

$$\begin{aligned}
 1 - \sin x \cos x \tan x &= 1 - \sin x \cos x \frac{\sin x}{\cos x} \\
 &= 1 - \sin^2 x \\
 &= \cos^2 x \quad \text{by identity (7)} \quad \blacksquare
 \end{aligned}$$

Example 3 Express $\frac{\tan \theta}{1 + \sec \theta} + \frac{1}{\tan \theta}$ in terms of a single trigonometric function.

Solution Using $(1 + \sec \theta)(\tan \theta)$ as a common denominator and adding, we have:

$$\begin{aligned}
 \frac{\tan \theta}{1 + \sec \theta} + \frac{1}{\tan \theta} &= \frac{\tan^2 \theta + 1 + \sec \theta}{(\tan \theta)(1 + \sec \theta)} \\
 &= \frac{\sec^2 \theta + \sec \theta}{(\tan \theta)(1 + \sec \theta)} \quad \text{by identity (8)} \\
 &= \frac{(\sec \theta)(\sec \theta + 1)}{(\tan \theta)(1 + \sec \theta)} \quad \text{by factoring the numerator} \\
 &= \frac{\sec \theta}{\tan \theta} \\
 &= \frac{1}{\frac{\cos \theta}{\sin \theta}} \quad \text{by identities (3) and (1)} \\
 &= \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta}, \text{ or } \csc \theta \quad \blacksquare
 \end{aligned}$$

EXERCISES 3-1

Express the following in terms of $\sin x$. Give your answers in simplest form.

- A 1. $\tan^2 x$ 2. $\cot^2 x$ 3. $\sec x \tan x$ 4. $\frac{\cot x}{\sec x}$
5. $\frac{\csc x}{1 + \cot^2 x}$ 6. $\frac{\sec^2 x - \tan^2 x}{\csc x}$ 7. $(\cos x)(\tan x + \cot x)$ 8. $\frac{1 + \cot x}{\sin x + \cos x}$

Express the following in terms of $\cos x$. Give your answers in simplest form.

9. $1 + \tan^2 x$ 10. $\sec^2 x - 1$
11. $(\sec x)(\cos x + \sin^2 x \sec x)$ 12. $(1 + \cot x)(1 - \cot x)$
13. $\frac{\sin^2 x}{1 + \cos x}$ 14. $(\csc^2 x)(1 - \cos x)$
15. $(\csc x)(\csc x + \cot x)$ 16. $\frac{\sec x + 1}{\sin^2 x \sec x}$

Express each of the following in terms of a single trigonometric function.

17. $\frac{\csc x - \sin x}{\cos x}$

18. $\sec x \csc x - \tan x$

19. $\frac{\sin t \cos t}{1 - \cos^2 t}$

20. $\frac{1 + \tan^2 t}{\csc t \sec t}$

21. $\frac{(\tan \theta)(1 + \cot^2 \theta)}{\cot \theta}$

22. $\frac{\sin x + \tan x}{1 + \sec x}$

23. $(\sin x)(\cos x + \sin x \tan x)$

24. $(\csc x)(\sec x - \cos x)$

25. $(\csc x)(1 - \cos x)(1 + \cos x)$

26. $\frac{(\sec x - \tan x)(\sec x + \tan x)}{\cos x}$

B 27. $\frac{\cos \theta}{1 + \sin \theta} + \tan \theta$

28. $\frac{\sec \theta + \csc \theta}{1 + \tan \theta}$

29. $\frac{\sec y}{\sin y} - \frac{\sec y}{\csc y}$

30. $\frac{\sin x + \tan x}{(\tan x)(\csc x + \cot x)}$

31. $1 - \frac{\cos^2 x}{1 + \sin x}$

32. $\frac{(\tan \theta + 1)^2}{\sec \theta} - \sec \theta$

33. $\frac{\sec x - \cos x}{\sin x}$

34. $\frac{\cot^2 x}{\csc x + 1} + 1$

35. $\frac{\sec \theta - \cos \theta}{\sin^2 \theta \sec^2 \theta}$

36. $\frac{\tan \theta + \cot \theta}{\sec^2 \theta}$

37. $\frac{1 - \tan^4 x}{\cos^2 x - \sin^2 x}$

38. $\frac{1}{2} \left(\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x} \right)$

Use identity (7) and any of identities (1)–(6) on page 86 to derive the following.

39. $1 + \tan^2 x = \sec^2 x$

40. $1 + \cot^2 x = \csc^2 x$

Show that each of the following expressions equals either 0 or 1.

C 41. $\frac{1 + \sec x}{\sec x - 1} + \frac{1 + \cos x}{\cos x - 1}$

42. $\frac{(\sec^2 x)(1 + \csc x) - (\tan x)(\sec x + \tan x)}{(\csc x)(1 + \sin x)}$

43. $\frac{\sec x}{1 - \cos x} - \frac{\sec x + 1}{\sin^2 x}$

44. $\frac{\tan x}{\tan x + \sin x} - \frac{1 - \cos x}{\sin^2 x}$

45. $\frac{\csc \theta}{1 + \sec \theta} - \frac{\cot \theta}{1 + \cos \theta}$

46. $\frac{\sin \theta + \cos \theta - 1}{\sin \theta - \cos \theta + 1} - \frac{\cos \theta}{\sin \theta + 1}$