$$\sin(a - b) = \sin a \cos b - \cos a \sin b \qquad (15)$$

Example 2 Find the exact value of $\sin \frac{13\pi}{12}$.

Solution Express $\frac{13\pi}{12}$ as a sum of numbers whose sines and cosines we know. One possible choice is

$$\frac{13\pi}{12} = \frac{5\pi}{6} + \frac{\pi}{4}.$$

(You may be able to discover this choice more easily by converting to degree measure: $195° = 150° + 45°$.) Using identity (14), we have:

$$\sin \frac{13\pi}{12} = \sin \frac{5\pi}{6} \cos \frac{\pi}{4} + \cos \frac{5\pi}{6} \sin \frac{\pi}{4}$$
$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right) \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4} \quad \blacksquare$$

Example 3 Find $\sin(a + b)$ if $\sin a = \frac{3}{5}$, $\sin b = \frac{5}{13}$, $0 < a < \frac{\pi}{2}$, and $\frac{\pi}{2} < b < \pi$.

Solution To use identity (14), we first use identity (7) to find $\cos a$ and $\cos b$.

$$\left(\frac{3}{5}\right)^2 + \cos^2 a = 1 \text{ so } \cos a = \frac{4}{5} \text{ since } 0 < a < \frac{\pi}{2}.$$

Similarly, $\cos b = -\frac{12}{13}$. Applying identity (14), we have

$$\sin(a + b) = \frac{3}{5}\left(-\frac{12}{13}\right) + \frac{4}{5}\left(\frac{5}{13}\right) = -\frac{16}{65} \quad \blacksquare$$

EXERCISES 3-3

Find the exact value of each trigonometric function.

A 1. $\sin 105°$
2. $\cos 165°$
3. $\sin 15°$
4. $\cos 345°$
5. $\cos 75°$
6. $\sin 165°$
7. $\cos(-75°)$
8. $\sin(-75°)$
9. $\cos\left(-\frac{\pi}{12}\right)$
10. $\sin \frac{5\pi}{12}$
11. $\cos \frac{13\pi}{12}$
12. $\cos\left(-\frac{5\pi}{12}\right)$

Find each of the following if $\sin r = \frac{4}{5}$, $\sin s = -\frac{12}{13}$, $0 < r < \frac{\pi}{2}$, and $\frac{3\pi}{2} < s < 2\pi$.

13. $\cos (r + s)$

14. $\sin (r + s)$

15. $\sin (r - s)$

16. $\cos (r - s)$

Find each of the following if $\cos s = -\frac{3}{5}$, $\cos t = -\frac{15}{17}$, $\frac{\pi}{2} < s < \pi$, and $\pi < t < \frac{3\pi}{2}$.

17. $\sin (s - t)$

18. $\cos (s - t)$

19. $\cos (s + t)$

20. $\sin (s + t)$

21. $\sin \left(\frac{\pi}{3} - s\right)$

22. $\cos \left(\frac{\pi}{3} + s\right)$

Prove each identity.

23. $\cos \left(\frac{3\pi}{2} - x\right) + \cos \left(\frac{3\pi}{2} + x\right) = 0$

24. $\sin (\pi + x) + \sin (\pi - x) = 0$

25. Use identity (10) on page 94 to prove that $\cos \left(\frac{\pi}{2} - x\right) = \sin x$.

26. Use identity (12) on page 95 to prove that $\sin \left(\frac{\pi}{2} - x\right) = \cos x$.

B 27. Illustrate the identity in Exercise 23 with a unit-circle diagram in which x is a number between 0 and $\frac{\pi}{2}$.

28. Illustrate the identity in Exercise 24 on a unit-circle diagram in which x is a number between $\frac{\pi}{2}$ and π.

29. Draw a unit-circle diagram illustrating angles of measure $45° - x$ and $45° + x$ for an arbitrarily chosen positive angle x of measure less than 45°. From the diagram, guess a relationship between $\sin (45° - x)$ and $\cos (45° + x)$. Prove this relationship using the formulas of this section.

30. Repeat Exercise 29 for angles of measure $135° - x$ and $135° + x$. Does the relationship that you guessed hold for an arbitrary angle x (for example, one whose measure is not necessarily less than 45°)?

31. Use identity (14) to prove that $\sin (a - b) = \sin a \cos b - \cos a \sin b$.

32. Prove that $\sec \left(\frac{\pi}{2} - x\right) = \csc x$.