk    30. linear    31. logarithmic    32. exponential    33. quadratic    34.  $y = 96e^{(\ln 0.38)x}$ ;  $y = 96e^{-0.968x}$



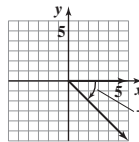
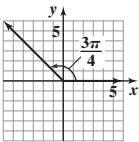
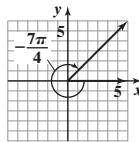
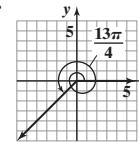
**Cumulative Review Exercises (Chapters P–3)**

1.  $\left\{\frac{2}{3}, 2\right\}$     2.  $\{-1 \pm 2i\}$     3.  $\{-2, -1, 1\}$     4.  $\left\{\frac{\ln 128}{5}\right\}$     5.  $\{3\}$     6.  $\emptyset$     7.  $(-\infty, 4]$     8.  $[1, 3]$
9.   $(x - 3)^2 + (y + 2)^2 = 4$
10.   $f(x) = (x - 2)^2 - 1$
11.   $f(x) = \frac{x^2 - 1}{x^2 - 4}$
12.   $f(x) = (x - 2)^2(x + 1)$
13.   $f(x) = 2x - 4$   
 $f^{-1}(x) = \frac{x + 4}{2}$
14.   $f(x) = \ln x$   
 $g(x) = \ln(x - 2) + 1$
15. using  $(1, 3)$ ,  $y - 3 = -3(x - 1)$ ;  $y = -3x + 6$   
 16.  $(f \circ g)(x) = (x + 2)^2$ ;  $(g \circ f)(x) = x^2 + 2$   
 17. You can expect to sleep 2 hours.  
 18. after 2 sec; 69 ft  
 19. 4.1 sec  
 20. \$12 per hr

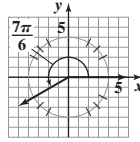
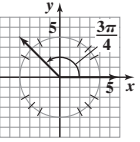
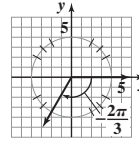
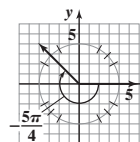
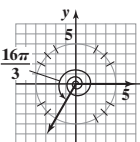
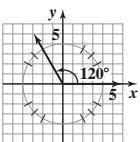
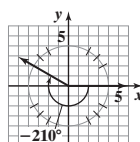
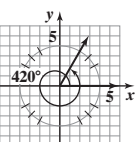
**CHAPTER 4**

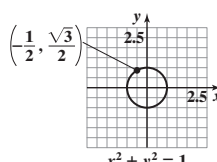
**Section 4.1**

**Check Point Exercises**

1. 3.5 radians    2. a.  $\frac{\pi}{3}$  radians    b.  $\frac{3\pi}{2}$  radians    c.  $-\frac{5\pi}{3}$  radians    3. a.  $45^\circ$     b.  $-240^\circ$     c.  $343.8^\circ$
4. a.     b.     c.     d. 
5. a.  $40^\circ$     b.  $225^\circ$     6. a.  $\frac{3\pi}{5}$     b.  $\frac{29\pi}{15}$     7. a.  $135^\circ$     b.  $\frac{5\pi}{3}$     c.  $\frac{11\pi}{6}$     8.  $\frac{3\pi}{2}$  in.  $\approx 4.71$  in.    9.  $135\pi$  in./min  $\approx 424$  in./min

**Exercise Set 4.1**

1. obtuse    3. acute    5. straight    7. 4 radians    9.  $\frac{4}{3}$  radians    11. 4 radians    13.  $\frac{\pi}{4}$  radians    15.  $\frac{3\pi}{4}$  radians    17.  $\frac{5\pi}{3}$  radians
19.  $-\frac{5\pi}{4}$  radians    21.  $90^\circ$     23.  $120^\circ$     25.  $210^\circ$     27.  $-540^\circ$     29. 0.31 radians    31.  $-0.70$  radians    33. 3.49 radians    35.  $114.59^\circ$
37.  $13.85^\circ$     39.  $-275.02^\circ$
41.  ; quadrant III    43.  ; quadrant II    45.  ; quadrant III
47.  ; quadrant II    49.  ; quadrant III    51.  ; quadrant II
53.  ; quadrant II    55.  ; quadrant I    57.  $35^\circ$     59.  $210^\circ$     61.  $315^\circ$     63.  $\frac{7\pi}{6}$     65.  $\frac{3\pi}{5}$
67.  $\frac{99\pi}{50}$     69.  $\frac{11\pi}{7}$     71.  $3\pi$  in.  $\approx 9.42$  in.    73.  $10\pi$  ft  $\approx 31.42$  ft    75.  $\frac{12\pi}{\text{second}}$  radians    77.  $-\frac{4\pi}{3}$  and  $\frac{2\pi}{3}$     79.  $-\frac{3\pi}{4}$  and  $\frac{5\pi}{4}$

81.  $-\frac{\pi}{2}$  and  $\frac{3\pi}{2}$     83.  $\frac{11\pi}{6}$     85.  $\frac{22\pi}{3}$     87.  $60^\circ; \frac{\pi}{3}$  radians    89.  $\frac{8\pi}{3}$  in.  $\approx 8.38$  in.    91.  $12\pi$  in.  $\approx 37.70$  in.
93. 2 radians;  $114.59^\circ$     95. 2094 mi    97. 1047 mph    99. 1508 ft/min    113.  $30.25^\circ$     115.  $30^\circ 25' 12''$     117. does not make sense
119. makes sense    121. smaller than a right angle    123. 1815 mi
124. 
125. domain:  $\{x | -1 \leq x \leq 1\}$  or  $[-1, 1]$ ; range:  $\{y | -1 \leq y \leq 1\}$  or  $[-1, 1]$
126.  $-\frac{\sqrt{3}}{3}$

## Section 4.2

### Check Point Exercises

1.  $\sin t = \frac{1}{2}$ ;  $\cos t = \frac{\sqrt{3}}{2}$ ;  $\tan t = \frac{\sqrt{3}}{3}$ ;  $\csc t = 2$ ;  $\sec t = \frac{2\sqrt{3}}{3}$ ;  $\cot t = \sqrt{3}$     2.  $\sin \pi = 0$ ;  $\cos \pi = -1$ ;  $\tan \pi = 0$ ;  $\csc \pi$  is undefined;  $\sec \pi = -1$ ;  $\cot \pi$  is undefined
3.  $\sqrt{2}$ ;  $\sqrt{2}$ ; 1    4. a.  $\sqrt{2}$     b.  $-\frac{\sqrt{2}}{2}$     5.  $\tan \theta = \frac{2\sqrt{5}}{5}$ ;  $\csc \theta = \frac{3}{2}$ ;  $\sec \theta = \frac{3\sqrt{5}}{5}$ ;  $\cot \theta = \frac{\sqrt{5}}{2}$     6.  $\frac{\sqrt{3}}{2}$
7. a. 1    b.  $\frac{\sqrt{2}}{2}$     8. a. 0.7071    b. 1.0025

### Exercise Set 4.2

1.  $\sin t = \frac{8}{17}$ ;  $\cos t = -\frac{15}{17}$ ;  $\tan t = -\frac{8}{15}$ ;  $\csc t = \frac{17}{8}$ ;  $\sec t = -\frac{17}{15}$ ;  $\cot t = -\frac{15}{8}$
3.  $\sin t = -\frac{\sqrt{2}}{2}$ ;  $\cos t = \frac{\sqrt{2}}{2}$ ;  $\tan t = -1$ ;  $\csc t = -\sqrt{2}$ ;  $\sec t = \sqrt{2}$ ;  $\cot t = -1$     5.  $\frac{1}{2}$     7.  $-\frac{\sqrt{3}}{2}$     9. 0    11. -2    13.  $\frac{2\sqrt{3}}{3}$     15. -1
17. undefined    19. a.  $\frac{\sqrt{3}}{2}$     b.  $\frac{\sqrt{3}}{2}$     21. a.  $\frac{1}{2}$     b.  $-\frac{1}{2}$     23. a.  $-\sqrt{3}$     b.  $\sqrt{3}$
25.  $\tan t = \frac{8}{15}$ ;  $\csc t = \frac{17}{8}$ ;  $\sec t = \frac{17}{15}$ ;  $\cot t = \frac{15}{8}$     27.  $\tan t = \frac{\sqrt{2}}{4}$ ;  $\csc t = 3$ ;  $\sec t = \frac{3\sqrt{2}}{4}$ ;  $\cot t = 2\sqrt{2}$     29.  $\frac{\sqrt{13}}{7}$     31.  $\frac{5}{8}$     33. 1    35. 1
37. 1    39.  $\frac{\sqrt{2}}{2}$     41.  $-\frac{\sqrt{2}}{2}$     43. 1    45. -1    47. -1    49.  $-\frac{\sqrt{2}}{2}$     51.  $\frac{\sqrt{2}}{2}$     53. a.  $\frac{\sqrt{2}}{2}$     b.  $\frac{\sqrt{2}}{2}$     55. a. 0    b. 0
57. a. 0    b. 0    59. a.  $-\frac{\sqrt{2}}{2}$     b.  $-\frac{\sqrt{2}}{2}$     61. 0.7174    63. 0.2643    65. 1.1884    67. 0.9511    69. 3.7321    71.  $-2a$     73.  $3b$
75.  $a - b + c$     77.  $-a - b + c$     79.  $3a + 2b - 2c$     81. a. 12 hr    b. 20.3 hr    c. 3.7 hr    83. a. 1; 0; -1; 0; 1    b. 28 days
97. makes sense    99. does not make sense    101. c    103.  $-\frac{1}{4}$     105.  $\frac{5}{13}$     106.  $\frac{\sqrt{2}}{2}$     107. 1

## Section 4.3

### Check Point Exercises

1.  $\sin \theta = \frac{3}{5}$ ;  $\cos \theta = \frac{4}{5}$ ;  $\tan \theta = \frac{3}{4}$ ;  $\csc \theta = \frac{5}{3}$ ;  $\sec \theta = \frac{5}{4}$ ;  $\cot \theta = \frac{4}{3}$     2.  $\sin \theta = \frac{1}{5}$ ;  $\cos \theta = \frac{2\sqrt{6}}{5}$ ;  $\tan \theta = \frac{\sqrt{6}}{12}$ ;  $\csc \theta = 5$ ;  $\sec \theta = \frac{5\sqrt{6}}{12}$ ;  $\cot \theta = 2\sqrt{6}$
3.  $\sqrt{2}$ ;  $\sqrt{2}$ ; 1    4.  $\sqrt{3}$ ;  $\frac{\sqrt{3}}{3}$     5. a.  $\cos 44^\circ$     b.  $\tan \frac{5\pi}{12}$     6. 333.9 yd    7.  $54^\circ$

### Exercise Set 4.3

1. 15;  $\sin \theta = \frac{3}{5}$ ;  $\cos \theta = \frac{4}{5}$ ;  $\tan \theta = \frac{3}{4}$ ;  $\csc \theta = \frac{5}{3}$ ;  $\sec \theta = \frac{5}{4}$ ;  $\cot \theta = \frac{4}{3}$
3. 20;  $\sin \theta = \frac{20}{29}$ ;  $\cos \theta = \frac{21}{29}$ ;  $\tan \theta = \frac{20}{21}$ ;  $\csc \theta = \frac{29}{20}$ ;  $\sec \theta = \frac{29}{21}$ ;  $\cot \theta = \frac{21}{20}$
5. 24;  $\sin \theta = \frac{5}{13}$ ;  $\cos \theta = \frac{12}{13}$ ;  $\tan \theta = \frac{5}{12}$ ;  $\csc \theta = \frac{13}{5}$ ;  $\sec \theta = \frac{13}{12}$ ;  $\cot \theta = \frac{12}{5}$     7. 28;  $\sin \theta = \frac{4}{5}$ ;  $\cos \theta = \frac{3}{5}$ ;  $\tan \theta = \frac{4}{3}$ ;  $\csc \theta = \frac{5}{4}$ ;  $\sec \theta = \frac{5}{3}$ ;  $\cot \theta = \frac{3}{4}$
9.  $\frac{\sqrt{3}}{2}$     11.  $\sqrt{2}$     13.  $\sqrt{3}$     15. 0    17.  $\frac{\sqrt{6}-4}{4}$     19.  $\frac{12\sqrt{3}+\sqrt{6}}{6}$     21.  $\cos 83^\circ$     23.  $\sec 65^\circ$     25.  $\cot \frac{7\pi}{18}$     27.  $\sin \frac{\pi}{10}$
29. 188 cm    31. 182 in.    33. 41 m    35.  $17^\circ$     37.  $78^\circ$     39. 1.147 radians    41. 0.395 radians    43. 0    45. 2    47. 1
49.  $\frac{2\sqrt{3}-1}{2}$     51.  $\frac{1}{4}$     53. 529 yd    55.  $36^\circ$     57. 2879 ft    59.  $37^\circ$     69. 0.92106, -0.19735; 0.95534, -0.148878; 0.98007, -0.099667; 0.99500, -0.04996; 0.99995, -0.005; 0.9999995, -0.0005; 0.99999995, -0.00005; 0.999999995, -0.000005;  $\frac{\cos \theta - 1}{\theta}$  approaches 0 as  $\theta$  approaches 0.
71. does not make sense    73. makes sense    75. true    77. true    79. As  $\theta$  approaches  $90^\circ$ ,  $\tan \theta$  increases without bound. At  $90^\circ$ ,  $\tan \theta$  is undefined.
81. a.  $\frac{y}{r}$     b.  $\frac{4}{5}$ ; positive    82. a.  $\frac{x}{r}$     b.  $-\frac{3\sqrt{34}}{34}$ ; negative    83. a.  $15^\circ$     b.  $\frac{\pi}{6}$

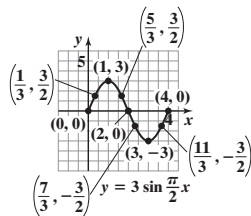
Section 4.4

Check Point Exercises

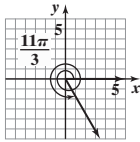
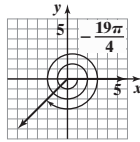
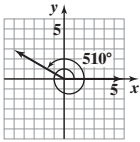
1.  $\sin \theta = -\frac{3\sqrt{10}}{10}$ ;  $\cos \theta = \frac{\sqrt{10}}{10}$ ;  $\tan \theta = -3$ ;  $\csc \theta = -\frac{\sqrt{10}}{3}$ ;  $\sec \theta = \sqrt{10}$ ;  $\cot \theta = -\frac{1}{3}$     2. a. 1; undefined    b. 0; 1    c. -1; undefined  
 d. 0; -1    3. quadrant III    4.  $\frac{\sqrt{10}}{10}$ ;  $-\frac{\sqrt{10}}{3}$     5. a.  $30^\circ$     b.  $\frac{\pi}{4}$     c.  $60^\circ$     d. 0.46    6. a.  $55^\circ$     b.  $\frac{\pi}{4}$     c.  $\frac{\pi}{3}$   
 7. a.  $-\frac{\sqrt{3}}{2}$     b. 1    c.  $\frac{2\sqrt{3}}{3}$     8. a.  $-\frac{\sqrt{3}}{2}$     b.  $\frac{\sqrt{3}}{2}$

Exercise Set 4.4

1.  $\sin \theta = \frac{3}{5}$ ;  $\cos \theta = -\frac{4}{5}$ ;  $\tan \theta = -\frac{3}{4}$ ;  $\csc \theta = \frac{5}{3}$ ;  $\sec \theta = -\frac{5}{4}$ ;  $\cot \theta = -\frac{4}{3}$   
 3.  $\sin \theta = \frac{3\sqrt{13}}{13}$ ;  $\cos \theta = \frac{2\sqrt{13}}{13}$ ;  $\tan \theta = \frac{3}{2}$ ;  $\csc \theta = \frac{\sqrt{13}}{3}$ ;  $\sec \theta = \frac{\sqrt{13}}{2}$ ;  $\cot \theta = \frac{2}{3}$     5.  $\sin \theta = -\frac{\sqrt{2}}{2}$ ;  $\cos \theta = \frac{\sqrt{2}}{2}$ ;  $\tan \theta = -1$ ;  $\csc \theta = -\sqrt{2}$ ;  
 $\sec \theta = \sqrt{2}$ ;  $\cot \theta = -1$     7.  $\sin \theta = -\frac{5\sqrt{29}}{29}$ ;  $\cos \theta = -\frac{2\sqrt{29}}{29}$ ;  $\tan \theta = \frac{5}{2}$ ;  $\csc \theta = -\frac{\sqrt{29}}{5}$ ;  $\sec \theta = -\frac{\sqrt{29}}{2}$ ;  $\cot \theta = \frac{2}{5}$     9. -1    11. -1  
 13. undefined    15. 0    17. quadrant I    19. quadrant III    21. quadrant II  
 23.  $\sin \theta = -\frac{4}{5}$ ;  $\tan \theta = \frac{4}{3}$ ;  $\csc \theta = -\frac{5}{4}$ ;  $\sec \theta = -\frac{5}{3}$ ;  $\cot \theta = \frac{3}{4}$     25.  $\cos \theta = -\frac{12}{13}$ ;  $\tan \theta = -\frac{5}{12}$ ;  $\csc \theta = \frac{13}{5}$ ;  $\sec \theta = -\frac{13}{12}$ ;  $\cot \theta = -\frac{12}{5}$   
 27.  $\sin \theta = -\frac{15}{17}$ ;  $\tan \theta = -\frac{15}{8}$ ;  $\csc \theta = -\frac{17}{15}$ ;  $\sec \theta = \frac{17}{8}$ ;  $\cot \theta = -\frac{8}{15}$     29.  $\sin \theta = \frac{2\sqrt{13}}{13}$ ;  $\cos \theta = -\frac{3\sqrt{13}}{13}$ ;  $\csc \theta = \frac{\sqrt{13}}{2}$ ;  $\sec \theta = -\frac{\sqrt{13}}{3}$ ;  $\cot \theta = -\frac{3}{2}$   
 31.  $\sin \theta = -\frac{4}{5}$ ;  $\cos \theta = -\frac{3}{5}$ ;  $\csc \theta = -\frac{5}{4}$ ;  $\sec \theta = -\frac{5}{3}$ ;  $\cot \theta = \frac{3}{4}$     33.  $\sin \theta = -\frac{2\sqrt{2}}{3}$ ;  $\cos \theta = -\frac{1}{3}$ ;  $\tan \theta = 2\sqrt{2}$ ;  $\csc \theta = -\frac{3\sqrt{2}}{4}$ ;  $\cot \theta = \frac{\sqrt{2}}{4}$   
 35.  $20^\circ$     37.  $25^\circ$     39.  $5^\circ$     41.  $\frac{\pi}{4}$     43.  $\frac{\pi}{6}$     45.  $30^\circ$     47.  $25^\circ$     49. 1.56    51.  $25^\circ$     53.  $\frac{\pi}{6}$     55.  $\frac{\pi}{4}$     57.  $\frac{\pi}{4}$     59.  $\frac{\pi}{6}$   
 61.  $-\frac{\sqrt{2}}{2}$     63.  $\frac{\sqrt{3}}{3}$     65.  $\sqrt{3}$     67.  $\frac{\sqrt{3}}{2}$     69. -2    71. 1    73.  $\frac{\sqrt{3}}{2}$     75. -1    77.  $-\sqrt{2}$     79.  $\sqrt{3}$     81.  $\frac{\sqrt{2}}{2}$     83.  $\frac{\sqrt{3}}{3}$   
 85.  $\frac{\sqrt{3}}{2}$     87.  $\frac{1-\sqrt{3}}{2}$     89.  $\frac{-\sqrt{6}-\sqrt{2}}{4}$  or  $-\frac{\sqrt{6}+\sqrt{2}}{4}$     91.  $-\frac{3}{2}$     93.  $\frac{-1-\sqrt{3}}{2}$  or  $-\frac{1+\sqrt{3}}{2}$     95. 1    97.  $\frac{2\sqrt{2}-4}{\pi}$     99.  $\frac{\pi}{4}$  and  $\frac{3\pi}{4}$   
 101.  $\frac{5\pi}{4}$  and  $\frac{7\pi}{4}$     103.  $\frac{2\pi}{3}$  and  $\frac{5\pi}{3}$     111. does not make sense    113. makes sense    114.  $\frac{1}{2}$ ; 0;  $-\frac{1}{2}$ ; 0;  $\frac{1}{2}$     115. 0; 4; 0; -4; 0  
 116.  $0$ ;  $\frac{3}{2}$ ;  $3$ ;  $\frac{3}{2}$ ;  $0$ ;  $-\frac{3}{2}$ ;  $-3$ ;  $-\frac{3}{2}$ ;  $0$



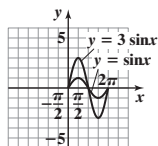
Mid-Chapter 4 Check Point

1.  $\frac{\pi}{18}$     2.  $-\frac{7\pi}{12}$     3.  $75^\circ$     4.  $-117^\circ$   
 5. a.  $\frac{5\pi}{3}$   
 b.   
 6. a.  $\frac{5\pi}{4}$   
 b.   
 7. a.  $150^\circ$   
 b.   
 c.  $\frac{\pi}{3}$     c.  $\frac{\pi}{4}$     c.  $30^\circ$   
 8.  $\sin t = -\frac{4}{5}$ ;  $\cos t = -\frac{3}{5}$ ;  $\tan t = \frac{4}{3}$ ;  $\csc t = -\frac{5}{4}$ ;  $\sec t = -\frac{5}{3}$ ;  $\cot t = \frac{3}{4}$     9.  $\sin \theta = \frac{5}{6}$ ;  $\cos \theta = \frac{\sqrt{11}}{6}$ ;  $\tan \theta = \frac{5\sqrt{11}}{11}$ ;  $\csc \theta = \frac{6}{5}$ ;  $\sec \theta = \frac{6\sqrt{11}}{11}$ ;  
 $\cot \theta = \frac{\sqrt{11}}{5}$     10.  $\sin \theta = -\frac{2\sqrt{13}}{13}$ ;  $\cos \theta = \frac{3\sqrt{13}}{13}$ ;  $\tan \theta = -\frac{2}{3}$ ;  $\csc \theta = -\frac{\sqrt{13}}{2}$ ;  $\sec \theta = \frac{\sqrt{13}}{3}$ ;  $\cot \theta = -\frac{3}{2}$     11.  $\sin \theta = \frac{3}{5}$ ;  $\cos \theta = -\frac{4}{5}$ ;  $\csc \theta = \frac{5}{3}$ ;  
 $\sec \theta = -\frac{5}{4}$ ;  $\cot \theta = -\frac{4}{3}$     12.  $\sin \theta = -\frac{2\sqrt{10}}{7}$ ;  $\tan \theta = -\frac{2\sqrt{10}}{3}$ ;  $\csc \theta = -\frac{7\sqrt{10}}{20}$ ;  $\sec \theta = \frac{7}{3}$ ;  $\cot \theta = -\frac{3\sqrt{10}}{20}$     13. 52 cm    14. 809 m  
 15.  $\sqrt{35}$     16.  $\frac{\sqrt{3}}{3}$     17.  $-\frac{\sqrt{3}}{3}$     18.  $\frac{1}{2}$     19.  $\frac{2\sqrt{3}}{3}$     20. 1    21.  $-\frac{\sqrt{3}}{2}$     22.  $-\frac{2\sqrt{3}}{3}$     23.  $-\frac{\sqrt{2}}{2}$     24.  $\frac{\sqrt{3}}{3}$     25. 2  
 26.  $-\frac{5\sqrt{3}}{6}$     27.  $8\pi$  cm  $\approx$  25.13 cm    28.  $160\pi$  ft/min  $\approx$  502 ft/min    29. 551.9 ft    30.  $40^\circ$

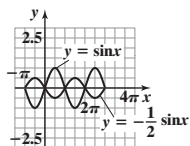
**Section 4.5**

**Check Point Exercises**

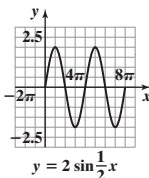
1. 3



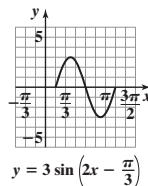
2.  $\frac{1}{2}$



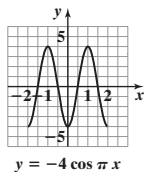
3. 2;  $4\pi$



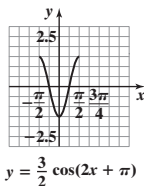
4. 3;  $\pi$ ;  $\frac{\pi}{6}$



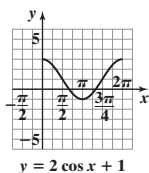
5. 4; 2



6.  $\frac{3}{2}$ ;  $\pi$ ;  $-\frac{\pi}{2}$



7.

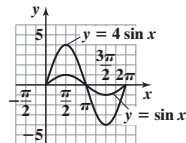


8.  $y = 4 \sin 4x$

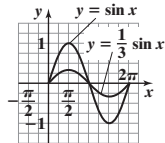
9.  $y = 2 \sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 12$

**Exercise Set 4.5**

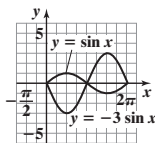
1. 4



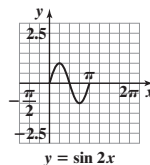
3.  $\frac{1}{3}$



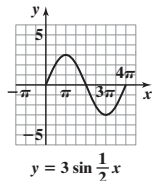
5. 3



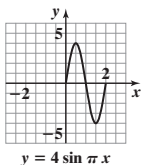
7. 1;  $\pi$



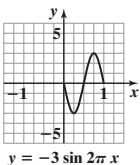
9. 3;  $4\pi$



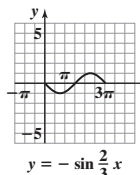
11. 4; 2



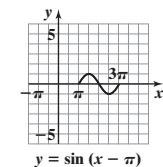
13. 3; 1



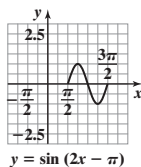
15. 1;  $3\pi$



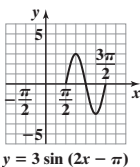
17. 1;  $2\pi$ ;  $\pi$



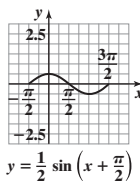
19. 1;  $\pi$ ;  $\frac{\pi}{2}$



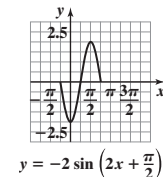
21. 3;  $\pi$ ;  $\frac{\pi}{2}$



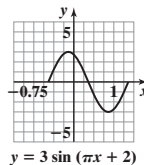
23.  $\frac{1}{2}$ ;  $2\pi$ ;  $-\frac{\pi}{2}$



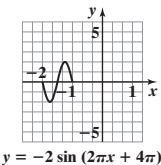
25. 2;  $\pi$ ;  $-\frac{\pi}{4}$



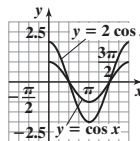
27. 3; 2;  $-\frac{2}{\pi}$



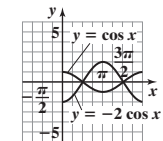
29. 2; 1; -2



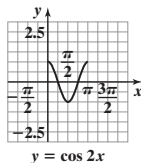
31. 2



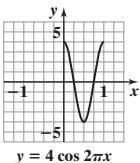
33. 2



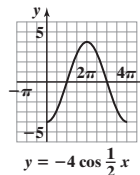
35. 1;  $\pi$



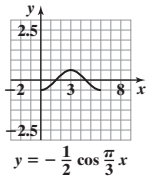
37. 4; 1



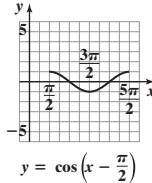
39. 4;  $4\pi$



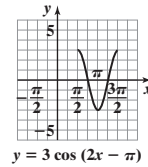
41.  $\frac{1}{2}; 6$



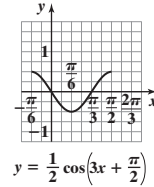
43.  $1; 2\pi, \frac{\pi}{2}$



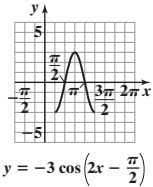
45.  $3; \pi; \frac{\pi}{2}$



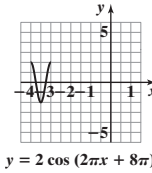
47.  $\frac{1}{2}; \frac{2\pi}{3}; -\frac{\pi}{6}$



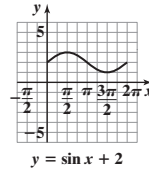
49.  $3; \pi; \frac{\pi}{4}$



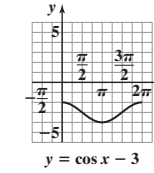
51.  $2; 1; -4$



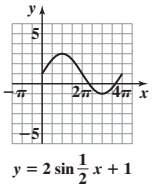
53.



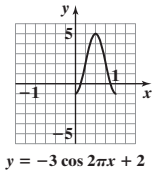
55.



57.



59.

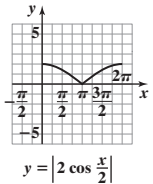


61. One possibility:  $y = 3 \cos(\frac{1}{2}x)$

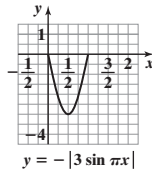
63. One possibility:  $y = -2 \sin(2x)$

65. One possibility:  $y = 2 \sin(\frac{\pi}{2}x)$

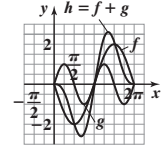
67.



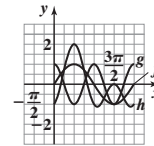
69.



71.



73.



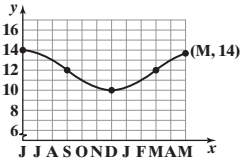
75. 33 days

77. 23 days

79. March 21

81. No

83.

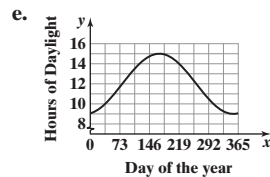


85. a. 3

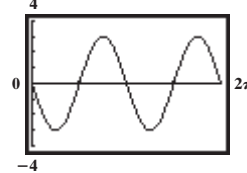
b. 365 days

c. 15 hours of daylight

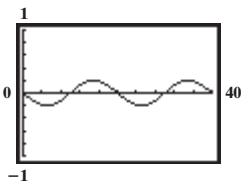
87.  $y = 3 \cos \frac{\pi x}{6} + 9$



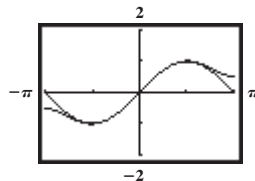
101.



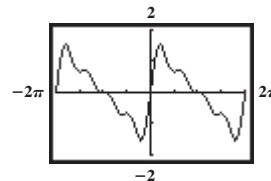
103.



105.

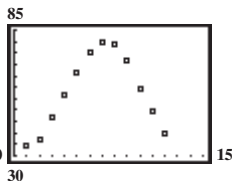


107.



The graph is similar to  $y = \sin x$ , except the amplitude is greater and the curve is less smooth.

109. a.



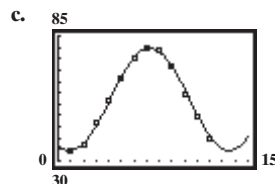
b.  $y = 22.61 \sin(0.50x - 2.04) + 57.17$

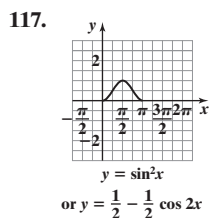
111. makes sense

113. makes sense

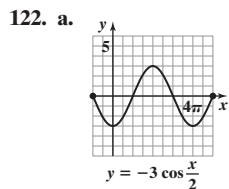
115. a. range:  $[-5, 1]$ ;  $[-\frac{\pi}{6}, \frac{23\pi}{6}, \frac{\pi}{6}]$  by  $[-5, 1, 1]$

b. range:  $[-3, -1]$ ;  $[-\frac{\pi}{6}, \frac{7\pi}{6}, \frac{\pi}{6}]$  by  $[-3, -1, 1]$





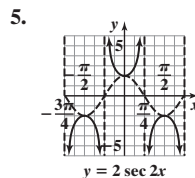
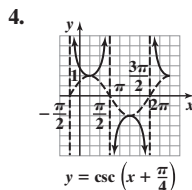
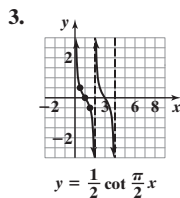
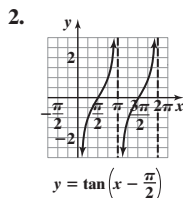
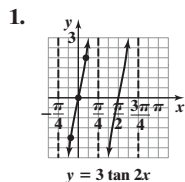
120.  $\left\{ x \mid -\frac{3\pi}{4} < x < \frac{\pi}{4} \right\}$  or  $\left( -\frac{3\pi}{4}, \frac{\pi}{4} \right)$     121.  $-\frac{\pi}{4}$



b. The reciprocal function is undefined.

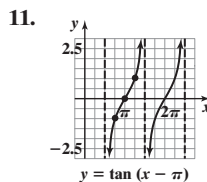
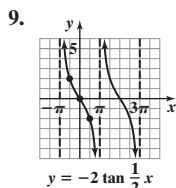
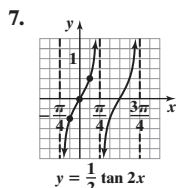
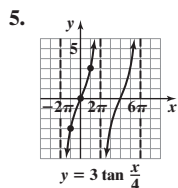
Section 4.6

Check Point Exercises



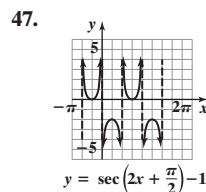
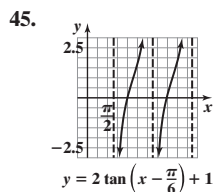
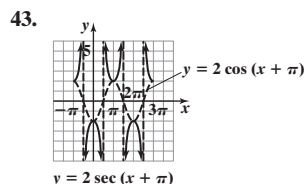
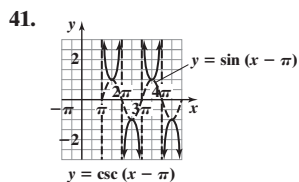
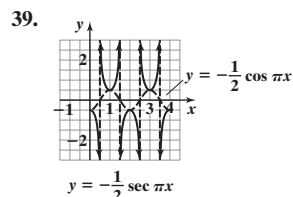
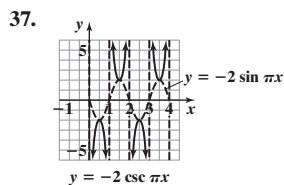
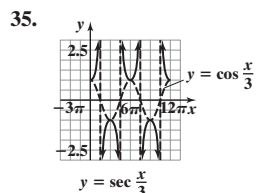
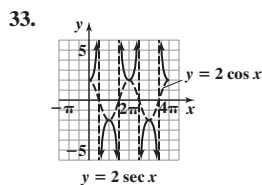
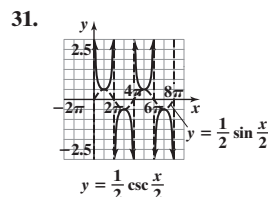
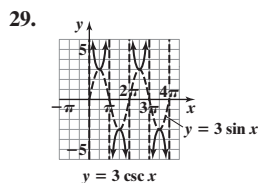
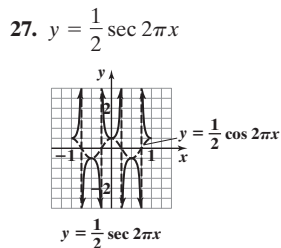
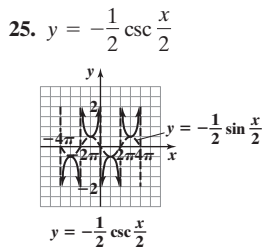
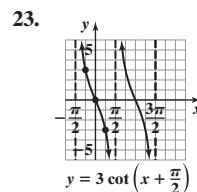
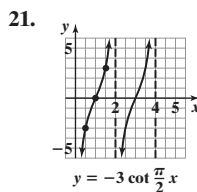
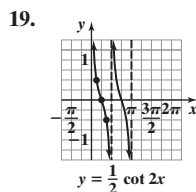
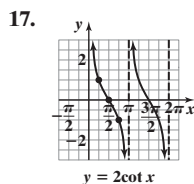
Exercise Set 4.6

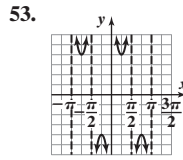
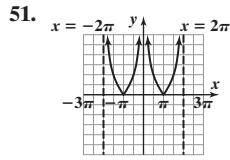
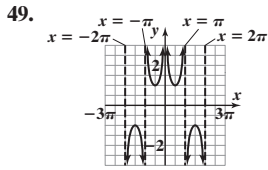
1.  $y = \tan(x + \pi)$     3.  $y = -\tan \left( x - \frac{\pi}{2} \right)$



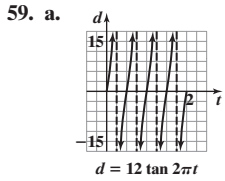
13.  $y = -\cot x$

15.  $y = \cot \left( x + \frac{\pi}{2} \right)$

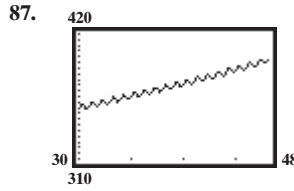
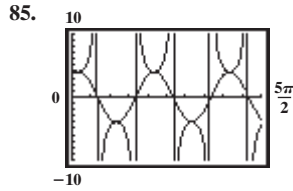
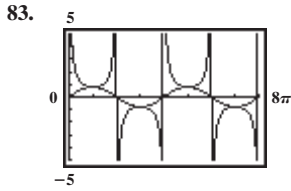
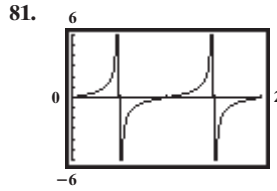
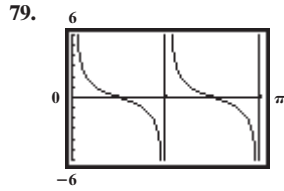
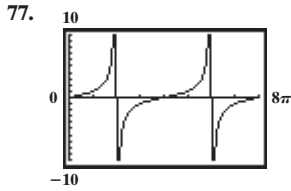
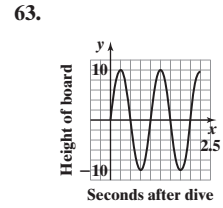
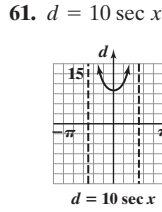




55.  $\left\{-\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}\right\}$     57.  $\left\{-\frac{3\pi}{2}, \frac{\pi}{2}\right\}$

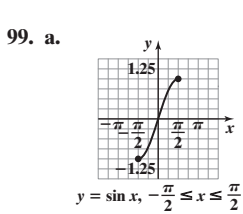


b. 0.25, 0.75, 1.25, 1.75;  
The beam of light is shining parallel to the wall at these times.

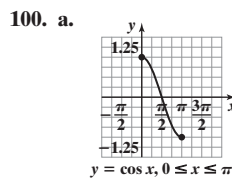


89. makes sense    91. does not make sense    93.  $y = \cot\frac{3x}{2}$     95.  $y = 2\csc\frac{3x}{2}$     97. a. range:  $(-\infty, -1] \cup [1, \infty)$ ;  $\left[-\frac{\pi}{6}, \pi, \frac{7\pi}{6}\right]$  by  $[-3, 3, 1]$

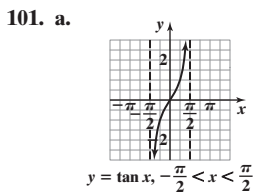
b. range:  $(-\infty, -3] \cup [3, \infty)$ ;  $\left[-\frac{1}{2}, \frac{7}{2}, 1\right]$  by  $[-6, 6, 1]$



b. yes    c.  $-\frac{\pi}{6}; \left(-\frac{\pi}{6}, -\frac{1}{2}\right)$



b. yes    c.  $\frac{5\pi}{6}; \left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2}\right)$



b. yes    c.  $-\frac{\pi}{3}; \left(-\frac{\pi}{3}, -\sqrt{3}\right)$

**Section 4.7**

**Check Point Exercises**

1.  $\frac{\pi}{3}$     2.  $-\frac{\pi}{4}$     3.  $\frac{2\pi}{3}$     4.  $-\frac{\pi}{4}$     5. a. 1.2310    b. -1.5429    6. a. 0.7    b. 0    c. not defined    7.  $\frac{3}{5}$     8.  $\frac{\sqrt{3}}{2}$     9.  $\sqrt{x^2 + 1}$

**Exercise Set 4.7**

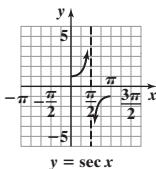
1.  $\frac{\pi}{6}$     3.  $\frac{\pi}{4}$     5.  $-\frac{\pi}{6}$     7.  $\frac{\pi}{6}$     9.  $\frac{3\pi}{4}$     11.  $\frac{\pi}{2}$     13.  $\frac{\pi}{6}$     15. 0    17.  $-\frac{\pi}{3}$     19. 0.30    21. -0.33    23. 1.19    25. 1.25

27. -1.52    29. -1.52    31. 0.9    33.  $\frac{\pi}{3}$     35.  $\frac{\pi}{6}$     37. 125    39.  $-\frac{\pi}{6}$     41.  $-\frac{\pi}{3}$     43. 0    45. not defined    47.  $\frac{3}{5}$     49.  $\frac{12}{5}$

51.  $-\frac{3}{4}$     53.  $\frac{\sqrt{2}}{2}$     55.  $\frac{4\sqrt{15}}{15}$     57.  $-2\sqrt{2}$     59. 2    61.  $\frac{3\sqrt{13}}{13}$     63.  $\frac{\sqrt{1-x^2}}{x}$     65.  $\sqrt{1-4x^2}$     67.  $\frac{\sqrt{x^2-1}}{x}$     69.  $\frac{\sqrt{3}}{x}$

71.  $\frac{\sqrt{x^2+4}}{2}$

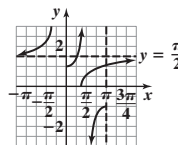
73. a.



$y = \sec x$

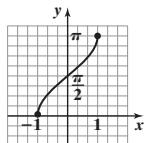
b. No horizontal line intersects the graph of  $y = \sec x$  more than once, so the function is one-to-one and has an inverse function.

c.



$y = \sec x$   
 $y = \sec^{-1} x$

75.

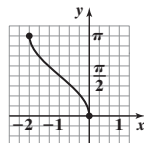


$f(x) = \sin^{-1}x + \frac{\pi}{2}$

domain:  $[-1, 1]$ ;

range:  $[0, \pi]$

77.

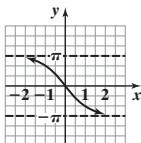


$g(x) = \cos^{-1}(x + 1)$

domain:  $[-2, 0]$ ;

range:  $[0, \pi]$

79.

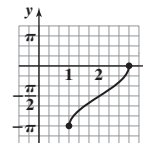


$f(x) = -2 \tan^{-1}x$

domain:  $(-\infty, \infty)$ ;

range:  $(-\pi, \pi)$

81.

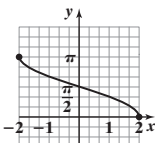


$f(x) = \sin^{-1}(x - 2) - \frac{\pi}{2}$

domain:  $[1, 3]$ ;

range:  $[-\pi, 0]$

83.



$g(x) = \cos^{-1} \frac{x}{2}$

domain:  $[-2, 2]$ ;

range:  $[0, \pi]$

85. domain:  $[-1, 1]$ ; range:  $[-1, 1]$

87. domain:  $(-\infty, \infty)$ ; range:  $[0, \pi]$

89. domain:  $(-\infty, \infty)$ ; range:  $[-\frac{\pi}{2}, \frac{\pi}{2}]$

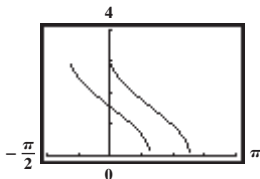
91. domain:  $[-1, 1]$ ; range:  $\{\frac{\pi}{2}\}$

93. 0.408 radians; 0.602 radians; 0.654 radians; 0.645 radians; 0.613 radians

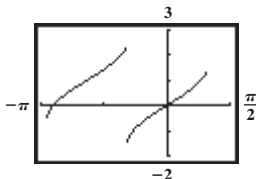
95. 1.3157 radians or 75.4°

97. 1.1071 sq units

111.

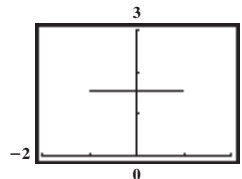


113.



Shifted left 2 units and up 1 unit

115.



It seems

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$  for  $-1 \leq x \leq 1$ .

117. does not make sense

119. does not make sense

121.  $x = \sin \frac{\pi}{8}$

123.  $\tan \alpha = \frac{8}{x}$ , so  $\tan^{-1} \frac{8}{x} = \alpha$ .  $\tan(\alpha + \theta) = \frac{33}{x}$ , so  $\tan^{-1} \frac{33}{x} = \alpha + \theta$ .  $\theta = \alpha + \theta - \alpha = \tan^{-1} \frac{33}{x} - \tan^{-1} \frac{8}{x}$ .

124.  $a \approx 4.96$ ;  $c \approx 13.08$

125. 35.8°    126. amplitude: 10; period: 12

## Section 4.8

### Check Point Exercises

1.  $B = 27.3^\circ$ ;  $b \approx 4.34$ ;  $c \approx 9.45$     2. 994 ft    3.  $29.0^\circ$     4. 60.3 ft    5. a. S  $25^\circ$ E    b. S  $15^\circ$ W    6. a. 4.2 m    b. S  $87.7^\circ$ W

7.  $d = -6 \cos \frac{\pi}{2}t$     8. a. 12 cm    b.  $\frac{1}{8}$  cm per sec    c. 8 sec

### Exercise Set 4.8

1.  $B = 66.5^\circ$ ;  $a \approx 4.35$ ;  $c \approx 10.90$     3.  $B = 37.4^\circ$ ;  $a \approx 42.90$ ;  $b \approx 32.80$     5.  $A = 73.2^\circ$ ;  $a \approx 101.02$ ;  $c \approx 105.52$

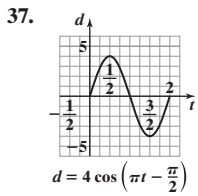
7.  $b \approx 39.95$ ;  $A \approx 37.3^\circ$ ;  $B \approx 52.7^\circ$     9.  $c \approx 26.96$ ;  $A \approx 23.6^\circ$ ;  $B \approx 66.4^\circ$     11.  $a \approx 6.71$ ;  $B \approx 16.6^\circ$ ;  $A \approx 73.4^\circ$     13. N  $15^\circ$  E

15. S  $80^\circ$  W    17.  $d = -6 \cos \frac{\pi}{2}t$     19.  $d = -3 \sin \frac{4\pi}{3}t$

21. a. 5 in.    b.  $\frac{1}{4}$  in. per sec    c. 4 sec    23. a. 6 in.    b. 1 in. per sec    c. 1 sec    25. a.  $\frac{1}{2}$  in.    b. 0.32 in. per sec    c. 3.14 sec

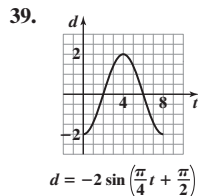
27. a. 5 in.    b.  $\frac{1}{3}$  in. per sec    c. 3 sec    29. 653 units    31. 39 units    33. 298 units    35. 257 units





$d = 4 \cos(\pi t - \frac{\pi}{2})$

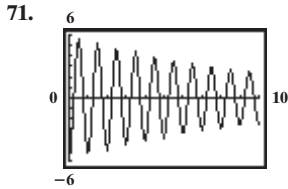
- a. 4 in.    b.  $\frac{1}{2}$  in. per sec  
c. 2 sec    d.  $\frac{1}{2}$



$d = -2 \sin(\frac{\pi}{4}t + \frac{\pi}{2})$

- a. 2 in.    b.  $\frac{1}{8}$  in. per sec  
c. 8 sec    d. -2

41. 2059 ft    43. 695 ft    45. 1376 ft    47.  $15.1^\circ$     49. 33.7 ft  
51. 90 mi north and 120 mi east    53. 13.2 mi    55. N  $53^\circ$  W  
57. N  $89.5^\circ$  E    59.  $d = 6 \sin \pi t$     61.  $d = \sin 528 \pi t$

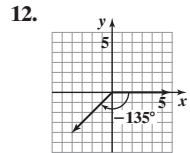
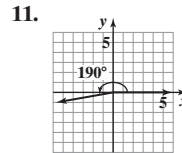
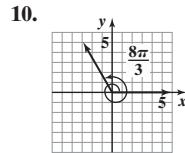
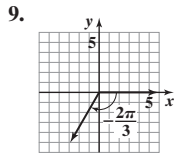
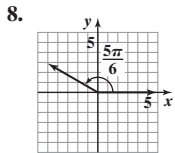


10 complete oscillations

73. does not make sense  
75. does not make sense  
77. 48 ft  
79.  $\csc x$   
80. 1  
81.  $\frac{1 + \sin x}{\cos x}$

Chapter 4 Review Exercises

1. 4.5 radians    2.  $\frac{\pi}{12}$  radians    3.  $\frac{2\pi}{3}$  radians    4.  $\frac{7\pi}{4}$  radians    5.  $300^\circ$     6.  $252^\circ$     7.  $-150^\circ$



13.  $40^\circ$     14.  $275^\circ$     15.  $\frac{5\pi}{4}$     16.  $\frac{7\pi}{6}$     17.  $\frac{4\pi}{3}$     18.  $\frac{15\pi}{2}$  ft  $\approx$  23.56 ft    19.  $20.6\pi$  radians per min    20. 42,412 ft per min

21.  $\sin t = -\frac{3}{5}$ ;  $\cos t = -\frac{4}{5}$ ;  $\tan t = \frac{3}{4}$ ;  $\csc t = -\frac{5}{3}$ ;  $\sec t = -\frac{5}{4}$ ;  $\cot t = \frac{4}{3}$

22.  $\sin t = -\frac{15}{17}$ ;  $\cos t = \frac{8}{17}$ ;  $\tan t = -\frac{15}{8}$ ;  $\csc t = -\frac{17}{15}$ ;  $\sec t = \frac{17}{8}$ ;  $\cot t = -\frac{8}{15}$

23.  $-\frac{2\sqrt{3}}{3}$     24.  $\sqrt{3}$     25. undefined    26. undefined    27.  $\cos t = \frac{\sqrt{21}}{7}$ ;  $\tan t = \frac{2\sqrt{3}}{3}$ ;  $\csc t = \frac{\sqrt{7}}{2}$ ;  $\sec t = \frac{\sqrt{21}}{3}$ ;  $\cot t = \frac{\sqrt{3}}{2}$     28. 1

29. 1    30. -1    31.  $\sin \theta = \frac{5\sqrt{89}}{89}$ ;  $\cos \theta = \frac{3\sqrt{89}}{89}$ ;  $\tan \theta = \frac{5}{8}$ ;  $\csc \theta = \frac{\sqrt{89}}{5}$ ;  $\sec \theta = \frac{\sqrt{89}}{8}$ ;  $\cot \theta = \frac{3}{5}$     32.  $\frac{7}{2}$     33.  $-\frac{1}{2}$     34. 1    35. 1

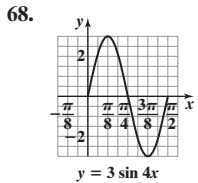
36.  $\cos 20^\circ$     37.  $\sin 0$     38. 42 mm    39. 23 cm    40. 37 in.    41.  $\sqrt{15}$     42. 772 ft    43. 31 m    44.  $56^\circ$     45.  $\sin \theta = -\frac{5\sqrt{26}}{26}$ ;  
 $\cos \theta = -\frac{\sqrt{26}}{26}$ ;  $\tan \theta = 5$ ;  $\csc \theta = -\frac{\sqrt{26}}{5}$ ;  $\sec \theta = -\sqrt{26}$ ;  $\cot \theta = \frac{1}{5}$     46.  $\sin \theta = -1$ ;  $\cos \theta = 0$ ;  $\tan \theta$  is undefined;  $\csc \theta = -1$ ;  $\sec \theta$  is undefined;

$\cot \theta = 0$     47. quadrant I    48. quadrant III    49.  $\sin \theta = -\frac{\sqrt{21}}{5}$ ;  $\tan \theta = -\frac{\sqrt{21}}{2}$ ;  $\csc \theta = -\frac{5\sqrt{21}}{21}$ ;  $\sec \theta = \frac{5}{2}$ ;  $\cot \theta = -\frac{2\sqrt{21}}{21}$

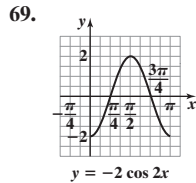
50.  $\sin \theta = \frac{\sqrt{10}}{10}$ ;  $\cos \theta = -\frac{3\sqrt{10}}{10}$ ;  $\csc \theta = \sqrt{10}$ ;  $\sec \theta = -\frac{\sqrt{10}}{3}$ ;  $\cot \theta = -3$     51.  $\sin \theta = -\frac{\sqrt{10}}{10}$ ;  $\cos \theta = -\frac{3\sqrt{10}}{10}$ ;  $\tan \theta = \frac{1}{3}$ ;  $\csc \theta = -\sqrt{10}$ ;

$\sec \theta = -\frac{\sqrt{10}}{3}$     52.  $85^\circ$     53.  $\frac{3\pi}{8}$     54.  $50^\circ$     55.  $\frac{\pi}{6}$     56.  $\frac{\pi}{3}$     57.  $-\frac{\sqrt{3}}{2}$     58.  $-\sqrt{3}$     59.  $\sqrt{2}$     60.  $\frac{\sqrt{3}}{2}$     61.  $-\sqrt{3}$

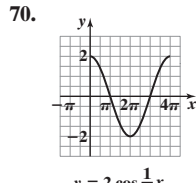
62.  $-\frac{2\sqrt{3}}{3}$     63.  $-\frac{\sqrt{3}}{2}$     64.  $\frac{\sqrt{2}}{2}$     65. 1    66.  $-\frac{\sqrt{3}}{2}$     67.  $\frac{\sqrt{3}}{2}$



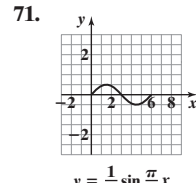
$y = 3 \sin 4x$



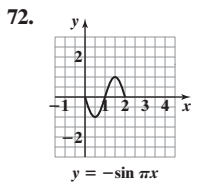
$y = -2 \cos 2x$



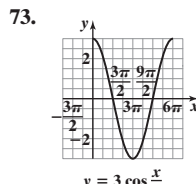
$y = 2 \cos \frac{1}{2}x$



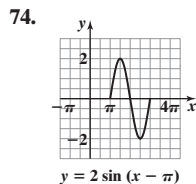
$y = \frac{1}{2} \sin \frac{\pi}{3}x$



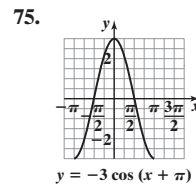
$y = -\sin \pi x$



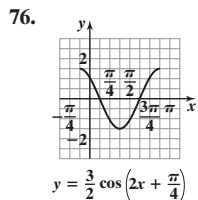
$y = 3 \cos \frac{x}{3}$



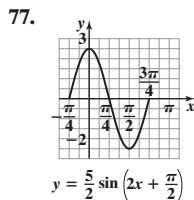
$y = 2 \sin(x - \pi)$



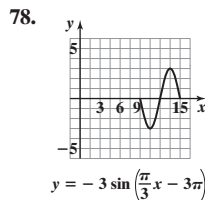
$y = -3 \cos(x + \pi)$



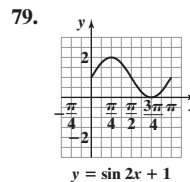
$y = \frac{3}{2} \cos\left(2x + \frac{\pi}{4}\right)$



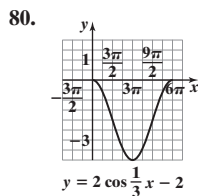
$y = \frac{5}{2} \sin\left(2x + \frac{\pi}{2}\right)$



$y = -3 \sin\left(\frac{\pi}{3}x - 3\pi\right)$

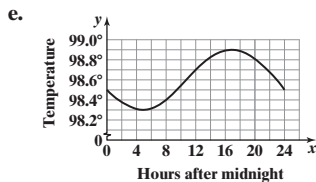


$y = \sin 2x + 1$

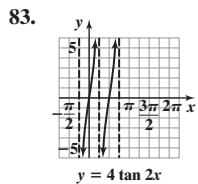


$y = 2 \cos\left(\frac{1}{3}x - 2\right)$

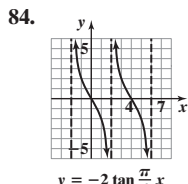
81. a.  $\approx 98.52^\circ$     b. 24 hr    c. 5:00 p.m.;  $98.9^\circ$     d. 5:00 a.m.;  $98.3^\circ$



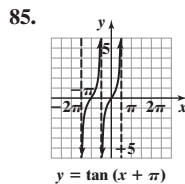
82. blue:  $y = \sin \frac{\pi}{240}x$ ; red:  $y = \sin \frac{\pi}{320}x$



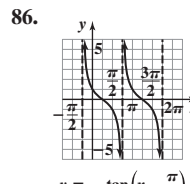
$y = 4 \tan 2x$



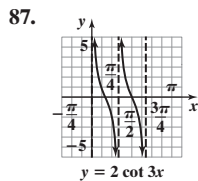
$y = -2 \tan \frac{\pi}{4}x$



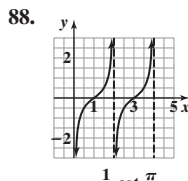
$y = \tan(x + \pi)$



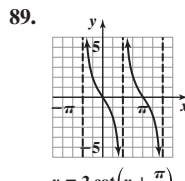
$y = -\tan\left(x - \frac{\pi}{4}\right)$



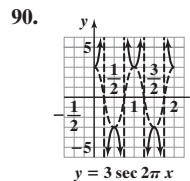
$y = 2 \cot 3x$



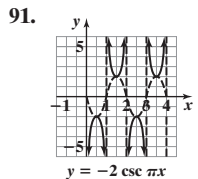
$y = -\frac{1}{2} \cot \frac{\pi}{2}x$



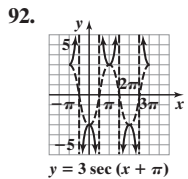
$y = 2 \cot\left(x + \frac{\pi}{2}\right)$



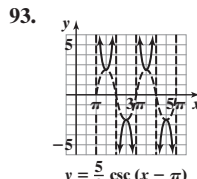
$y = 3 \sec 2\pi x$



$y = -2 \csc \pi x$



$y = 3 \sec(x + \pi)$



$y = \frac{5}{2} \csc(x - \pi)$

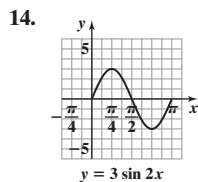
94.  $\frac{\pi}{2}$     95. 0    96.  $\frac{\pi}{4}$   
 97.  $-\frac{\pi}{3}$     98.  $\frac{2\pi}{3}$     99.  $-\frac{\pi}{6}$   
 100.  $\frac{\sqrt{2}}{2}$     101. 1    102.  $-\frac{\sqrt{3}}{3}$   
 103.  $-\frac{\sqrt{3}}{3}$     104. 2    105.  $\frac{4}{5}$     106.  $\frac{4}{5}$

107.  $-\frac{3}{4}$     108.  $-\frac{3}{4}$     109.  $-\frac{\sqrt{10}}{10}$     110.  $\frac{\pi}{3}$     111.  $\frac{\pi}{3}$     112.  $-\frac{\pi}{6}$     113.  $\frac{2\sqrt{x^2+4}}{x^2+4}$     114.  $\frac{x\sqrt{x^2-1}}{x^2-1}$

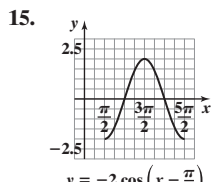
115.  $B \approx 67.7^\circ; a \approx 37.9; b \approx 9.25$     116.  $A \approx 52.6^\circ; a \approx 7.85; c \approx 9.88$     117.  $A \approx 16.6^\circ; B \approx 73.4^\circ; b \approx 6.71$   
 118.  $A \approx 21.3^\circ; B \approx 68.7^\circ; c \approx 3.86$     119. 38 ft    120. 90 yd    121. 21.7 ft    122. N  $35^\circ$  E    123. S  $35^\circ$  W    124. 24.6 mi  
 125. a. 1282.2 mi    b. S  $74^\circ$  E    126. a. 20 cm    b.  $\frac{1}{8}$  cm per sec    c. 8 sec    127. a.  $\frac{1}{2}$  cm    b. 0.64 cm per sec    c. 1.57 sec  
 128.  $d = -30 \cos \pi t$     129.  $d = -\frac{1}{4} \sin \frac{2\pi}{5}t$

Chapter 4 Test

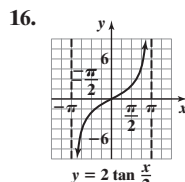
1.  $\frac{3\pi}{4}$  radians    2.  $\frac{25\pi}{3}$  ft  $\approx 26.18$  ft    3. a.  $\frac{4\pi}{3}$     b.  $\frac{\pi}{3}$   
 4.  $\sin \theta = \frac{5\sqrt{29}}{29}; \cos \theta = -\frac{2\sqrt{29}}{29}; \tan \theta = -\frac{5}{2}; \csc \theta = \frac{\sqrt{29}}{5}; \sec \theta = -\frac{\sqrt{29}}{2}; \cot \theta = -\frac{2}{5}$     5. quadrant III  
 6.  $\sin \theta = -\frac{2\sqrt{2}}{3}; \tan \theta = -2\sqrt{2}; \csc \theta = -\frac{3\sqrt{2}}{4}; \sec \theta = 3; \cot \theta = -\frac{\sqrt{2}}{4}$     7.  $\frac{\sqrt{3}}{6}$     8.  $-\sqrt{3}$     9.  $-\frac{\sqrt{2}}{2}$     10. -2    11.  $\frac{\sqrt{3}}{3}$   
 12.  $\sqrt{3}$     13. a.  $-a + b$  or  $b - a$     b.  $\frac{a}{b} - \frac{1}{b}$  or  $\frac{a-1}{b}$



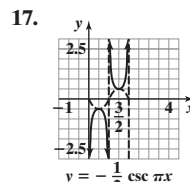
$y = 3 \sin 2x$



$y = -2 \cos\left(x - \frac{\pi}{2}\right)$



$y = 2 \tan \frac{\pi}{2}x$

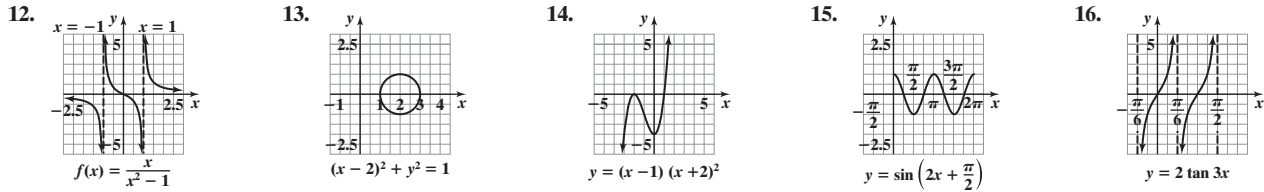


$y = -\frac{1}{2} \csc \pi x$

18.  $-\sqrt{3}$  19.  $\frac{\sqrt{9-x^2}}{3}$  20.  $B = 69^\circ; a = 4.7; b = 12.1$  21. 23 yd 22.  $36.1^\circ$  23. N  $80^\circ$  W 24. a. 6 in. b.  $\frac{1}{2}$  in. per sec  
 c. 2 sec 25. Trigonometric functions are periodic.

**Cumulative Review Exercises (Chapters P–4)**

1.  $\{-3, 6\}$  2.  $\{-5, -2, 2\}$  3.  $\{4\}$  4.  $\{7\}$  5.  $\{-1, 2, 3\}$  6.  $-3 \leq x \leq 8$  7.  $f^{-1}(x) = x^2 + 6$   
 8.  $4x^2 - \frac{14}{5}x - \frac{17}{25} + \frac{284}{125x + 50}$  9.  $\log 1000 = 3$  10.  $280^\circ$  11. 3 positive real roots; 1 negative real root



17. 48 performances 18. a.  $A = 110e^{0.1251t}$  where  $t$  is the number of years after 2000 b. 8 years after 2000, or 2008 19. 1540 Btu per hr  
 20.  $76^\circ$

**CHAPTER 5**

**Section 5.1**

**Check Point Exercises**

1.  $\csc x \tan x = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \sec x$

2.  $\cos x \cot x + \sin x = \cos x \cdot \frac{\cos x}{\sin x} + \sin x = \frac{\cos^2 x}{\sin x} + \sin x = \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$

3.  $\sin x - \sin x \cos^2 x = \sin x(1 - \cos^2 x) = \sin x \cdot \sin^2 x = \sin^3 x$  4.  $\frac{1 + \cos \theta}{\sin \theta} = \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta} = \csc \theta + \cot \theta$

5.  $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = \frac{\sin x(\sin x)}{(1 + \cos x)(\sin x)} + \frac{(1 + \cos x)(1 + \cos x)}{(\sin x)(1 + \cos x)} = \frac{\sin^2 x + 1 + 2 \cos x + \cos^2 x}{(1 + \cos x)(\sin x)}$   
 $= \frac{\sin^2 x + \cos^2 x + 1 + 2 \cos x}{(1 + \cos x)(\sin x)} = \frac{1 + 1 + 2 \cos x}{(1 + \cos x)(\sin x)} = \frac{2 + 2 \cos x}{(1 + \cos x)(\sin x)} = \frac{2(1 + \cos x)}{(1 + \cos x)(\sin x)} = \frac{2}{\sin x} = 2 \csc x$

6.  $\frac{\cos x}{1 + \sin x} = \frac{\cos x(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} = \frac{\cos x(1 - \sin x)}{1 - \sin^2 x} = \frac{\cos x(1 - \sin x)}{\cos^2 x} = \frac{1 - \sin x}{\cos x}$  7.  $\frac{\sec x + \csc(-x)}{\sec x \csc x} = \frac{\sec x - \csc x}{\sec x \csc x}$   
 $= \frac{\frac{1}{\cos x} - \frac{1}{\sin x}}{\frac{1}{\cos x} \cdot \frac{1}{\sin x}} = \frac{\frac{\sin x - \cos x}{\cos x \cdot \sin x}}{\frac{1}{\cos x \cdot \sin x}} = \frac{\sin x - \cos x}{\cos x \cdot \sin x} \cdot \frac{\cos x \cdot \sin x}{1} = \sin x - \cos x$

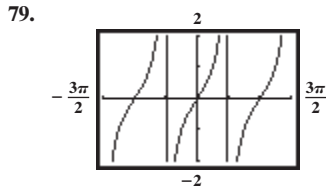
8. Left side:  $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = \frac{1(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)} + \frac{1(1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} = \frac{1 - \sin \theta + 1 + \sin \theta}{(1 + \sin \theta)(1 - \sin \theta)} = \frac{2}{1 - \sin^2 \theta}$   
 Right side:  $2 + 2 \tan^2 \theta = 2 + 2 \left(\frac{\sin^2 \theta}{\cos^2 \theta}\right) = \frac{2 \cos^2 \theta}{\cos^2 \theta} + \frac{2 \sin^2 \theta}{\cos^2 \theta} = \frac{2 \cos^2 \theta + 2 \sin^2 \theta}{\cos^2 \theta} = \frac{2(\cos^2 \theta + \sin^2 \theta)}{\cos^2 \theta} = \frac{2}{\cos^2 \theta} = \frac{2}{1 - \sin^2 \theta}$

**Exercise Set 5.1**

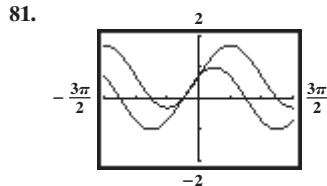
For Exercises 1–59, proofs may vary.

61.  $\cos x$ ; Proofs may vary. 63.  $2 \sin x$ ; Proofs may vary. 65.  $2 \sec x$ ; Proofs may vary. 67.  $\frac{1}{\cos x}$  69.  $\frac{1}{\cos x}$  71.  $2 \csc^2 x - 1$

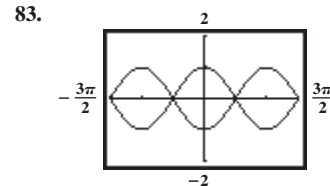
73.  $\sec x \tan x$



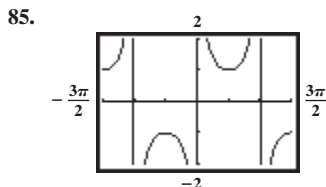
Proofs may vary.



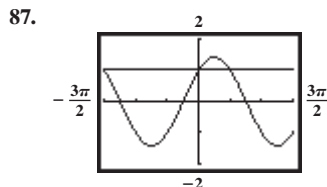
Values for  $x$  may vary.



Values for  $x$  may vary.



Proofs may vary.



Values for  $x$  may vary.

89. makes sense 91. does not make sense

For Exercises 92–95, proofs may vary.

98.  $\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{3}}{2}; 0; 1$     99. a. no    b. yes    100. a. no    b. yes

**Section 5.2**

**Check Point Exercises**

1.  $\frac{\sqrt{3}}{2}$     2.  $\frac{\sqrt{3}}{2}$     3.  $\frac{\cos(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\cos \alpha \cos \beta + \sin \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\cos \alpha}{\cos \alpha} \cdot \frac{\cos \beta}{\cos \beta} + \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta} = 1 + \tan \alpha \tan \beta$

4.  $\frac{\sqrt{2} + \sqrt{6}}{4}$     5. a.  $\cos \alpha = -\frac{3}{5}$     b.  $\cos \beta = \frac{\sqrt{3}}{2}$     c.  $\frac{-3\sqrt{3} - 4}{10}$     d.  $\frac{4\sqrt{3} - 3}{10}$     6. a.  $y = \sin x$

b.  $\cos\left(x + \frac{3\pi}{2}\right) = \cos x \cos \frac{3\pi}{2} - \sin x \sin \frac{3\pi}{2} = \cos x \cdot 0 - \sin x \cdot (-1) = \sin x$

7.  $\tan(x + \pi) = \frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} = \frac{\tan x + 0}{1 - \tan x \cdot 0} = \frac{\tan x}{1} = \tan x$

**Exercise Set 5.2**

1.  $\frac{\sqrt{6} + \sqrt{2}}{4}$     3.  $\frac{\sqrt{2} - \sqrt{6}}{4}$     5. a.  $\alpha = 50^\circ, \beta = 20^\circ$     b.  $\cos 30^\circ$     c.  $\frac{\sqrt{3}}{2}$     7. a.  $\alpha = \frac{5\pi}{12}, \beta = \frac{\pi}{12}$     b.  $\cos \frac{\pi}{3}$     c.  $\frac{1}{2}$   
 For Exercises 9 and 11, proofs may vary.    13.  $\frac{\sqrt{6} - \sqrt{2}}{4}$     15.  $\frac{\sqrt{6} + \sqrt{2}}{4}$     17.  $-\frac{\sqrt{6} + \sqrt{2}}{4}$     19.  $\frac{\sqrt{6} - \sqrt{2}}{4}$     21.  $\frac{\sqrt{3} + 1}{\sqrt{3} - 1}$     23.  $\frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

25.  $\sin 30^\circ; \frac{1}{2}$     27.  $\tan 45^\circ; 1$     29.  $\sin \frac{\pi}{6}; \frac{1}{2}$     31.  $\tan \frac{\pi}{6}; \frac{\sqrt{3}}{3}$

For Exercises 33–55, proofs may vary.

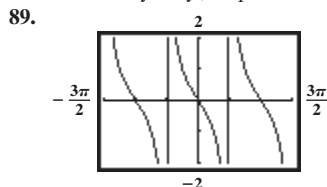
57. a.  $-\frac{63}{65}$     b.  $-\frac{16}{65}$     c.  $\frac{16}{63}$     59. a.  $-\frac{4 + 6\sqrt{2}}{15}$     b.  $\frac{3 - 8\sqrt{2}}{15}$     c.  $\frac{54 - 25\sqrt{2}}{28}$   
 61. a.  $-\frac{8\sqrt{3} + 15}{34}$     b.  $\frac{15\sqrt{3} - 8}{34}$     c.  $\frac{480 - 289\sqrt{3}}{33}$     63. a.  $-\frac{4 + 3\sqrt{15}}{20}$     b.  $\frac{-3 + 4\sqrt{15}}{20}$     c.  $\frac{3 - 4\sqrt{15}}{4 + 3\sqrt{15}}$

65. a.  $y = \sin x$     b.  $\sin(\pi - x) = \sin \pi \cos x - \cos \pi \sin x = 0 \cdot \cos x - (-1) \sin x = \sin x$     67. a.  $y = 2 \cos x$

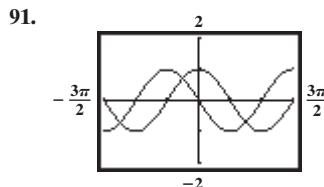
b.  $\sin\left(x + \frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2} - x\right) = \sin x \cos \frac{\pi}{2} + \cos x \sin \frac{\pi}{2} + \sin \frac{\pi}{2} \cos x - \cos \frac{\pi}{2} \sin x = \sin x \cdot 0 + \cos x \cdot 1 + 1 \cdot \cos x - 0 \cdot \sin x$

$= \cos x + \cos x = 2 \cos x$     69.  $\cos \alpha$     71.  $\tan \beta$     73.  $\cos \frac{\pi}{3} = \frac{1}{2}$     75.  $\cos 3x$ ; Proofs may vary.    77.  $\sin \frac{x}{2}$ ; Proofs may vary.

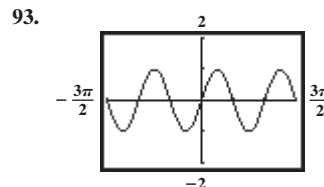
79. Proofs may vary.; amplitude is  $\sqrt{13}$ ; period is  $2\pi$



Proofs may vary.



Values for  $x$  may vary.



Proofs may vary.

95. makes sense    97. makes sense.    99.  $\frac{4\sqrt{3} + 3}{10}$     101.  $-\frac{33}{65}$     103.  $y\sqrt{1 - x^2} + x\sqrt{1 - y^2}$     105.  $\frac{xy + (\sqrt{1 - x^2})(\sqrt{1 - y^2})}{y\sqrt{1 - x^2} - x\sqrt{1 - y^2}}$

107.  $\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{3}}{2}$     108. a. no    b. yes    109. a. no    b. yes

**Section 5.3**

**Check Point Exercises**

1. a.  $-\frac{24}{25}$     b.  $-\frac{7}{25}$     c.  $\frac{24}{7}$     2.  $\frac{\sqrt{3}}{2}$     3.  $\sin 3\theta = \sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta = 2 \sin \theta \cos \theta \cos \theta + (2\cos^2 \theta - 1)\sin \theta = 2 \sin \theta \cos^2 \theta + 2 \sin \theta \cos^2 \theta - \sin \theta = 4 \sin \theta \cos^2 \theta - \sin \theta = 4 \sin \theta(1 - \sin^2 \theta) - \sin \theta = 4 \sin \theta - 4 \sin^3 \theta - \sin \theta = 3 \sin \theta - 4 \sin^3 \theta$

4.  $\sin^4 x = (\sin^2 x)^2 = \left(\frac{1 - \cos 2x}{2}\right)^2 = \frac{1 - 2 \cos 2x + \cos^2 2x}{4} = \frac{1}{4} - \frac{1}{2} \cos 2x + \frac{1}{4} \cos^2 2x$

$= \frac{1}{4} - \frac{1}{2} \cos 2x + \frac{1}{4} \left(\frac{1 + \cos 2(2x)}{2}\right) = \frac{1}{4} - \frac{1}{2} \cos 2x + \frac{1}{8} + \frac{1}{8} \cos 4x = \frac{3}{8} - \frac{1}{2} \cos 2x + \frac{1}{8} \cos 4x$     5.  $-\frac{\sqrt{2 - \sqrt{3}}}{2}$

6.  $\frac{\sin \theta}{1 + \cos \theta} = \frac{2 \sin \theta \cos \theta}{1 + (1 - 2 \sin^2 \theta)} = \frac{2 \sin \theta \cos \theta}{2 - 2 \sin^2 \theta} = \frac{2 \sin \theta \cos \theta}{2(1 - \sin^2 \theta)} = \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta$

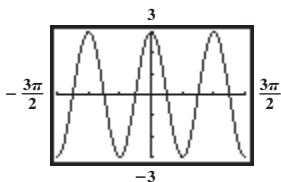
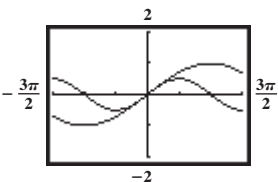
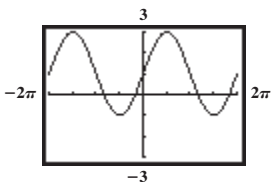
7.  $\frac{\sec \alpha}{\sec \alpha \csc \alpha + \csc \alpha} = \frac{1}{\cos \alpha} \cdot \frac{1}{\frac{1}{\sin \alpha} + \frac{1}{\sin \alpha}} = \frac{1}{\cos \alpha} \cdot \frac{1}{\frac{2}{\sin \alpha}} = \frac{1}{\cos \alpha} \cdot \frac{\sin \alpha}{2} = \frac{\sin \alpha}{2 \cos \alpha} = \frac{1}{2} \tan \alpha$

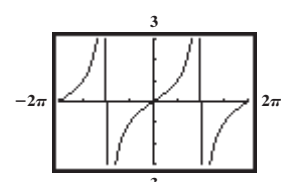
**Exercise Set 5.3**

1.  $\frac{24}{25}$    3.  $\frac{24}{7}$    5.  $\frac{527}{625}$    7. a.  $-\frac{240}{289}$    b.  $-\frac{161}{289}$    c.  $\frac{240}{161}$    9. a.  $-\frac{336}{625}$    b.  $\frac{527}{625}$    c.  $-\frac{336}{527}$   
 11. a.  $\frac{4}{5}$    b.  $\frac{3}{5}$    c.  $\frac{4}{3}$    13. a.  $\frac{720}{1681}$    b.  $\frac{1519}{1681}$    c.  $\frac{720}{1519}$    15.  $\frac{1}{2}$    17.  $-\frac{\sqrt{3}}{2}$    19.  $\frac{\sqrt{2}}{2}$    21.  $\frac{\sqrt{3}}{3}$

- For Exercises 23–33, proofs may vary.   35.  $\frac{9}{4} - 3 \cos 2x + \frac{3}{4} \cos 4x$    37.  $\frac{1}{8} - \frac{1}{8} \cos 4x$    39.  $\frac{\sqrt{2} - \sqrt{3}}{2}$    41.  $\frac{\sqrt{2 + \sqrt{2}}}{2}$   
 43.  $2 + \sqrt{3}$    45.  $-\sqrt{2} + 1$    47.  $\frac{\sqrt{10}}{10}$    49.  $\frac{1}{3}$    51.  $\frac{7\sqrt{2}}{10}$    53.  $\frac{3}{5}$    55. a.  $\frac{2\sqrt{5}}{5}$    b.  $-\frac{\sqrt{5}}{5}$    c.  $-2$    57. a.  $\frac{3\sqrt{13}}{13}$    b.  $\frac{2\sqrt{13}}{13}$    c.  $\frac{3}{2}$

- For Exercises 59–67, proofs may vary.  
 69.  $\cos 2x$ ; Proofs may vary.   71.  $1 + \sin x$ ; Proofs may vary.   73.  $\sec x$ ; Proofs may vary.   75.  $2 \csc 2x$ ; Proofs may vary.  
 77.  $\sin 3x$ ; Proofs may vary.   79. a.  $\frac{v_0^2}{32} \cdot \sin 2\theta$    b.  $\theta = \frac{\pi}{4}$    81.  $\sqrt{2 - \sqrt{2}} \cdot (2 + \sqrt{2}) \approx 2.6$

95.    97.    99.   
 Proofs may vary.   Values for x may vary.   a.  $y = 1 + 2 \sin x$   
 b. Proofs may vary.

101.    a.  $y = \tan \frac{x}{2}$    b. Proofs may vary.   103. does not make sense   105. does not makes sense  
 107.  $\frac{\sqrt{3}}{2}$    109.  $\frac{9}{10}$    111.  $2x\sqrt{1 - x^2}$    113. Both sides equal  $\frac{\sqrt{3}}{4}$ .   114. Both sides equal 0.  
 115. Both sides equal 0.

**Mid-Chapter 5 Check Point**

- For Exercises 1–18, proofs may vary.  
 19.  $\frac{33}{65}$    20.  $-\frac{16}{63}$    21.  $-\frac{24}{25}$    22.  $-\frac{\sqrt{26}}{26}$    23.  $-\frac{\sqrt{6} + \sqrt{2}}{4}$    24.  $\frac{\sqrt{3}}{2}$    25.  $\frac{1}{2}$    26.  $\sqrt{\frac{\sqrt{2} - 1}{\sqrt{2} + 1}}$

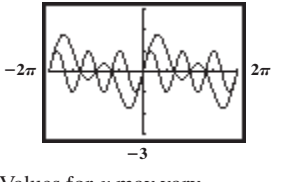
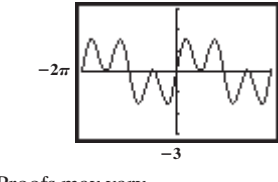
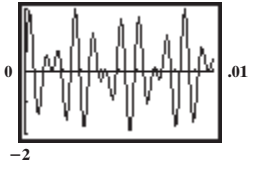
**Section 5.4**

**Check Point Exercises**

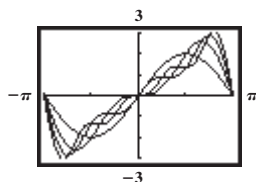
1. a.  $\frac{1}{2}[\cos 3x - \cos 7x]$    b.  $\frac{1}{2}[\cos 6x + \cos 8x]$    2. a.  $2 \sin 5x \cos 2x$    b.  $2 \cos \frac{5x}{2} \cos \frac{x}{2}$   
 3.  $\frac{\cos 3x - \cos x}{\sin 3x + \sin x} = \frac{-2 \sin \left(\frac{3x+x}{2}\right) \sin \left(\frac{3x-x}{2}\right)}{2 \sin \left(\frac{3x+x}{2}\right) \cos \left(\frac{3x-x}{2}\right)} = \frac{-2 \sin \left(\frac{4x}{2}\right) \sin \left(\frac{2x}{2}\right)}{2 \sin \left(\frac{4x}{2}\right) \cos \left(\frac{2x}{2}\right)} = \frac{-2 \sin 2x \sin x}{2 \sin 2x \cos x} = -\frac{\sin x}{\cos x} = -\tan x$

**Exercise Set 5.4**

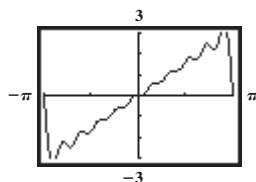
1. product; difference   3. product; sum   5.  $\frac{1}{2}[\cos 4x - \cos 8x]$    7.  $\frac{1}{2}[\cos 4x + \cos 10x]$    9.  $\frac{1}{2}[\sin 3x - \sin x]$    11.  $\frac{1}{2}[\sin 2x - \sin x]$   
 13. sum; product   15. sum; product   17.  $2 \sin 4x \cos 2x$    19.  $2 \sin 2x \cos 5x$    21.  $2 \cos 3x \cos x$    23.  $2 \sin \frac{3x}{2} \cos \frac{x}{2}$   
 25.  $2 \cos x \cos \frac{x}{2}$    27.  $\frac{\sqrt{6}}{2}$    29.  $-\frac{\sqrt{2}}{2}$    For Exercises 31–37, proofs may vary.  
 39. a.  $y = \cos x$    b. Proofs may vary.   41. a.  $y = \tan 2x$    b. Proofs may vary.  
 43. a.  $y = -\cot 2x$    b. Proofs may vary.   45. a.  $y = \sin 1704\pi t + \sin 2418\pi t$    b.  $2 \sin 2061\pi t \cdot \cos 357\pi t$   
 53.   55.   57.

53.    55.    57.   
 Values for x may vary.   Proofs may vary.

59. a.



b.



c.  $\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \dots$

61. makes sense    63. makes sense

For Exercises 65–69, proofs may vary.

71.  $\left\{-\frac{1}{2}, 2\right\}$     72.  $\{-\sqrt{3}, 0, \sqrt{3}\}$     73.  $\left\{\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right\}$

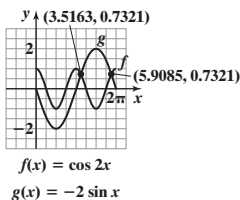
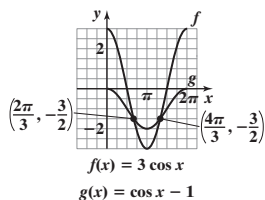
### Section 5.5

#### Check Point Exercises

1.  $x = \frac{\pi}{3} + 2n\pi$  or  $x = \frac{2\pi}{3} + 2n\pi$ , where  $n$  is any integer.    2.  $\frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}$     3.  $\frac{\pi}{2}$     4.  $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$     5.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$     6.  $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}$   
 7.  $\frac{\pi}{3}, \frac{5\pi}{3}$     8.  $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$     9.  $\frac{3\pi}{4}, \frac{7\pi}{4}$     10.  $\frac{\pi}{2}, \pi$     11. a. 1.2592, 4.4008    b. 3.3752, 6.0496    12. 2.3423, 3.9409

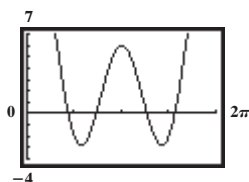
#### Exercise Set 5.5

1. Solution    3. Not a solution    5. Solution    7. Solution    9. Not a solution    11.  $x = \frac{\pi}{3} + 2n\pi$  or  $x = \frac{2\pi}{3} + 2n\pi$ , where  $n$  is any integer.  
 13.  $x = \frac{\pi}{4} + n\pi$ , where  $n$  is any integer.    15.  $x = \frac{2\pi}{3} + 2n\pi$  or  $x = \frac{4\pi}{3} + 2n\pi$ , where  $n$  is any integer.    17.  $x = n\pi$ , where  $n$  is any integer.  
 19.  $x = \frac{5\pi}{6} + 2n\pi$  or  $x = \frac{7\pi}{6} + 2n\pi$ , where  $n$  is any integer.    21.  $\theta = \frac{\pi}{6} + 2n\pi$  or  $\theta = \frac{5\pi}{6} + 2n\pi$ , where  $n$  is any integer.  
 23.  $\theta = \frac{3\pi}{2} + 2n\pi$ , where  $n$  is any integer.    25.  $\frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$     27.  $\frac{5\pi}{24}, \frac{7\pi}{24}, \frac{17\pi}{24}, \frac{19\pi}{24}, \frac{29\pi}{24}, \frac{31\pi}{24}, \frac{41\pi}{24}, \frac{43\pi}{24}$     29.  $\frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$   
 31. 0    33. no solution    35.  $\frac{4\pi}{9}, \frac{8\pi}{9}, \frac{16\pi}{9}$     37.  $0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}$     39.  $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$     41.  $\frac{2\pi}{3}, \pi, \frac{4\pi}{3}$     43.  $\frac{3\pi}{2}$     45.  $\frac{\pi}{2}, \frac{3\pi}{2}$   
 47.  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$     49.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$     51.  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$     53.  $\frac{\pi}{4}, \pi, \frac{5\pi}{4}$     55.  $\frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$     57.  $\frac{\pi}{4}, \frac{5\pi}{4}$     59.  $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$   
 61.  $0, \pi$     63.  $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$     65.  $\pi$     67.  $\frac{\pi}{6}, \frac{5\pi}{6}$     69.  $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$     71.  $0, \frac{2\pi}{3}, \frac{4\pi}{3}$     73.  $\frac{2\pi}{3}, \frac{4\pi}{3}$     75.  $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$     77.  $0, \frac{\pi}{2}$   
 79.  $\frac{\pi}{4}, \frac{3\pi}{4}$     81.  $\frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}$     83. 0    85. 0.9695, 2.1721    87. 1.9823, 4.3009    89. 1.8925, 5.0341    91. 2.2370, 4.0461  
 93. 0.4636, 0.9828, 3.6052, 4.1244    95. 0.3876, 2.7540, 3.5292, 5.8956    97.  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$     99.  $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$     101.  $\frac{\pi}{6}, \frac{11\pi}{6}$   
 103. 1.7798, 4.9214    105.  $\frac{\pi}{2}$     107.  $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$     109.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$     111. 0.7495, 5.5337    113.  $\frac{7\pi}{6}, \frac{11\pi}{6}$   
 115. 2.1588,  $\frac{3\pi}{4}, 5.3004, \frac{7\pi}{4}$     117.  $\left(\frac{2\pi}{3}, -\frac{3}{2}\right), \left(\frac{4\pi}{3}, -\frac{3}{2}\right)$     119. (3.5163, 0.7321), (5.9085, 0.7321)    121.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$



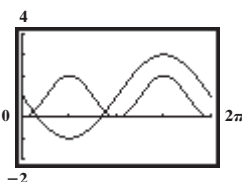
129. 0.4 sec and 2.1 sec    131. 49 days and 292 days    133.  $t = 2 + 6n$  or  $t = 4 + 6n$  where  $n$  is any nonnegative integer.    135.  $21^\circ$  or  $69^\circ$ .

147.



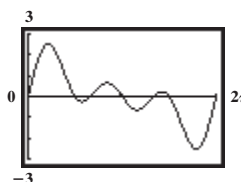
$x = 1.37, x = 2.30, x = 3.98,$   
 or  $x = 4.91$

149.



$x = 0.37$  or  $x = 2.77$

151.



$x = 0, x = 1.57, x = 2.09, x = 3.14, x = 4.19,$  or  $x = 4.71$

153. makes sense    155. does not make sense    157. false    159. false    161.  $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$     163.  $a \approx 45.2$

164.  $B \approx 31.5^\circ$     165. no solution or  $\emptyset$

#### Chapter 5 Review Exercises

For Exercises 1–13, proofs may vary.

14.  $\frac{\sqrt{6} - \sqrt{2}}{4}$     15.  $\frac{\sqrt{2} - \sqrt{6}}{4}$     16.  $2 - \sqrt{3}$     17.  $\sqrt{3} + 2$     18.  $\frac{1}{2}$     19.  $\frac{1}{2}$

For Exercises 19–31, proofs may vary.

32. a.  $y = \cos x$     b.  $\sin\left(x - \frac{3\pi}{2}\right) = \sin x \cos \frac{3\pi}{2} - \cos x \sin \frac{3\pi}{2} = \sin x \cdot 0 - \cos x \cdot -1 = \cos x$

33. a.  $y = -\sin x$     b.  $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos \frac{\pi}{2} - \sin x \sin \frac{\pi}{2} = \cos x \cdot 0 - \sin x \cdot 1 = -\sin x$

34. a.  $y = \tan x$     b.  $y = \frac{\tan x - 1}{1 - \cot x} = \frac{\frac{\sin x}{\cos x} - 1}{1 - \frac{\cos x}{\sin x}} = \frac{\frac{\sin x - \cos x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}} = \frac{\sin x - \cos x}{\cos x} \cdot \frac{\sin x}{\sin x - \cos x} = \frac{\sin x}{\cos x} = \tan x$

35. a.  $\frac{33}{65}$     b.  $\frac{16}{65}$     c.  $-\frac{33}{56}$     d.  $\frac{24}{25}$     e.  $\frac{2\sqrt{13}}{13}$     36. a.  $-\frac{63}{65}$     b.  $-\frac{56}{65}$     c.  $\frac{63}{16}$     d.  $\frac{24}{25}$     e.  $\frac{5\sqrt{26}}{26}$

37. a. 1    b.  $-\frac{3}{5}$     c. undefined    d.  $-\frac{3}{5}$     e.  $\frac{\sqrt{10+3\sqrt{10}}}{2\sqrt{5}}$     38. a. 1    b.  $\frac{4\sqrt{2}}{9}$     c. undefined    d.  $\frac{4\sqrt{2}}{9}$     e.  $-\frac{\sqrt{3}}{3}$

39.  $\frac{\sqrt{3}}{2}$     40.  $-\frac{\sqrt{3}}{3}$     41.  $\frac{\sqrt{2-\sqrt{2}}}{2}$     42.  $2 - \sqrt{3}$     43.  $\frac{1}{2}[\cos 2x - \cos 10x]$     44.  $\frac{1}{2}[\sin 10x + \sin 4x]$     45.  $-2 \sin x \cos 3x$

46.  $\frac{\sqrt{6}}{2}$     47. Proofs may vary.    48. Proofs may vary.    49. a.  $y = \cot x$     b. Proofs may vary.

50.  $x = \frac{2\pi}{3} + 2n\pi$  or  $x = \frac{4\pi}{3} + 2n\pi$ , where  $n$  is any integer.    51.  $x = \frac{\pi}{4} + 2n\pi$  or  $x = \frac{3\pi}{4} + 2n\pi$ , where  $n$  is any integer.

52.  $x = \frac{7\pi}{6} + 2n\pi$  or  $x = \frac{11\pi}{6} + 2n\pi$ , where  $n$  is any integer.    53.  $x = \frac{\pi}{6} + n\pi$ , where  $n$  is any integer.    54.  $\frac{\pi}{2}, \frac{3\pi}{2}$     55.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}$

56.  $\frac{3\pi}{2}$     57.  $0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$     58.  $\pi$     59.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$     60.  $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$     61.  $0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$     62.  $0, \frac{\pi}{6}, \pi, \frac{11\pi}{6}$     63.  $0, \pi$

64. 3.7890, 5.6358    65. 0.6847, 2.4569, 3.8263, 5.5985    66.  $\frac{\pi}{4}, 1.2490, \frac{5\pi}{4}, 4.3906$     67. 0.8959, 2.2457

68.  $t = \frac{2}{3} + 4n$  or  $t = \frac{10}{3} + 4n$ , where  $n$  is any integer.    69.  $12^\circ$  or  $78^\circ$

**Chapter 5 Test**

1.  $-\frac{63}{65}$     2.  $\frac{56}{33}$     3.  $-\frac{24}{25}$     4.  $\frac{3\sqrt{13}}{13}$     5.  $\frac{\sqrt{6} + \sqrt{2}}{4}$     6.  $\cos x \csc x = \cos x \cdot \frac{1}{\sin x} = \frac{\cos x}{\sin x} = \cot x$

7.  $\frac{\sec x}{\cot x + \tan x} = \frac{\frac{1}{\cos x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}} = \frac{1}{\cos x} \cdot \frac{\sin x \cos x}{1} = \sin x$

8.  $1 - \frac{\cos^2 x}{1 + \sin x} = 1 - \frac{(1 - \sin^2 x)}{1 + \sin x} = 1 - \frac{(1 + \sin x)(1 - \sin x)}{1 + \sin x} = 1 - (1 - \sin x) = \sin x$

9.  $\cos\left(\theta + \frac{\pi}{2}\right) = \cos \theta \cos \frac{\pi}{2} - \sin \theta \sin \frac{\pi}{2} = \cos \theta \cdot 0 - \sin \theta \cdot 1 = -\sin \theta$

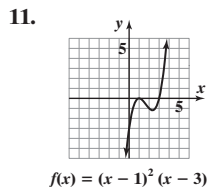
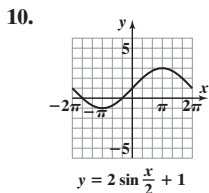
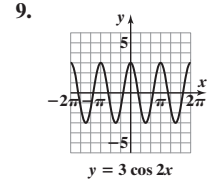
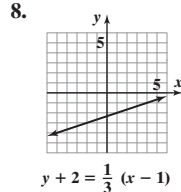
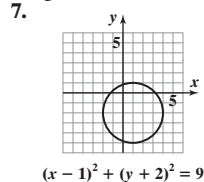
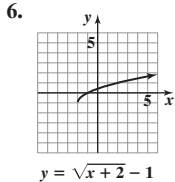
10.  $\frac{\sin(\alpha - \beta)}{\sin \alpha \cos \beta} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\sin \alpha \cos \beta} = \frac{\sin \alpha \cos \beta}{\sin \alpha \cos \beta} - \frac{\cos \alpha \sin \beta}{\sin \alpha \cos \beta} = 1 - \cot \alpha \tan \beta$

11.  $\sin t \cos t(\tan t + \cot t) = \sin t \cos t \left(\frac{\sin t}{\cos t} + \frac{\cos t}{\sin t}\right) = \sin^2 t + \cos^2 t = 1$     12.  $\frac{7\pi}{18}, \frac{11\pi}{18}, \frac{19\pi}{18}, \frac{23\pi}{18}, \frac{31\pi}{18}$ , and  $\frac{35\pi}{18}$

13.  $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$     14.  $0, \frac{\pi}{3}, \frac{5\pi}{3}$     15.  $0, \frac{2\pi}{3}, \frac{4\pi}{3}$     16. 2.5136, 3.7696    17. 1.2340,  $\frac{\pi}{2}, \frac{3\pi}{2}, 5.0522$     18. 1.2971, 2.6299, 4.4387, 5.7715

**Cumulative Review Exercises (Chapters P–5)**

1.  $-3, 1 + 2i$ , and  $1 - 2i$     2.  $x = \frac{\log 125}{\log 11} + 1$  or  $x \approx 3.01$     3.  $(-\infty, -4] \cup [2, \infty)$     4.  $\frac{\pi}{3}, \frac{5\pi}{3}$     5.  $\frac{\pi}{4}, 2.0344, \frac{5\pi}{4}, 5.1760$



12.  $2a + h + 3$     13.  $-\frac{\sqrt{2}}{2}$     14. Proofs may vary.

15.  $\frac{16\pi}{9}$  radians    16.  $t \approx 19.1$  yr

17.  $f^{-1}(x) = \frac{3x+1}{x-2}$     18.  $B = 67^\circ, b = 28.27, c = 30.71$

19. 106 mg    20.  $h \approx 15.9$  ft