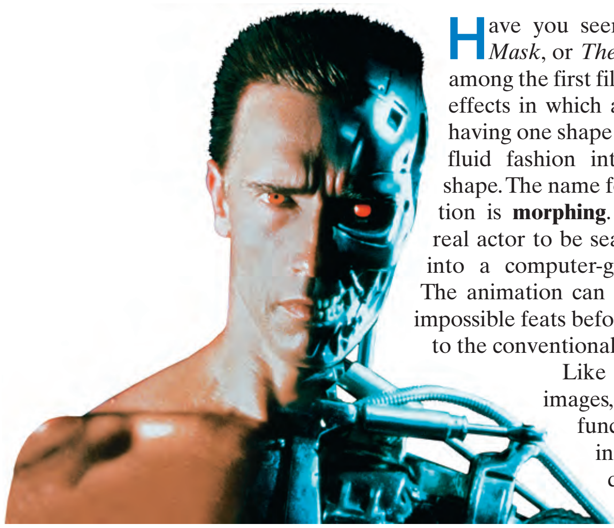
1.6 Transformations of Functions

Objectives

1. Recognize graphs of common functions.
2. Use vertical shifts to graph functions.
3. Use horizontal shifts to graph functions.
4. Use reflections to graph functions.
5. Use vertical stretching and shrinking to graph functions.
6. Use horizontal stretching and shrinking to graph functions.
7. Graph functions involving a sequence of transformations.

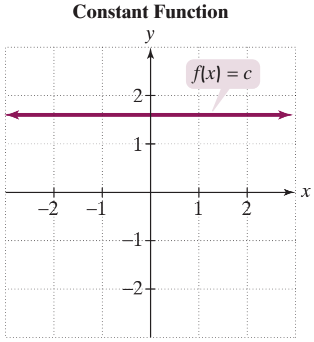


Have you seen *Terminator 2, The Mask*, or *The Matrix?* These were among the first films to use spectacular effects in which a character or object having one shape was transformed in a fluid fashion into a quite different shape. The name for such a transformation is **morphing**. The effect allows a real actor to be seamlessly transformed into a computer-generated animation. The animation can be made to perform impossible feats before it is morphed back to the conventionally filmed image.

Like transformed movie images, the graph of one function can be turned into the graph of a different function. To do this, we need to rely on a function's equation. Knowing that a graph is a transformation of a familiar graph makes graphing easier.

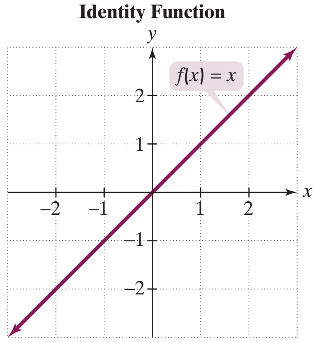
Table 1.3 Algebra's Common Graphs

Constant Function



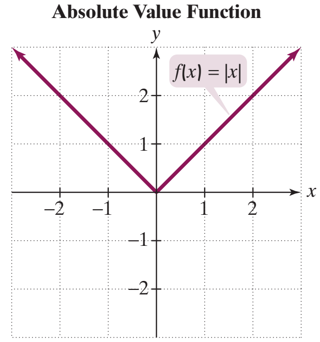
- Domain: $(-\infty, \infty)$
- Range: the single number c
- Constant on $(-\infty, \infty)$
- Even function

Identity Function



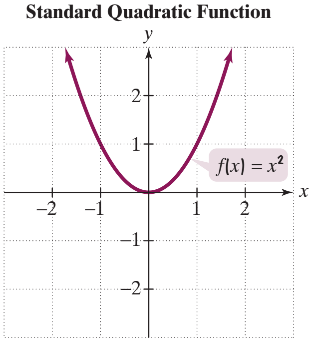
- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

Absolute Value Function



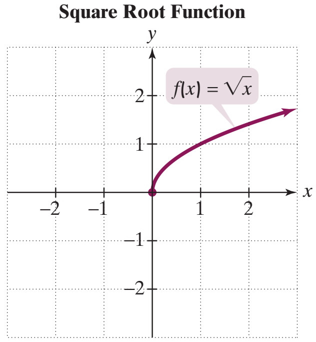
- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
- Even function

Standard Quadratic Function



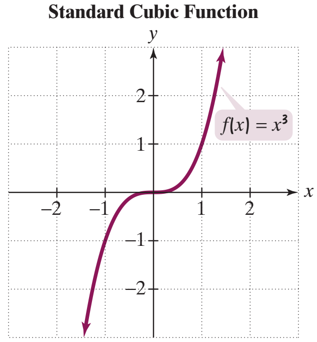
- Domain: $(-\infty, \infty)$
- Range: $[0, \infty)$
- Decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$
- Even function

Square Root Function



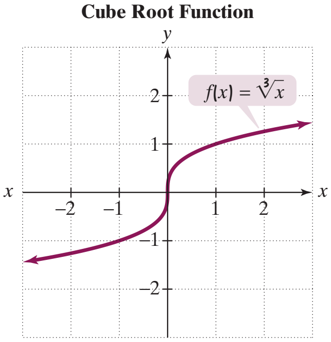
- Domain: $[0, \infty)$
- Range: $[0, \infty)$
- Increasing on $(0, \infty)$
- Neither even nor odd

Standard Cubic Function



- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

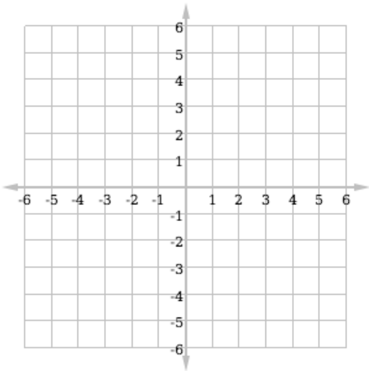
Cube Root Function



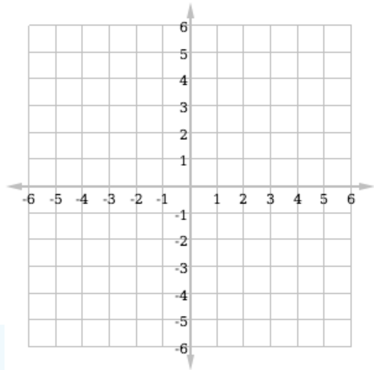
- Domain: $(-\infty, \infty)$
- Range: $(-\infty, \infty)$
- Increasing on $(-\infty, \infty)$
- Odd function

EXAMPLE 1 Vertical Shift Downward

Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = |x| - 4$.



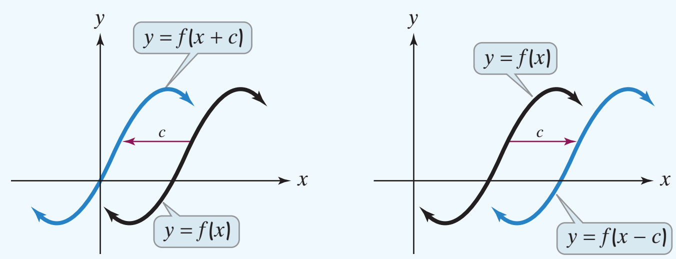
Check Point 1 Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = |x| + 3$.



Horizontal Shifts

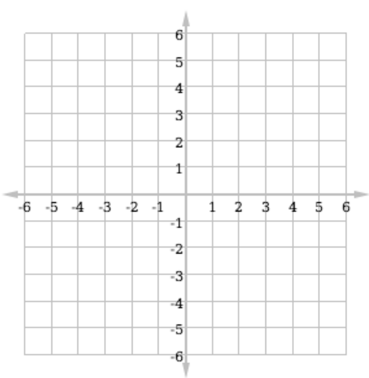
Let f be a function and c a positive real number.

- The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted to the left c units.
- The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted to the right c units.



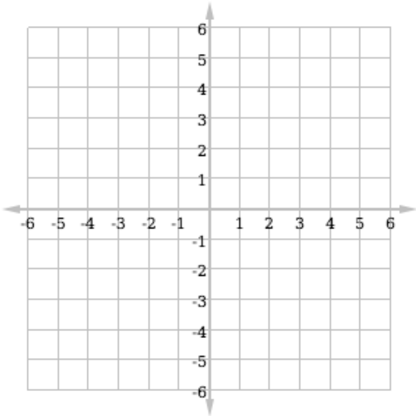
EXAMPLE 2 Horizontal Shift to the Left

Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $g(x) = \sqrt{x + 5}$.

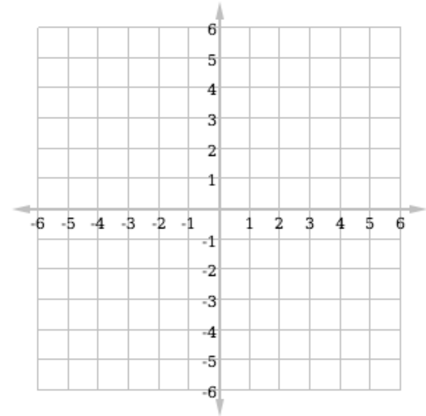


EXAMPLE 3 Combining Horizontal and Vertical Shifts

Use the graph of $f(x) = x^2$ to obtain the graph of $h(x) = (x + 1)^2 - 3$.



Check Point 3 Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $h(x) = \sqrt{x - 1} - 2$.

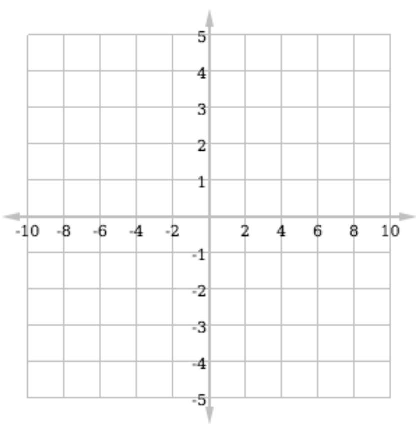


Reflection about the *x*-Axis

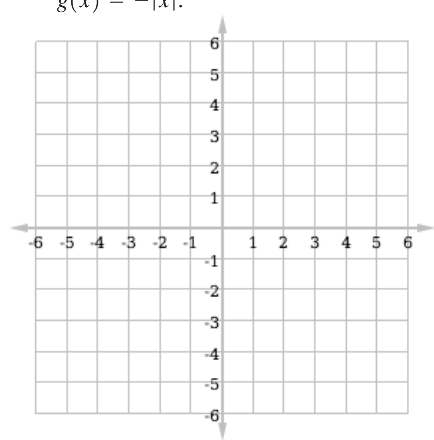
The graph of $y = -f(x)$ is the graph of $y = f(x)$ reflected about the x-axis.

EXAMPLE 4 Reflection about the *x*-Axis

Use the graph of $f(x) = \sqrt[3]{x}$ to obtain the graph of $g(x) = -\sqrt[3]{x}$.



Check Point 4 Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = -|x|$.

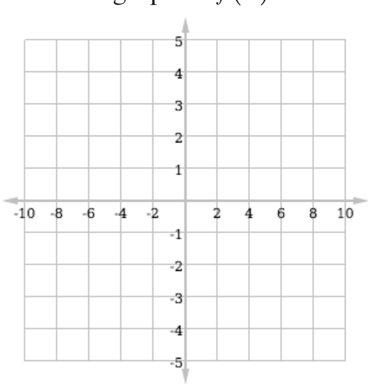


Reflection about the *y*-Axis

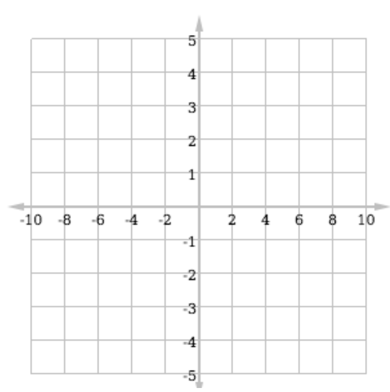
The graph of $y = f(-x)$ is the graph of $y = f(x)$ reflected about the y-axis.

EXAMPLE 5 Reflection about the *y*-Axis

Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $h(x) = \sqrt{-x}$.



Check Point 5 Use the graph of $f(x) = \sqrt[3]{x}$ to obtain the graph of $h(x) = \sqrt[3]{-x}$.



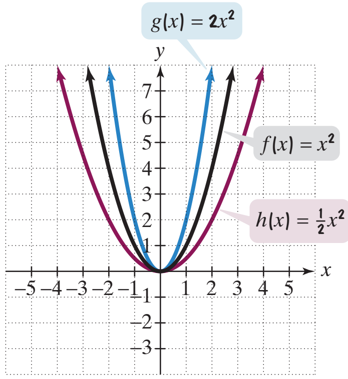
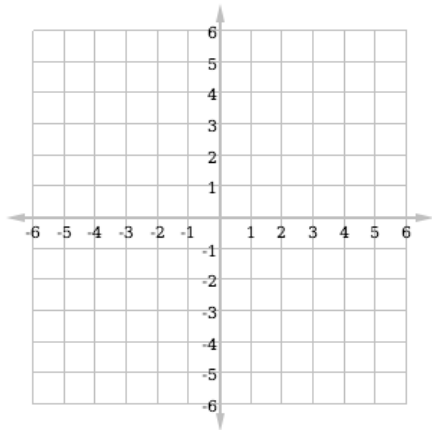


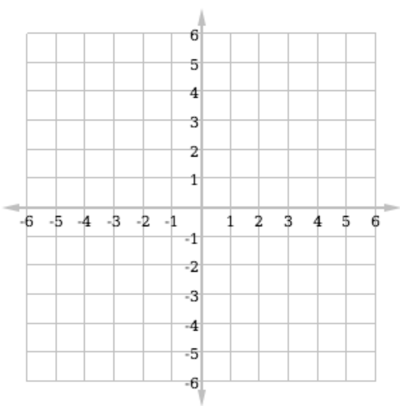
Figure 1.58 Vertically stretching and shrinking $f(x) = x^2$

EXAMPLE 6 Vertically Shrinking a Graph

Use the graph of $f(x) = x^3$ to obtain the graph of $h(x) = \frac{1}{2}x^3$.



Check Point 6 Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = 2|x|$.



EXAMPLE 7 Horizontally Stretching and Shrinking a Graph

Use the graph of $y = f(x)$ in **Figure 1.59** to obtain each of the following graphs:

a. $g(x) = f(2x)$ **b.** $h(x) = f\left(\frac{1}{2}x\right)$.

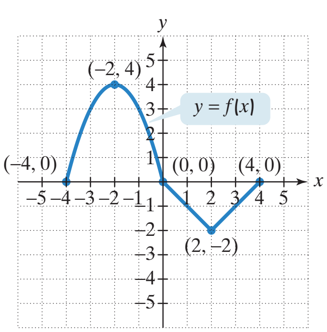
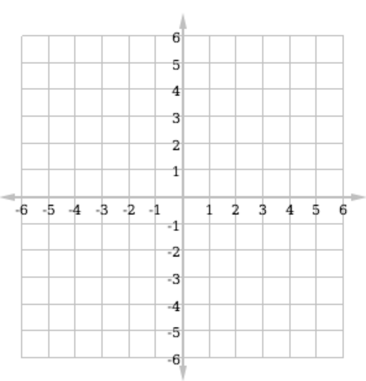
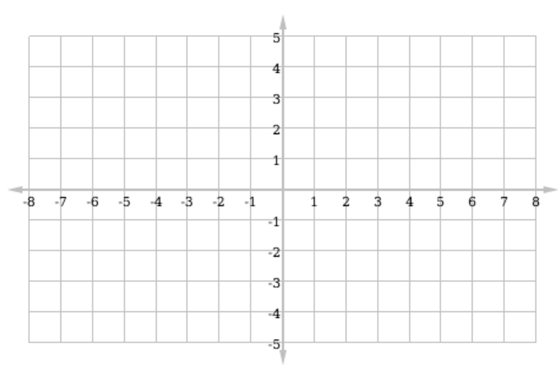


Figure 1.59





Check Point 7 Use the graph of $y = f(x)$ in **Figure 1.60** to obtain each of the following graphs:

a. $g(x) = f(2x)$ **b.** $h(x) = f\left(\frac{1}{2}x\right)$.

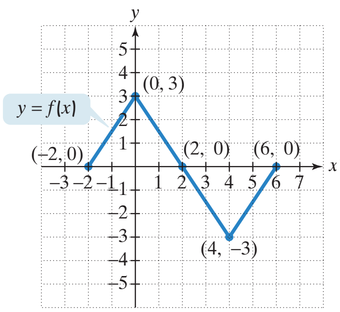
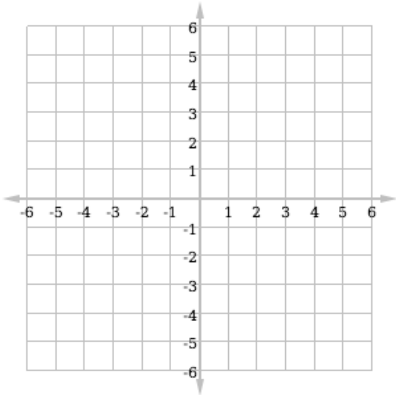
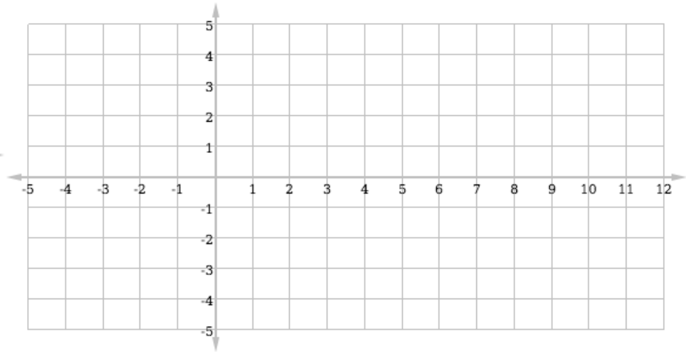


Figure 1.60





EXAMPLE 8 Graphing Using a Sequence of Transformations

Use the graph of $y = f(x)$ given in **Figure 1.59** of Example 7 on page 212, and repeated below, to graph $y = -\frac{1}{2}f(x - 1) + 3$.

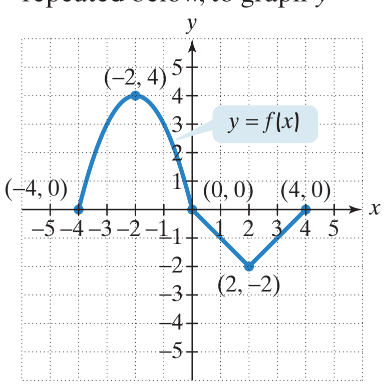
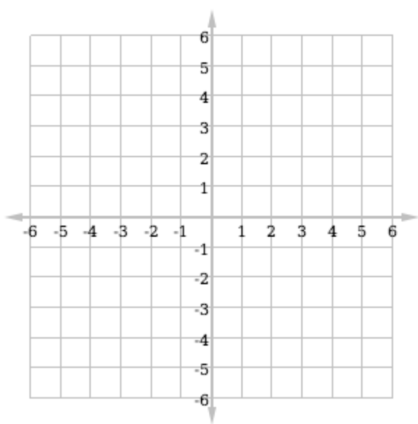
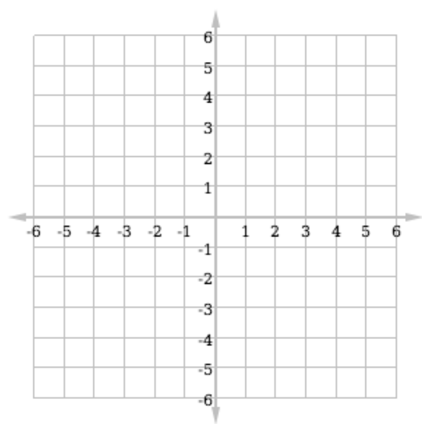
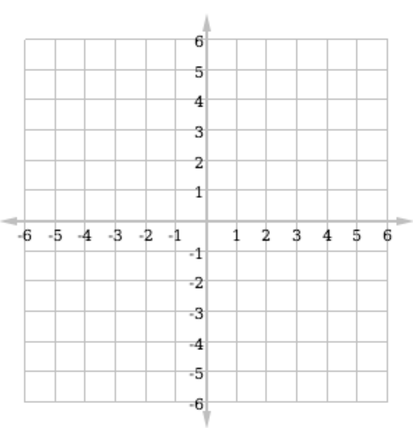
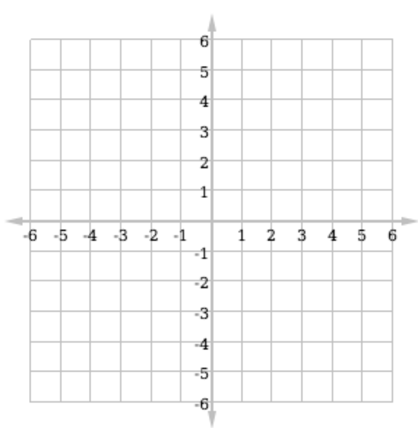


Figure 1.59









EXAMPLE 9 Graphing Using a Sequence of Transformations

Use the graph of $f(x) = x^2$ to graph $g(x) = 2(x + 3)^2 - 1$.