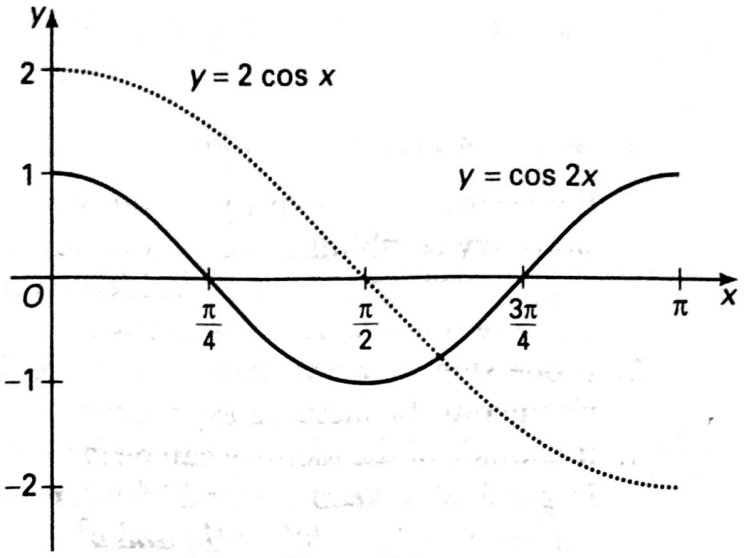

Figure 3-1

Figure 3-2 shows the graph of $y = \cos 2x$ together with that of $y = \cos^2 x - \sin^2 x$ (raised a bit for purposes of clarity). Notice that the graphs would appear to coincide for all points plotted. In Section 3-4, you will see a trigonometric proof that $\cos 2x = \cos^2 x - \sin^2 x$ is an identity.

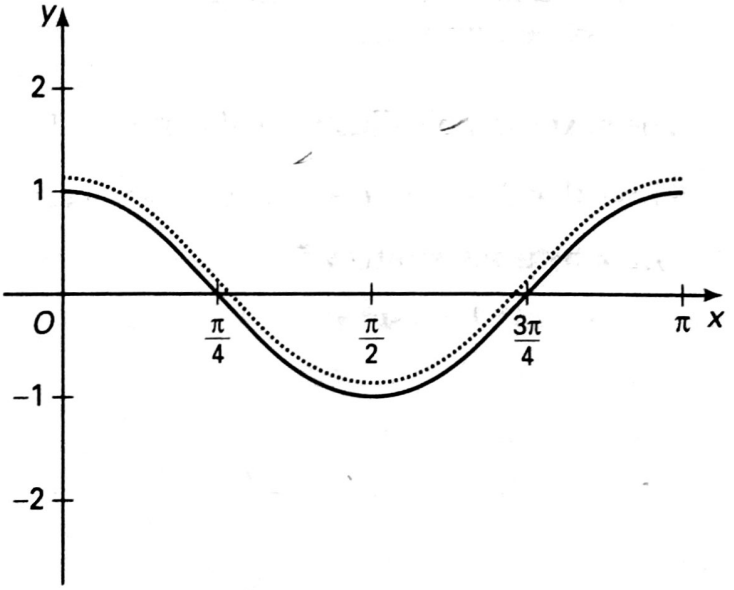

Figure 3-2

EXERCISES 3-2

Prove each identity.

A 1. $\dfrac{x^2 - 1}{x} - \dfrac{2x - 1}{2} = \dfrac{1}{2} - \dfrac{1}{x}$

2. $\dfrac{1}{x - 1} - \dfrac{1}{x + 1} = \dfrac{2}{x^2 - 1}$

3. $\dfrac{1}{1 + y} - \dfrac{y^2}{1 + y} = 1 - y$

4. $\dfrac{1 + t^2}{1 - t^2} - \dfrac{t}{1 - t} = \dfrac{1}{1 + t}$

5. $\frac{1}{x}(x-1)^2 = x - 2 + \frac{1}{x}$

6. $\frac{x^2 - 2x + 1}{x^2 - 1} = \frac{x-1}{x+1}$

7. $\frac{x^2 + y^2}{x^4 - y^4} \cdot \left(\frac{1}{y} - \frac{1}{x}\right) = \frac{1}{xy(x+y)}$

8. $x(x+1) - y(y+1) = (x + 1 + y)(x - y)$

In Exercises 9–16, is the given equation an identity? Give an algebraic or graphical justification of your answer.

9. $\sin 2x = \sin x + \sin x$

10. $\sin(x - \pi) = -\sin x$

11. $\frac{\csc x}{\sin x} + \frac{\sec x}{\cos x} = 1$

12. $\cos\left(\frac{\pi}{4} + x\right) = \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x$

13. $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

14. $-\sin(-x) = \sin x$

15. $(\sin x + \cos x)^2 = 1$

16. $\sin(x + \pi) = \sin(x - \pi)$

Prove each identity.

17. $(\sec^2 x)(1 - \cos^2 x) = \tan^2 x$

18. $\frac{\sin^2 x + \cos^2 x}{\tan x} = \cot x$

19. $(\cos x)(\sec x - \cos x) = \sin^2 x$

20. $(\sin x)(\csc x + \sin x \sec^2 x) = \sec^2 x$

21. $\cos\theta + \sin\theta \tan\theta = \sec\theta$

22. $(\sec\theta)(\csc\theta - \cot\theta\cos\theta) = \tan\theta$

23. $\frac{1}{1 + \tan^2 x} + \frac{1}{1 + \cot^2 x} = 1$

24. $\frac{\sec^2 x - 1}{\csc^2 x - 1} = \tan^4 x$

25. $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

26. $\csc t - \sin t = \cos t \cot t$

27. $(\tan x + \sin x)(1 - \cos x) = \sin^2 x \tan x$

28. $(\cot x - \cos x)(\csc x + 1) = \cos x \cot^2 x$

29. $(1 - \cos\theta)(\csc\theta + \cot\theta) = \sin\theta$

30. $(\sec\theta + 1)(\csc\theta - \cot\theta) = \tan\theta$

31. $\cos\theta + \frac{\sin\theta}{\cot\theta} = \frac{\cot\theta + \tan\theta}{\csc\theta}$

32. $\frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} = 2\tan x$

33. $\frac{1}{\sec t - 1} + \frac{1}{\sec t + 1} = 2\cot t \csc t$

34. $\frac{(\sin x + \cos x)^2}{\sin x} = \csc x + 2\cos x$

35. $(\cos x)(1 + \tan x)^2 = \sec x + 2\sin x$

36. $\tan\theta + \cot\theta = \sec\theta \csc\theta$

37. $\frac{\sin A}{\sec A - 1} - \frac{\sin A}{\sec A + 1} = 2\cos A \cot A$

B 38. $\dfrac{1 + \sec x}{\tan x} + \dfrac{\tan x}{1 + \sec x} = 2 \csc x$

39. $\dfrac{\cos x}{1 + \sin x} + \dfrac{1 + \sin x}{\cos x} = 2 \sec x$

40. $\dfrac{\sec \theta}{\sec \theta - 1} - \dfrac{\sec \theta + 1}{\tan^2 \theta} = 1$

41. $\dfrac{1}{1 - \sin x} = \dfrac{\cot x}{\cot x - \cos x}$

42. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$

43. $\dfrac{\csc x - 1}{\cot x} + \dfrac{\cot x}{\csc x + 1} = \dfrac{2 \cos x}{1 + \sin x}$

44. $\dfrac{\sec x - 1}{\tan x} + \dfrac{\tan x}{\sec x + 1} = \dfrac{2 \sin x}{1 + \cos x}$

45. $\dfrac{1 - \cos x}{\sin x} = \dfrac{\sin x}{1 + \cos x}$ $\left(\text{Hint: Show that } \dfrac{1 - \cos x}{\sin x} - \dfrac{\sin x}{1 + \cos x} = 0.\right)$

46. $\dfrac{\tan t}{1 + \sec t} = \dfrac{\sec t - 1}{\tan t}$ (Hint: See the hint for Exercise 45.)

47. $\dfrac{\sec x - \tan x}{\cos x} - \dfrac{\cos x}{\sec x + \tan x} = \dfrac{\sin^2 x}{1 + \sin x}$

48. $(\cot \theta + \tan \theta)^2 = \csc^2 \theta \sec^2 \theta$

49. $\dfrac{\cos \theta}{\sec \theta - \tan \theta} + \dfrac{\sin \theta \cos \theta}{\sec \theta + \tan \theta} = \cos^2 \theta + 2 \sin \theta$

50. $(\sec \theta - \cos \theta)^2 = \tan^2 \theta - \sin^2 \theta$

51. $\dfrac{\cos \theta}{1 - \tan \theta} + \dfrac{\sin \theta}{1 - \cot \theta} = \sin \theta + \cos \theta$

52. $\dfrac{\tan^2 x}{1 - \cos x} = \sec x + \sec^2 x$

53. $\dfrac{1 + \sin x}{1 - \sin x} = (\sec x + \tan x)^2$

C 54. $\dfrac{\sec x + \tan x}{\sec x - \tan x} = \dfrac{1 + 2 \sin x + \sin^2 x}{\cos^2 x}$

55. $(\sin x + \csc x)^2 + (\cos x - \sec x)^2 = \sec^2 x \csc^2 x + 1$

56. $(1 + \tan t)^2 + (1 + \cot t)^2 = (\sec t + \csc t)^2$

57. Use the given unit-circle diagram to give a geometric proof of the identity in Exercise 56 for an acute angle t. (Hint: Show that $AN = \tan t$, $AO = \sec t$, $CM = \cot t$, and $CO = \csc t$.)