1.2 Basics of Functions and Their Graphs

Objectives

1. Find the domain and range of a relation.
2. Determine whether a relation is a function.
3. Determine whether an equation represents a function.
4. Evaluate a function.
5. Graph functions by plotting points.
6. Use the vertical line test to identify functions.
7. Obtain information about a function from its graph.
8. Identify the domain and range of a function from its graph.
9. Identify intercepts from a function’s graph.

Definition of a Relation

A relation is any set of ordered pairs. The set of all first components of the ordered pairs is called the domain of the relation and the set of all second components is called the range of the relation.

Definition of a Function

A function is a correspondence from a first set, called the domain, to a second set, called the range, such that each element in the domain corresponds to exactly one element in the range.

Example 2  Determining Whether a Relation Is a Function

Determine whether each relation is a function:

a. \(\{(1, 6), (2, 6), (3, 8), (4, 9)\}\)
   b. \(\{(6, 1), (6, 2), (8, 3), (9, 4)\}\).

Example 3  Determining Whether an Equation Represents a Function

Determine whether each equation defines \(y\) as a function of \(x\):

a. \(x^2 + y = 4\)
   b. \(x^2 + y^2 = 4\).

Example 4  Evaluating a Function

If \(f(x) = x^2 + 3x + 5\), evaluate each of the following:

a. \(f(2)\)
   b. \(f(x + 3)\)
   c. \(f(-x)\).

The Vertical Line Test for Functions

If any vertical line intersects a graph in more than one point, the graph does not define \(y\) as a function of \(x\).

Example 6  Using the Vertical Line Test

Use the vertical line test to identify graphs in which \(y\) is a function of \(x\).

a. 
   b. 
   c. 
   d.
EXAMPLE 7 Analyzing the Graph of a Function

The human immunodeficiency virus, or HIV, infects and kills helper T cells. Because T cells stimulate the immune system to produce antibodies, their destruction disables the body's defenses against other pathogens. By counting the number of T cells that remain active in the body, the progression of HIV can be monitored. The fewer helper T cells, the more advanced the disease. Figure 1.20 shows a graph that is used to monitor the average progression of the disease. The average number of T cells, \( f(x) \), is a function of time after infection, \( x \).

a. Explain why \( f \) represents the graph of a function.
b. Use the graph to find \( f(8) \).
c. For what value of \( x \) is \( f(x) = 350 \)?
d. Describe the general trend shown by the graph.

Figure 1.20

EXAMPLE 8 Identifying the Domain and Range of a Function from Its Graph

Use the graph of each function to identify its domain and its range.

a. \( y = f(x) \)
b. \( y = f(x) \)
c. \( y = f(x) \)
d. \( y = f(x) \)
e. \( y = f(x) \)