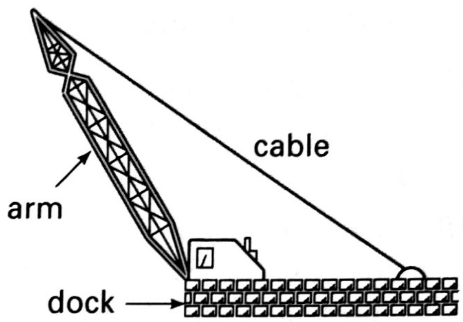
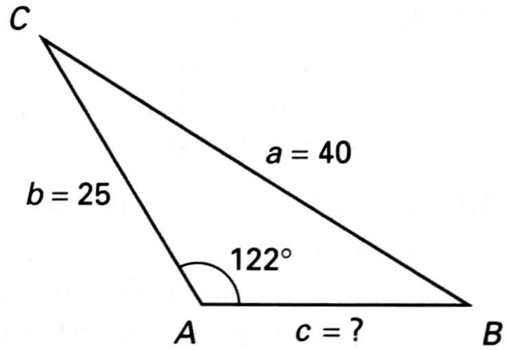
Complete solutions of ΔABC in parts (b) and (c) can be obtained by using $\angle C = 180° - (\angle A + \angle B)$ and $c = \dfrac{a \sin C}{\sin A}$. There will, of course, be two complete solutions in part (b).

In most applications involving triangle solving it is not necessary to find a complete solution. Example 5 is an exception because all parts of a triangle must be found in order to find the one part needed.

Example 5 A derrick at the end of a dock has an arm 25 m long that makes an angle of 122° with the floor of the dock. The arm is to be braced with a cable 40 m long from the end of the arm back to the dock. How far from the edge of the dock will the cable be fastened?



Solution Draw and label a sketch for this SSA problem. The goal is to find c.



Step 1 Find $\angle B$.

$$\sin B = \frac{b \sin A}{a} = \frac{25 \sin 122°}{40} = 0.5300$$

$$\angle B = 32.0°$$

Step 2 Find $\angle C$.

$$\angle C = 180° - (\angle A + \angle B) = 180° - (122° + 32°) = 26°$$

Step 3 Find c: $c = \dfrac{a \sin C}{\sin A} = \dfrac{40 \sin 26°}{\sin 122°} = 20.7$

Therefore, the cable is fastened 20.7 m from the edge of the dock. ■

EXERCISES 4-3

Solve ΔABC. If no solution exists, so state. If there are two solutions, find both.

A 1. $a = 21$; $c = 30$; $\angle B = 42°$

2. $a = 16$; $\angle B = 32°$; $\angle C = 50°$

3. $b = 14$; $\angle B = 25°$; $\angle C = 110°$

4. $a = 5$; $b = 8$; $c = 10$

5. $a = 2.3$; $b = 3.7$; $c = 5.0$

6. $b = 120$; $c = 145$; $\angle A = 100°$

7. $b = 20$; $c = 15$; $\angle B = 115°$

8. $a = 30$; $b = 20$; $\angle A = 130°$

9. $a = 12$; $b = 15$; $\angle A = 55°$

10. $a = 12$; $b = 7$; $\angle B = 35°$

11. $a = 5.2$; $b = 3.9$; $c = 6.5$

12. $b = 13.4$; $c = 6.7$; $\angle C = 30°$

13. $b = 15$; $c = 13$; $\angle C = 50°$

14. $b = 1.1$; $c = 1.8$; $\angle B = 40°$

15. If $\angle B$ is acute, what condition must b, c, and $\angle B$ satisfy in order that there be at least one triangle having these parts?

16. Draw diagrams similar to those in Figure 4-5 to illustrate the two SSA cases where $\angle A$ is obtuse.

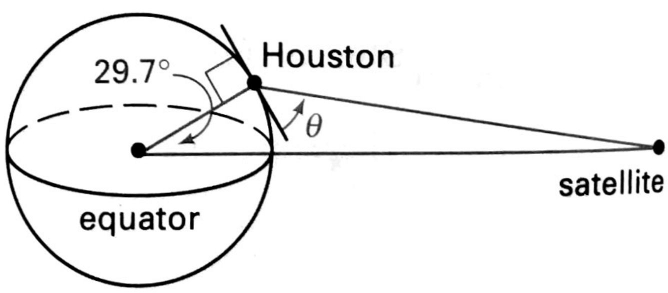
17. A monument consists of a 20 m flagpole standing on a mound in the shape of a cone with vertex angle 140°. How long a shadow does the pole cast on the cone when the angle of elevation of the sun is 58°?

18. Ann is flying a plane on a triangular course at 400 km/h. She flies due east for two hours and then turns left through a 15° angle measured clockwise from north. How long after turning will she be exactly northeast of where she started?

19. John is flying a plane from Upton to Vista, a distance of 500 km. Because of a storm between the two cities he has flown 17.5° off course for 300 km. How far is he now from Vista and through what angle should he turn to fly directly there?

20. Maria hears the 4:00 P.M. whistle of Wilson Industries at 10 seconds after the hour and she hears the 4:00 P.M. whistle of Ramos Manufacturing 8 seconds later. If the angle between Maria's lines of sight to the two plants is 56°, how far apart are they? (The speed of sound is 340 m/s.)

B 21. A communication satellite is in orbit 35,800 km above the equator. It completes one orbit every 24 hours, so that from Earth it appears to be stationary above a point on the equator. If this point has the same longitude as Houston, find the measure of θ, the satellite's angle of elevation from Houston. The latitude of Houston is 29.7° N; take the radius of Earth to be 6400 km.

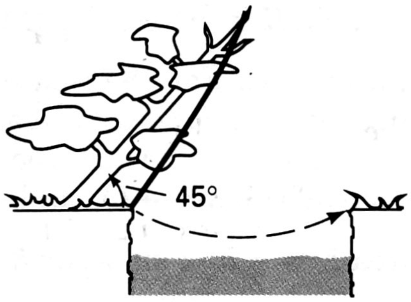


22. What is the greatest latitude from which a signal can travel to the satellite of Exercise 21 in a straight line?

23. A kite 2.5 m long is a quadrilateral having two sides each 1 m long and two sides each 2 m long. How wide is the kite? (That is, what is the length of the shorter diagonal?)

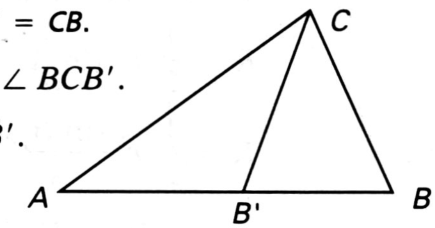
24. From the top of a tower 80 m above sea level, an observer sights a sailboat at an angle of depression of 9°. Turning in a different direction he sights another sailboat at an angle of depression of 12°. The angle between these lines of sight is 36°. How far apart are the boats?

25. To cross a river, an explorer swings on a 100-foot vine attached to a tree leaning over the river at a 45° angle, as shown at the right. The vine is attached to the tree 120 feet from its base. How wide is the river?



26. Show that in any triangle ABC, $c = a\cos B + b\cos A$. (Hint: Consider separately the cases where both $\angle A$ and $\angle B$ are acute and where one of them is obtuse. Draw figures.)

Exercises 27–30 refer to the figure at the right where $CB' = CB$.



Exercises 27-30

27. Given that $a = 8$, $b = 13$, and $\angle A = 30°$, find $\angle BCB'$.

28. Given the measures in Exercise 27, find $\angle ACB'$.

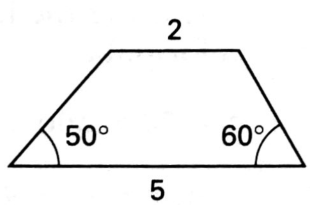
C 29. Show that $\dfrac{\text{area } \triangle ABC}{\text{area } \triangle AB'C} = \dfrac{\sin \angle ACB}{\sin \angle ACB'}$.

30. Show that the ratio in Exercise 29 equals $\dfrac{\sin (B + A)}{\sin (B - A)}$.

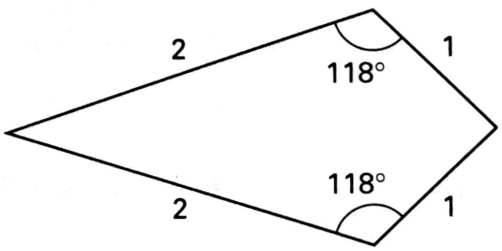
31. In $\triangle ABC$, $a = 3$, $b = 5$, and $\angle C = 120°$. Find the length of the median to the longest side.

32. In $\triangle ABC$, $c = 10$, and $\angle A = \angle B = 50°$. Find the length of the median to $\overline{AC}$.

33. Find the lengths of the diagonals of the trapezoid shown below.



34. Find the lengths of the diagonals of the quadrilateral shown below.



35. Given $\triangle ABC$ with $\angle C \neq 90°$, show that $\dfrac{\cos A}{\cos B} = \dfrac{b}{a}$ implies that $\triangle ABC$ is isosceles. (Hints: Explain why the given equation implies that $\angle A$ and $\angle B$ are acute. Then combine the equation with the law of sines and use a double-angle formula.)