

3.5 Problems in the book

EXERCISES 3-5

Find the exact value of each expression.

- A** 1. $\tan 75^\circ$ 2. $\tan 165^\circ$ 3. $\tan(-15^\circ)$ 4. $\tan 67.5^\circ$

Given that $\tan x = \frac{1}{3}$ and $\tan y = \frac{6}{5}$, find each of the following.

5. $\tan(x + y)$ 6. $\tan(x - y)$ 7. $\tan 2x$ 8. $\tan 4x$

Given that $\sin x = -\frac{4}{5}$ and $\pi < x < \frac{3\pi}{2}$, find each of the following.

9. $\tan \frac{x}{2}$ 10. $\tan 2x$ 11. $\tan 3x$ 12. $\tan\left(x + \frac{x}{2}\right)$

In Exercises 13–16, find the value of each of the expressions in Exercises 9–12, given that $\cos x = \frac{1}{3}$ and $0 < x < \frac{\pi}{2}$.

17. Use the fact that tangent is an odd function to prove that

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

18. Derive identity (22) from identity (20).

Prove each identity.

B 19. $\tan\left(x + \frac{\pi}{4}\right) = \frac{\cos x + \sin x}{\cos x - \sin x}$ 20. $\tan\left(x - \frac{\pi}{4}\right) = \frac{\sin x - \cos x}{\cos x + \sin x}$

21. $\frac{1}{1 - \tan x} - \frac{1}{1 + \tan x} = \tan 2x$ 22. $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

23. $\tan \frac{\theta}{2} + \cot \frac{\theta}{2} = 2 \csc \theta$ (Hint: $\cot \frac{\theta}{2} = \frac{1}{\tan \frac{\theta}{2}}$)

24. $\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$

C 25. $(\cos \theta - \sin \theta)(\sec \theta + \csc \theta) = 2 \cot 2\theta$

26. $\frac{2 \tan\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)} = \sin x$ (Hint: Use identity (8).)

27. $\tan\left(\frac{x}{2} + \frac{\pi}{4}\right) = \sec x + \tan x$

(Hint: Use $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and see Exercise 47 of Section 3-4.)

28. Find a formula for $\tan \frac{x}{2}$ in terms of $\tan x$ only by using the following method:

(a) Rewrite identity (22) in the form $\tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}$.

- (b) Regarding (a) as a quadratic equation in $\tan \frac{x}{2}$ solve the equation for $\tan \frac{x}{2}$.